Receding Horizon Optimization Approach to PID Controller Parameters Auto-tuning[1)]

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Abstract A novel supervised receding horizon optimal scheme is presented for discrete time systems in the process control. In the employing level, PID controller is used, while the receding horizon approach is applied to the optimized level. The considered problem is to optimize the employing level PID controller parameters through minimizing a generalized predictive control criterion. Compared with a fixed parameters PID controller, the proposed algorithm provides well performance over a range of operating condition.

Key words Receding horizon optimization, auto-tuning, PID controller, generalized predictive control

1 Introduction

With the development of complex and nonlinear systems, a fixed and linear controller could not meet the requirement of practice. Especially, system performance at different operating conditions may vary, and the use of constant-gain controllers is not always sufficient because many processes might be drifted all over the operating condition[1]. Auto-tuning controller has started appearing in the industrial scene, especially in process control areas mainly because of their capability of on-line tuning of the controller parameters[2~3]. However, most papers on a single loop control adopt PID controllers with Ziegler-Nichols tuning rules as a benchmark. This is very unsatisfactory situation because the Ziegler-Nichols rules are known to give a poor result in many cases[4]. Hence, a more complex control structure or new ways of tuning PID controllers beyond the classical methods should be explored.

Model predictive control (MPC) refers to a family of control algorithms that employ an explicit model to predict the future behavior of the process over an extended prediction horizon. The core of all MPC algorithms is the receding horizon strategy, also known as the open-loop optimal feedback approach. An identified process model predicts the future response, and then the control action is determined so as to obtain the desired performance over a finite time horizon[5].

To improve system performance of over a wide range of operating conditions, a supervised auto tuning strategy is presented. In the employing level, a incremental PID controller is often adopted in practice. Because PID controller with fixed parameters will deteriorate the performance, as the operating level moves away from the original design. Therefore, the optimized level aims at tuning the parameters to span the range of large-scale operating condition through minimizing the generalized predictive control criterion.

2 The employing level PID controller

A classical discrete incremental PID controller can be written as a time series, *i.e.*,

$$\triangle u(k) = w_0 e(k) + w_1 e(k-1) + w_2 e(k-2) \tag{1}$$

where $w_0 = K_p + K_i + K_d,\ w_1 = -K_p - 2K_d,\ w_3 = K_d$ and K_p, K_d, K_d are PID parameters.

In the classical controller, parameters w_0, w_1 and w_2 are fixed over the whole operating condition. In order to fit the large-scale operating condition, the parameters in the vector $[w]$ can be time varying through the control horizon N.

$$w_{k+j} \neq w_k,\ j = 1, 2, \cdots, N \tag{2}$$

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Therefore, this time series is the output of a linear time varying adaptive FIR differential filter

$$\triangle u(k) = \sum_{i=0}^{2} w_i(k)e(k-i) = \boldsymbol{W}^{\mathrm{T}}(k)\boldsymbol{e}(k) \tag{3}$$

where $\boldsymbol{e}(k)$ represents the error input to the FIR filter and the time series parameters $w_i(k)$ are assumed to be the weights of the adaptive FIR filter and contained in the vector $\boldsymbol{W}(k)$. Then the output of adaptive filter $\triangle u(k)$ is the convolution of the weight vector $\boldsymbol{W}(k)$ and the input vector $\boldsymbol{e}(k)$, defined as

$$\boldsymbol{W}^{\mathrm{T}}(k) = [w_0(k) \quad w_1(k) \quad w_2(k)] \quad \text{and} \quad \boldsymbol{e}(k) = [e(k) \quad e(k-1) \quad e(k-2)]^{\mathrm{T}}$$

The future control action can be expressed as

$$\triangle u(k_p) = \sum_{i=0}^{2} w_i(k_p)e(k_p-i) = \boldsymbol{W}^{\mathrm{T}}(k_p)\boldsymbol{e}(k_p) \tag{4}$$

For convenience, the time index $k_p = (k+j-1)$ is introduced as the initial point for the receding horizon prediction. The error input to the adaptive FIR filter is defined as

$$e(k_p) = r(k_p) - y(k_p) \tag{5}$$

3 The optimization level with multi-step ahead predictive control

The time-varying dynamics of the process control model is represented by a controlled auto-regressive integrated moving average (CARIMA) model:

$$A(q^{-1})y(k) = B(q^{-1})u(k-1) + C(q^{-1})v(k)/\triangle \tag{6}$$

where A, B and C are polynomials in the backward shift operator q^{-1} of degrees n_a, n_b, n_c, respectively. $\nu(k)$ is an external discrete white noise source with zero mean and a variance σ^2 driving coloring filter.

The j-step output for the process is derived by using two Diophantine equations:

$$y(k+j) = G_j\triangle u(k+j-1) + (H_j/C)\triangle u(k-1) + (F_j/C)y(k) + E_j\nu(k+j)$$

where $\boldsymbol{E}, \boldsymbol{F}, \boldsymbol{G}$ and $\boldsymbol{H}$ are polynomials in the backward shift operator q^{-1} (See [6]). Taking the expectation, the j-step for the process output becomes

$$\hat{y}(k+j) = G_j\triangle u(k+j-1) + H_j\triangle\bar{u}(k-1) + F_j\bar{y}(k) \tag{7}$$

where $\triangle\bar{u}(k-1) = \triangle u(k-1)/C$ and $\bar{y} = y(k)/C$, C is assumed to be minimum phase.

Substituting (4) into (7) yields

$$\hat{y}(k+j) = G_j\left(\sum_{i=0}^{2} w_i(k_p)\hat{e}(k_p-i|k)\right) + H_j\triangle\bar{u}(k-1) + F_j\bar{y}(k) \tag{8}$$

Consider the GPC performance criterion to be minimized

$$J = \sum_{j=1}^{N}(r(k+j) - \hat{y}(k+j))^2 + \sum_{j=1}^{N}\lambda_j(\triangle u(k+j-1))^2 \tag{9}$$

where λ that is a scalar quantity or a polynomial represents weighting elements acting on feedback control signal $u(k)$. The problem to be solved is achieved by minimizing the above criterion with specified structure controller.

From (4) and (8), a GPC parametric optimization problem is rewritten as

$$J = \sum_{j=1}^{N}\left(L_j - G_j\sum_{i=0}^{2} w_i(k_p)\hat{e}(k_p-i|k)\right)^2 + \sum_{j=1}^{N}\lambda_j\left(\sum_{i=0}^{2} w_i(k_p)\hat{e}(k_p-i|k)\right)^2 \tag{10}$$

Denote

$$\begin{cases} L_j = r(k+j) - H_j\triangle\bar{u}(k-1) - F_j\bar{y}(k) \\ \sum_{i=0}^{N} w_i(k+j-1)\boldsymbol{e}(k-i+j-1) = \boldsymbol{W}(k+j-1)\boldsymbol{e}(k+j-1) \end{cases}$$

Then

$$J = \sum_{j=1}^{N}(L_j - G_j\boldsymbol{W}(k_p)\boldsymbol{e}(k_p))^2 + \sum_{j=1}^{N}\lambda_j(\boldsymbol{W}(k_p)\boldsymbol{e}(k_p))^2 \tag{11}$$

Re-writing (11) in matrix form and discarding some subscripts for convenience yield

$$\boldsymbol{J} = [\boldsymbol{L} - \boldsymbol{G}\boldsymbol{W}\boldsymbol{e}]^{\mathrm{T}}[\boldsymbol{L} - \boldsymbol{G}\boldsymbol{W}\boldsymbol{e}] + \lambda[\boldsymbol{W}\boldsymbol{e}]^{\mathrm{T}}[\boldsymbol{W}\boldsymbol{e}] \tag{12}$$

Differentiating (12) with respect to vector $\boldsymbol{W}$, leads to

$$\frac{\partial \boldsymbol{J}}{\partial \boldsymbol{W}} = 2(\boldsymbol{I}_N \otimes \boldsymbol{e}^{\mathrm{T}})\frac{\partial \boldsymbol{W}^{\mathrm{T}}}{\partial \boldsymbol{W}}[\boldsymbol{I}_{3N} \otimes (\boldsymbol{G}^{\mathrm{T}}\boldsymbol{G} + \lambda I)\boldsymbol{W}\boldsymbol{e}] - 2(\boldsymbol{I}_N \otimes \boldsymbol{L}^{\mathrm{T}}\boldsymbol{G})\frac{\partial \boldsymbol{W}^{\mathrm{T}}}{\partial \boldsymbol{W}}(\boldsymbol{I}_{3N} \otimes \boldsymbol{e}) \tag{13}$$

where $\otimes$ is Kronecker products. Setting $\partial \boldsymbol{J}/\partial \boldsymbol{W} = 0$, the optimal parameters of the controller are given

$$\boldsymbol{W} = [\boldsymbol{e}^{\mathrm{T}}(\boldsymbol{G}\boldsymbol{G} + \lambda I)\boldsymbol{e}]^{-1}(\boldsymbol{e}^{\mathrm{T}}\boldsymbol{G}^{\mathrm{T}}\boldsymbol{L}) \tag{14}$$

At time k, only the vector of controller parameters $W(k)$ is calculated and applied to the process. Then, at time $k+1$, a new control action $\triangle u(k+1)$ is counted with receding horizon optimization strategy.

4 On-line identification based on receding horizon windows data

The problem of nonlinear dynamic system identification comes back to the problem of a static (receding moving) mapping identification. Within every receding horizon window (RHW) the system can be described as local linear models.

Therefore, (6) is described as a locally valid linear state space model, and may be written as follows.

$$\boldsymbol{y}(k) = \boldsymbol{\theta}^{\mathrm{T}}\boldsymbol{\varphi}(k) + \boldsymbol{\varepsilon}(k) \tag{15}$$

where

$$\boldsymbol{\theta}^{\mathrm{T}} = [a_1, \cdots, a_{n_b}, b_1, \cdots, b_{n_b}, c_1, \cdots, c_{n_c}]^{\mathrm{T}},\ \varepsilon(k) = y(k) - \boldsymbol{\theta}^{\mathrm{T}}\boldsymbol{\varphi}(k)\ (\varepsilon \in R^{n_y})$$
$$\boldsymbol{\varphi}(k) = [-y(k-1), \cdots, -y(k-n_a), u(k), \cdots, u(k-n_b+1), e(k-1), \cdots, e(k-n_c)]^{\mathrm{T}}$$

Iterative prediction error (IPE) method, which allows the direct estimation of model parameters from the on-line input-output data sequences $\{u(k), y(k)\}$, is proposed based on receding horizon windows data. Given N distinct data samples $\{\boldsymbol{y}(i), \boldsymbol{\varphi}(i)\}, i = 1, 2, \cdots, N$, the least squares estimate of the parameter matrix $\boldsymbol{\Theta}$ is given by the following equation (forgetting factor $0 \leqslant \lambda_f \leqslant 1$ to enhance model adaptation):

$$\tilde{\boldsymbol{R}}\boldsymbol{\Theta} = \Gamma$$

where
$$\begin{cases} \tilde{R} = \sum_{s=1}^{p}\lambda_f^{p-s}\displaystyle\sum_{t=(s-1)N+1}^{sN}\boldsymbol{\varphi}(t)\boldsymbol{\varphi}^{\mathrm{T}}(t) \\ \Gamma = \displaystyle\sum_{s=1}^{p}\lambda_f^{p-s}\sum_{t=(s-1)N+1}^{sN}\boldsymbol{\varphi}(t)\boldsymbol{y}^{\mathrm{T}}(t) \end{cases}.$$

5 Simulation

The distillation column shown in Fig. 1 is a simulation model with highly nonlinear characteristics. The goal of the distillation column is to keep separating benzene and toluene at the constant pressure. There are two loops, which contain two measured process variables and two manipulated variables. On the top, the reflux rate controls the composition of benzene in the distillated steam, while the composition of the benzene is controlled by the steam rate on the bottom[7].

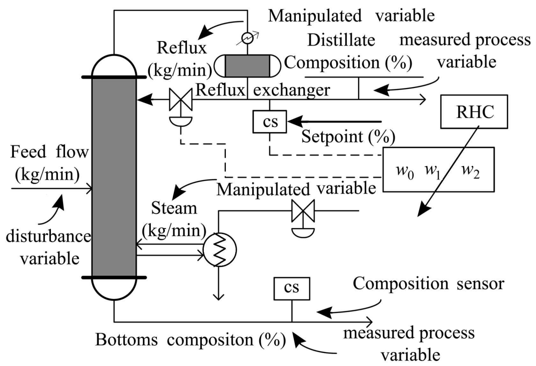

Fig. 1 Distlliation column graphic

Given one local control loop, where the output and control variables are top distillate compositions and valve open position, respectively. The operation of the building is divided into three levels: the lower level, the middle level, and the upper level. Different products are obtained at different level based on the percent of the distillate composition. Therefore, the nonlinear model is described as some linear model over the large operating regions. Through performing dynamic test, three models parameters are obtained in every level.

Table 1 lists the first order plus dead time (FOPDT) model parameters, when the distillate compositions are 89.3%, 94.4% and 98.7%. Generally speaking, fixed parameters PID controller based on the middle level model is designed through the whole operating condition. However, the fixed parameters PID controller has unsatisfactory performance at all levels.

Table 1 FOPDT model parameters for the distillation column simulation of top process variable

FOPDT model parameters	Process variable value (%)		
	89.3	94.4	98.7
k	1.1	0.94	0.11
τ_p (min)	43.6	64.8	50.6
θ_p (min)	21.9	27.9	16.7

The set point is stepped from 96% till 90.5%, and then the output of the system should track the set point with no more than a 5% peak overshoot ratio (POR). Response of both fixed parameters controller and auto tuning controller is shown in Fig. 2. As the set point is stepped from 96.5% to 94.5%, response of the fixed parameter PID controller has a very long setting time compared with the performance of auto tuning parameters controller. At the stage that set point is stepped from 94.5% to 92.5%, the fix parameter PID controller has more than 15% POR contrast to the performance of the auto tuning parameter PID controller. For the set point is stepped form 92.5% to 90.5%, the fixed parameter PID controller has large oscillation. Therefore, it is concluded that auto tuning parameters PID controller has a fast rise time and setting time with no more than 5% POR over a large range of work operations.

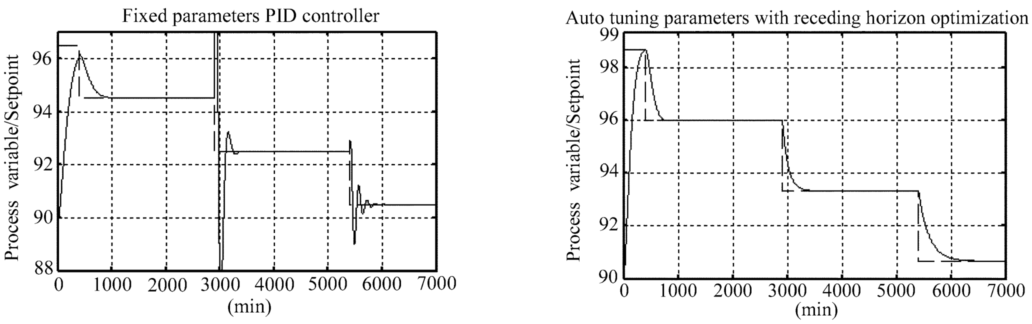

Fig. 2 Response of the top process variable for the rigorous distillation column simulation

6 Conclusion

A supervised auto tuning method for PID regulators according to the generalized predictive control criterion was proposed. The design was based on the minimization of a performance index, which includes prediction errors and control efforts over a period of time. The nonlinear model can be estimated on-line using a recursive identification algorithm. And the problem to be solved adopted a receding horizon optimization, which allowed an updating of the PID parameters at each sampling time for a plant-wide work operation. Simulation results on distillation column showed that the novel approach was able to adept a wide range of operation conditions and obtained the better performance compared with the fixed parameter PID controller.

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