Robust Stabilization of Uncertain Networked Control Systems with Data Packet Dropouts[1)]

HUANG Jian GUAN Zhi-Hong WANG Zhong-Dong

(*Department of Control Science & Engineering,*
Huazhong University of Science & Technology, Wuhan 430074)
(E-mail: huang_jan531@sina.com)

Abstract The data packet dropouts phenomenon is usually inevitable when information transmitted among communication networks. In this paper, the robust stabilization problem for uncertain networked control systems with data packet dropouts is studied. First, an uncertain discrete-time switching system model is presented to describe these networked control systems. The stability equivalence is then proved between this switching system and an uncertain impulsive difference system. Moreover, a sufficient condition is obtained for the asymptotical stability of the nonlinear impulsive difference system. From this condition the robust stabilization problem is dealt with for the uncertain impulsive system. Main results are given in linear matrix inequalities. Finally a numerical example is given to illustrate the theoretical results.

Key words Networked control systems, impulsive difference system, linear matrix inequality, robust stabilization

1 Introduction

The application of networked control systems (NCSs) have brought up many new issues, such as transmission delays which could make the closed loop control systems unstable. On the other hand, perturbations always exist in communication channels. Theses perturbations often cause data packet dropouts that might lead to communication failures. Obviously, an NCS may be no longer stable when suffered from continual packet dropouts, even if it is stable when the communication procedure is always normal. This paper is focused on the modeling and robust stabilization problem of NCSs with inevitable data packet dropouts in communication channels.

There have been plenty of theoretical results about NCSs so far. Nilsson[1] analyzed NCSs in the discrete-time domain. He further modeled the network delays as constant, independently random, and random but governed by an underlying Markov chain. Yu *et al.*[2] presented a method to transform the random network induced delays into constant ones. A state observer with delay compensation has been studied in their work. Hassibi *et al.*[3] modeled NCSs with data packet dropouts as asynchronous dynamical systems (ADSs). Zhang *et al.*[4] discussed detailedly the problem that how to guarantee the stability of NCSs when various transmission delay or data packet dropout occurs. Zhivoglyadov *et al.*[5] studied a systematic networked control method designed specifically to handle the constraints of the networked realization of an LTI control system. However there have been seldom results about robust control of NCSs with data packet dropouts up to now.

Less conservative stability conditions for impulsive differential systems were presented by Ye in [6]. We extended these results to impulsive difference systems by applying methods in [7]. Linear matrix inequalities (LMIs) are widely used in robust control problems[8]. Yu[9] solved the robust stabilization problem of uncertain discrete-time linear systems with the LMI technique. Based on the theoretical results mentioned above, the robust stabilization problem is discussed for uncertain NCSs with data packet dropouts in this paper.

This paper is organized as follows. Considering the inevitable data packet dropouts in communication networks, an uncertain discrete-time switching system model for NCSs is presented in Section 2. Moreover, the equivalence with respect to stability is proved between this model and an uncertain impulsive difference system. In Section 3, the stability condition is obtained for the equivalent impul-

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sive difference system, from which the robust stabilization problem is studied in Section 4. Finally, a numerical example is given to illustrate the theoretical results.

The following notations will be used throughout the paper. $R = (-\infty, \infty)$ denotes the real numbers, $N = \{1, 2, 3, \cdots\}$ is the set of positive integers, $Z^+ = \{0, 1, 2, \cdots\}$ is the set of nonnegative integers. $T = \{t_0, t_0 + 1, t_0 + 2, \cdots\} \subseteq Z^+$ is the set of discrete time instants. $\boldsymbol{x}(t) \in R^n, t \in T$ is the discrete n dimensional vector sequence of set T. $\|\bullet\|$ refers to the Euclidean vector norm or the induced matrix norm.

2 Problem formulation

A type of uncertain NCSs with data packet dropouts can be depicted as Fig. 1.

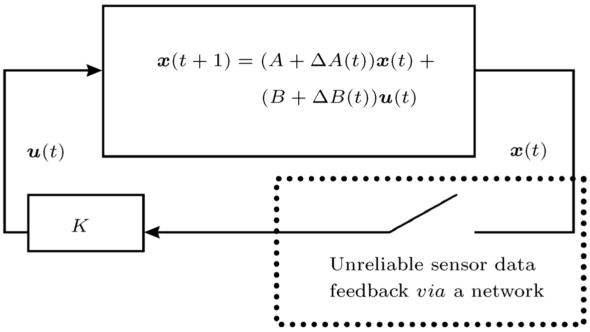

Fig. 1 A model of networked control systems with data packet dropouts

Before the detailed discussion, some basic assumptions for the system model are given as follows.

1) an NCS comprises an uncertain discrete-time closed loop control system with sensor data being communicated to the controller *via* a real-time network. A full state feedback controller is assumed, which has a static gain matrix K.

2) the network has a sufficient large bandwidth such that transmission delays can be omitted. This assumption is reasonable in real-time control networks.

3) a sampling switch in the feedback loop indicates whether the network works normally. When the switch is closed, sensor data arrive at the controller without any loss and the NCS works like a conventional discrete-time feedback control system. Otherwise, if the switch is open, the network is congested somehow and the controller receives no data. In this case, we assume that control law is $u(t) = 0$ for simplicity.

4) since the network can′t be abnormal for ever, we assume that there exists an upper bound for the duration of data packet dropouts every time. A constant $N_L(0 < N_L < +\infty)$ is used to denote this upper bound. Thus, the controller receives no data at most N_L steps between two successive successful communication procedures. Note that N_L is a positive integer.

Let $\varGamma = \{t_0, t_1, t_2, \cdots : t_0 < t_1 < t_2 < \cdots\} \subset T \subseteq Z^+$ denote the set of beginning and ending instants of data packet dropouts durations, where t_0 is also the initial time of the NCS. Without loss of generality, we assume that the network works normally between instants t_{2k-2} and t_{2k-1} but becomes congested between instants t_{2k-1} and t_{2k}, where $k \in N$. Then we can introduce two notations, that is, $\varOmega_1 = \bigcup_{k=1}^{\infty}\{t_{2k-1}, t_{2k-1} + 1, \cdots, t_{2k} - 1\}$ and $\varOmega_2 = \bigcup_{k=1}^{\infty}\{t_{2k-2}, t_{2k-2} + 1, \cdots, t_{2k-1} - 1\}$. Obviously, $\varOmega_1$ denotes the set of time instants at which sensor data are always lost, while $\varOmega_2$ denotes the set of time instants at which sensor data is communicated to the controller normally. Note that we have $T = \varOmega_1 \cup \varOmega_2$.

Based on aforementioned assumptions, we present an uncertain discrete-time switching system model for an NCS with data packet dropouts. This switching system is described by

$$
\begin{array}{l}
\boldsymbol{x}(t+1) = (A + \Delta A(t))\boldsymbol{x}(t), \quad t \in \varOmega_1 \\
\left\{\begin{array}{l} \boldsymbol{x}(t+1) = (A + \Delta A(t))\boldsymbol{x}(t) + (B + \Delta B(t))\boldsymbol{u}(t) \\ \boldsymbol{u}(t) = K\boldsymbol{x}(t) \end{array}\right., \ t \in \varOmega_2
\end{array}
\tag{1}
$$

where $\boldsymbol{x} \in R^n$, $\boldsymbol{u} \in R^m$, $t \in T$, A, B and K are matrices with appropriate dimensions, $\Delta A(t)$ and $\Delta B(t)$ are norm-bounded matrices to describe the system uncertainties, $\boldsymbol{x}(t_0) = \boldsymbol{x}_0$ is the initial state.

It might be difficult to discuss the robust stabilization problem of system (1) directly. We can rewrite it as

$$\boldsymbol{x}(t+1) = \bar{A}(t)\boldsymbol{x}(t),\ t \in \Omega_2 \qquad \boldsymbol{x}(t_{2k}) = I_k\boldsymbol{x}(t_{2k-1}),\ k \in N \tag{2}$$

where $\bar{A}(t) = A + \Delta A(t) + (B + \Delta B(t))K$, $\boldsymbol{x}(t_0) = \boldsymbol{x}_0$ is the initial state. I_k satisfies

$$I_k = \prod_{t=t_{2k}-1}^{t_{2k-1}} (A + \Delta A(t)) = (A + \Delta A(t_{2k}-1))(A + \Delta A(t_{2k}-2)) \cdots (A + \Delta A(t_{2k-1})) \tag{3}$$

$$N_k = t_{2k} - t_{2k-1} < N_L, \quad k \in N$$

Let $\Omega_3 = \bigcup_{k=1}^{\infty}\{t_{2k-2}, t_{2k-2}+1, \cdots, t_{2k-1}\}$, and we have the following definition.

Definition 1. Let $g : \Omega_3 \mapsto T$, which maps elements of Ω_3 to T in turn. It can be depicted as Fig. 2. Apparently, g is invertible.

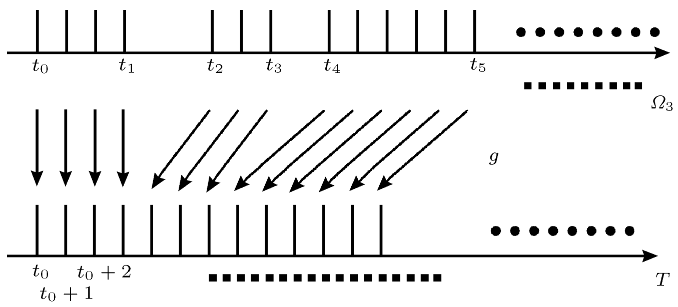

Fig. 2 Function $g : \Omega_3 \mapsto T$

The function g can be described by

$$g(t) = \begin{cases} t, & t \in \{t_0, t_0+1, \cdots, t_1-1\} \\ \tau_k, & t = t_{2k-1}, k \in N \\ \tau_k + 1 + t - t_{2k}, & t \in \{t_{2k}, t_{2k}+1, \cdots, t_{2k+1}-2, t_{2k+1}-1\}, k \in N \end{cases} \tag{4}$$

Note that $\{t_0, t_0+1, \cdots, \tau_1-1, \tau_1, \cdots, \tau_2-1, \tau_2, \cdots, \tau_k-1, \tau_k, \cdots\} = \{t_0, t_0+1, t_0+2, \cdots\} = T$, where $\tau_k = g(t_{2k-1}), k \in N$ and $\tau_k + 1 = g(t_{2k}), k \in N$.

Consider an uncertain impulsive difference system as follows:

$$\boldsymbol{x}(t+1) = \tilde{A}(t)\boldsymbol{x}(t),\ t \in T \text{ and } t \neq \tau_k \qquad \boldsymbol{x}(\tau_k+1) = I_k\boldsymbol{x}(\tau_k),\ k \in N \tag{5}$$

where $\tilde{A}(t) = \bar{A}(g^{-1}(t)) = A + \Delta A(g^{-1}(t)) + (B + \Delta B(g^{-1}(t)))K$, time instants τ_k satisfy $\tau_k = g(t_{2k-1}) \in T, k \in N$, I_k is given by (3). Since uncertain matrices $\Delta A(t)$ are norm-bounded and the duration of data packet dropouts is limited ($< N_L$) for every time, it follows from (3) that there exists a constant $\beta > 0$ such that $\|I_k\| < \beta$.

Theorem 1. The trivial solution of system (1) is stable if and only if the trivial solution of system (5) is stable.

Proof. Assume that both system (1) and system (5) have a same initial condition $\boldsymbol{x}(t_0) = \boldsymbol{x}_0$. Let $\boldsymbol{x}(t)|_{(1)}$ and $\boldsymbol{x}(t)|_{(5)}$ represent the solutions of systems (1) and (5) respectively. We have

$$\boldsymbol{x}(t)|_{(5)} = \tilde{A}(t-1)\tilde{A}(t-2)\cdots\tilde{A}(t_0)\boldsymbol{x}_0 = \bar{A}(t-1)\bar{A}(t-2)\cdots\bar{A}(t_0)\boldsymbol{x}_0 = \boldsymbol{x}(g^{-1}(t))|_{(1)}$$

when $t_0 < t \leqslant t_1 = \tau_1$. Otherwise when $t = \tau_k + 1, k \in N$, we have

$$\boldsymbol{x}(t)|_{(5)} = \left[\prod_{j=k}^{2} I_j\tilde{A}(\tau_j-1)\tilde{A}(\tau_j-2)\cdots\tilde{A}(\tau_{j-1}+1)\right]I_1\tilde{A}(t_1-1)\tilde{A}(t_1-2)\cdots\tilde{A}(t_0)\boldsymbol{x}_0 =$$

$$\left[\prod_{j=k}^{2}(A+\Delta A(t_{2j}-1))(A+\Delta A(t_{2j}-2))\cdots(A+\Delta A(t_{2j-1}))\bar{A}(t_{2j-1}-1)\bar{A}(t_{2j-1}-2)\cdots\right.$$

$$\left.\bar{A}(t_{2j-2})\right](A+\Delta A(t_2-1))(A+\Delta A(t_2-2))\cdots(A+\Delta A(t_1))\bar{A}(t_1-1)\bar{A}(t_1-2)\cdots\bar{A}(t_0)\boldsymbol{x}_0=$$

$$\boldsymbol{x}(t_{2k})|_{(1)}=\boldsymbol{x}(g^{-1}(t))|_{(1)}$$

When $\tau_k+1<t\leqslant\tau_{k+1}, k\in N$, we can obtain $\boldsymbol{x}(t)|_{(5)}=\boldsymbol{x}(g^{-1}(t))|_{(1)}$ similarly. Thus we can conclude that $\boldsymbol{x}(t)|_{(5)}=\boldsymbol{x}(g^{-1}(t))|_{(1)}, t\in T$.

The rest of this proof is omitted because it is apparent. □

Remark 1. The stability equivalence between system (1) and system (5) can be extended to the asymptotical stability, the exponential stability and so on.

From now on, we transform the stability problem of NCS (1) to the stability problem of an uncertain impulsive difference system (5).

3 Stability results of impulsive difference systems

New stability results were obtained by Ye in [6] for the impulsive differential system. We extend these results to the discrete-time domain, *i.e.*, to the impulsive difference system.

Consider a nonlinear impulsive difference system described by

$$\begin{cases}\boldsymbol{x}(t+1)=f(\boldsymbol{x}(t),t), & t\in T,\ t\neq\tau_k\in E\\ \boldsymbol{x}(t+1)=\boldsymbol{\varphi}_k(\boldsymbol{x}(t)), & t=\tau_k\in E,\ k\in Z^+\end{cases}\tag{6}$$

where $\boldsymbol{x}\in R^n, \boldsymbol{f}: R^n\times T\to R^n$ is continuous with respect to $\boldsymbol{x}$ and satisfies $\boldsymbol{f}(0,t)=0$. The set $E=\{\tau_0,\tau_1,\cdots:\tau_0<\tau_1<\cdots\}\subset Z^+$ is an unbounded, closed, discrete subset of Z^+ which denotes the set of times when jumps occur. $\boldsymbol{\varphi}_k: R^n\to R^n$ denote the transitions of the state at time instants τ_k and satisfy $\boldsymbol{\varphi}_k(0)=0$ for all $k\in Z^+$. Assume that system (6) has a unique solution $\{\boldsymbol{x}(t;x_0,t_0)\}$ for all $\boldsymbol{x}_0\in R^n$ and $t_0\geqslant 0$ with initial condition $(\boldsymbol{x}_0,t_0)$. Obviously, $\{\mathbf{0}\}$ is a solution of system (6). For the sake of simplicity, we further assume that $\boldsymbol{\varphi}_0(\boldsymbol{x})=\boldsymbol{x},\tau_0=t_0-1,\boldsymbol{x}(t_0)=\boldsymbol{x}(\tau_0)$.

Theorem 2. Assume that there exists a function $V(\boldsymbol{x}(t),t)$ which is continuous with respect to $\boldsymbol{x}$, and $a(r),b(r),c(r)\in K$ such that

i) $a(\|\boldsymbol{x}\|)\leqslant V(\boldsymbol{x},t)\leqslant b(\|\boldsymbol{x}\|)$ for all $(\boldsymbol{x}(t),t)\in R^n\times T$

ii) for all $t\in\{\tau_k+1,\tau_k+2,\cdots,\tau_{k+1}-1\}$, $k\in Z^+$, the difference of $V(\boldsymbol{x}(t),t)$ along the solution of system (6) satisfies

$$\Delta V(\boldsymbol{x}(t),t)=V(\boldsymbol{x}(t+1),t+1)-V(\boldsymbol{x}(t),t)\leqslant 0\tag{7}$$

iii) for all $k\in Z^+$,

$$V(\boldsymbol{x}(\tau_{k+1}+1),\tau_{k+1}+1)-V(\boldsymbol{x}(\tau_k+1),\tau_k+1)\leqslant -c(\|\boldsymbol{x}(\tau_k+1)\|)\tag{8}$$

Then the trivial solution of system (6) is asymptotically stable.

Proof. 1) stability

From (7) we have

$$V(\boldsymbol{x}(t_0),t_0)=V(\boldsymbol{x}(\tau_0+1),\tau_0+1)\geqslant V(\boldsymbol{x}(t),t)\tag{9}$$

for all $t\in\{\tau_0+1,\tau_0+2,\cdots,\tau_1\}$. Moreover, inequality (8) implies that

$$V(\boldsymbol{x}(t_0),t_0)\geqslant V(\boldsymbol{x}(\tau_1+1),\tau_1+1)\geqslant\cdots\geqslant V(\boldsymbol{x}(\tau_j+1),\tau_j+1)\geqslant V(\boldsymbol{x}(t),t)\tag{10}$$

for all $t\in\{\tau_j+1,\tau_j+2,\cdots,\tau_{j+1}\}, j\in N$.

Therefore it follows from (9) and (10) that $V(\boldsymbol{x}(t_0),t_0)\geqslant V(\boldsymbol{x}(t),t)$, $\forall t\in T$. Note that $V(\mathbf{0},t)=0$. Thus, for any $\varepsilon>0$ there is $\delta=\delta(\varepsilon,t_0)>0$ such that $V(\boldsymbol{x}(t_0),t_0)<a(\varepsilon)$ if $\|\boldsymbol{x}(t_0)\|<\delta$, which leads to $a(\|\boldsymbol{x}(t)\|)\leqslant V(\boldsymbol{x}(t),t)\leqslant V(\boldsymbol{x}(t_0),t_0)<a(\varepsilon)$ for all $t\in T$. It follows from $a(r)\in K$ that $\|\boldsymbol{x}(t)\|<\varepsilon$. This proves the stability.

2) attraction

From (8) we have $V(\boldsymbol{x}(\tau_k+1),\tau_k+1)-V(\boldsymbol{x}(t_0),t_0)\leqslant-\sum_{j=0}^{k-1}c(\|\boldsymbol{x}(\tau_j+1)\|)$. It implies $0<V(\boldsymbol{x}(\tau_k+1),\tau_k+1)\leqslant V(\boldsymbol{x}(t_0),t_0)-\sum_{j=0}^{k-1}c(\|\boldsymbol{x}(\tau_j+1)\|)$. Taking the limit with respect to k gives $0\leqslant\lim_{k\to\infty}V(\boldsymbol{x}(\tau_k+1),\tau_k+1)\leqslant V(\boldsymbol{x}(t_0),t_0)-\sum_{j=0}^{\infty}c(\|\boldsymbol{x}(\tau_j+1)\|)$. Note that $\|\boldsymbol{x}(t_0)\|$ can be selected arbitrary small. Thus the series $\sum_{j=0}^{\infty}c(\|\boldsymbol{x}(\tau_j+1)\|)$ is convergent, which implies that $\lim_{k\to\infty}c(\|\boldsymbol{x}(\tau_k+1)\|)=0$. Therefore, it follows from $c(r)\in K$ that $\lim_{k\to\infty}\boldsymbol{x}(\tau_k+1)=0$.

For any $\varepsilon>0$, there is $\delta=\delta(\varepsilon),\ 0<\delta\leqslant\varepsilon$ such that $b(\delta)<a(\varepsilon)$. Moreover, there is a $n=n(\delta)$ such that $\|\boldsymbol{x}(\tau_k+1)\|<\delta\leqslant\varepsilon$ for all $k>n$. It follows that

$$a(\|\boldsymbol{x}(t)\|)\leqslant V(\boldsymbol{x}(t),t)\leqslant V(\boldsymbol{x}(\tau_k+1),\tau_k+1)\leqslant b(\|\boldsymbol{x}(\tau_k+1)\|)<b(\delta)<a(\varepsilon) \tag{11}$$

for all $t\in\{\tau_k+2,\tau_k+3,\cdots,\tau_{k+1}\}, k>n$. Thus we have $\|\boldsymbol{x}(t)\|<\varepsilon$ for all $t\in\{\tau_k+2,\tau_k+3,\cdots,\tau_{k+1}\}$, $k>n$. Let $M=\tau_{n+1}+1$; it is apparent that $\|\boldsymbol{x}(t)\|<\varepsilon$ for all $t>M$. This proves the attraction. Both stability and attraction imply that the conclusion of Theorem 2 holds. □

4 Robust stabilization of uncertain NCSs with data dropouts

First, a sufficient condition for the asymptotical stability of system (5) is obtained as follows.

Theorem 3. Assume that for system (5) there exist a constant $\alpha>0$ and a positive definite diagonal matrix $P\in R^{n\times n}$ whose diagonal elements are the same, such that

$$\boldsymbol{x}^{\mathrm{T}}(t)[\gamma\tilde{A}^{\mathrm{T}}(t)P\tilde{A}(t)-P]\boldsymbol{x}(t)\leqslant-\alpha\|\boldsymbol{x}(t)\|^2 \tag{12}$$

for all $\boldsymbol{x}\in R^n$, $t\in T$ where positive constant γ satisfies

$$\gamma=\max(1,\beta^2),\quad \|I_k\|<\beta \tag{13}$$

Then the trivial solution of system (5) is asymptotically stable.

Proof. For system (5), construct the Lyapunov function $V(\boldsymbol{x}(t),t)=\boldsymbol{x}^{\mathrm{T}}(t)P\boldsymbol{x}(t)$. Apparently, this function satisfies the first condition of Theorem 2.

It follows from (12) and (13) that for all $t\in\{\tau_k+1,\tau_k+2,\cdots,\tau_{k+1}-1\},\ k\in Z^+$,

$$\Delta V(\boldsymbol{x}(t),t)=V(\boldsymbol{x}(t+1),t+1)-V(\boldsymbol{x}(t),t)=\boldsymbol{x}^{\mathrm{T}}(t)(\tilde{A}^{\mathrm{T}}(t)P\tilde{A}(t))\boldsymbol{x}(t)-\boldsymbol{x}^{\mathrm{T}}(t)P\boldsymbol{x}(t)\leqslant 0 \tag{14}$$

Thus $V(\boldsymbol{x}(t),t)$ satisfies the second condition in Theorem 2.

For all $t\in\{\tau_1+1,\tau_2+1,\cdots,\tau_{k+1}+1,\cdots\}, k\in Z^+$, the solution of system (5) can be described by $\boldsymbol{x}(\tau_{k+1}+1)=I_{k+1}\boldsymbol{x}(\tau_{k+1})=I_{k+1}\left\{\sum_{t=\tau_{k+1}-1}^{\tau_k+1}\tilde{A}(t)\right\}\boldsymbol{x}(\tau_k+1)$. It follows that

$$\begin{aligned}&V(\boldsymbol{x}(\tau_{k+1}+1),\tau_{k+1}+1)-V(\boldsymbol{x}(\tau_k+1),\tau_k+1)=\\&\boldsymbol{x}^{\mathrm{T}}(\tau_k+1)\left\{\left[\prod_{t=\tau_k+1}^{\tau_{k+1}-1}\tilde{A}^{\mathrm{T}}(t)\right]I_{k+1}^{\mathrm{T}}PI_{k+1}\left[\prod_{t=\tau_{k+1}-1}^{\tau_k+1}\tilde{A}(t)\right]-P\right\}\boldsymbol{x}(\tau_k+1)\end{aligned} \tag{15}$$

Note that all elements in the diagonal of P have the same value. Thus for all $\boldsymbol{x}\in R^n, k\in Z^+$ we have $\boldsymbol{x}^{\mathrm{T}}(I_{k+1}^{\mathrm{T}}PI_{k+1})\boldsymbol{x}=\boldsymbol{x}^{\mathrm{T}}(I_{k+1}^{\mathrm{T}}I_{k+1}P)\boldsymbol{x}\leqslant\|I_{k+1}\|^2\boldsymbol{x}^{\mathrm{T}}P\boldsymbol{x}\leqslant\gamma\boldsymbol{x}^{\mathrm{T}}P\boldsymbol{x}$. Moreover, it follows that

$$\begin{aligned}&\left[\prod_{t=\tau_k+1}^{\tau_{k+1}-1}\tilde{A}^{\mathrm{T}}(t)\right]I_{k+1}^{\mathrm{T}}PI_{k+1}\left[\prod_{t=\tau_{k+1}-1}^{\tau_k+1}\tilde{A}(t)\right]\leqslant\\&\gamma\left[\prod_{t=\tau_k+1}^{\tau_{k+1}-1}\tilde{A}^{\mathrm{T}}(t)\right]P\left[\prod_{t=\tau_{k+1}-1}^{\tau_k+1}\tilde{A}(t)\right]\leqslant\gamma\tilde{A}^{\mathrm{T}}(\tau_{k+1}-1)P\tilde{A}(\tau_{k+1}-1)\leqslant P\end{aligned} \tag{16}$$

which leads to $V(\boldsymbol{x}(\tau_{k+1}+1),\tau_{k+1}+1)-V(\boldsymbol{x}(\tau_k+1),\tau_k+1)\leqslant -\alpha\|\boldsymbol{x}(\tau_k+1)\|^2, k\in Z^+$. Let $c(r)=\alpha r^2$; then the third condition of Theorem 2 is satisfied. Since system (5) is a special case of system (6), it is asymptotically stable from Theorem 2. This completes the proof. □

To study the robust stabilization problem of system (5), we assume further that the uncertainty matrices in the system are described by

$$[\Delta A(t) \quad \Delta B(t)] = DF(t)[E_1 \quad E_2] \tag{17}$$

where time-varying part $F(t)$ satisfies $F^{\mathrm{T}}(t)F(t)\leqslant I$.

Theorem 4. For uncertain impulsive difference system (5), if there exist a positive definite diagonal matrix $\tilde{X}\in R^{n\times n}$ whose diagonal elements are the same and a matrix $Q\in R^{m\times n}$, such that the LMI

$$\begin{bmatrix} -\tilde{X}+\gamma DD^{\mathrm{T}} & \sqrt{\gamma}[A\tilde{X}+BQ] & 0 \\ \sqrt{\gamma}[A\tilde{X}+BQ]^{\mathrm{T}} & -\tilde{X} & [E_1\tilde{X}+E_2Q]^{\mathrm{T}} \\ 0 & E_1\tilde{X}+E_2Q & -I \end{bmatrix} < 0 \tag{18}$$

holds, where γ is given by (13), then system (5) is stabilizable *via* a linear static controller $\boldsymbol{u}(t)=Q\tilde{X}^{-1}\boldsymbol{x}(t)$.

Proof. The proof is similar to Theorem 2 in [9] and thus is omitted. □

5 Numerical examples

For system (1), we assume $\boldsymbol{x}\in R^2$, $\boldsymbol{u}\in R^2$, $A=\begin{bmatrix}0.9 & 0.6\\ 0.2 & 0.2\end{bmatrix}$, $\boldsymbol{B}=\begin{bmatrix}0\\1\end{bmatrix}$, $D=\begin{bmatrix}0.1 & 0\\ 0 & 0.1\end{bmatrix}$, $F(t)=\begin{bmatrix}\sin(t) & 0\\ 0 & \sin(t)\end{bmatrix}$, $E_1=\begin{bmatrix}0.2 & 0.3\\ 0.1 & 0.4\end{bmatrix}$. Further, we assume $N_L=3$ such that $\|I_k\|\leqslant\|A+\Delta A(\tau_k)\|^{N_L}<1.65$. Selecting $\beta=1.65$, $\gamma=\max\{1,\beta^2\}=2.7225$, from Theorem 4 we can obtain

$$\tilde{\boldsymbol{X}}=\begin{bmatrix}0.9504 & 0\\ 0 & 0.9504\end{bmatrix},\quad \boldsymbol{Q}=[-0.6887 \quad -0.4759],\quad \boldsymbol{K}=Q\tilde{X}^{-1}=[-0.7246 \quad -0.5007]$$

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HUANG Jian Received his master degree from Huazhong University of Science and Technology in 2000. Now he is a doctor candidate. His research interests include networked control systems, intelligent control systems, and biological control systems.

GUAN Zhi-Hong Received his Ph. D. degree from South China University of Technology in 1994. Now he is a professor at Huazhong University of Science and Technology. His current research interests include impulsive and hybrid control systems, chaos control and synchronization, nonlinear and complex dynamic networks and systems.

WANG Zhong-Dong He is currently a professor at Huazhong University of Science and Technology. His research interests include intelligent instruments design and distributed control systems.