Integration of Weights Model of Interval Numbers Comparison Matrix[1)]

ZHU Jian-Jun[1] LIU Shi-Xin[2] WANG Meng-Guang[2]

[1](*School of Information Science and Engineering, Northeastern University, Shenyang* 110004)

[2](*College of Economics and Management, Nanjing University of Aeronautics and Astronautics, Nanjing* 210016)

(E-mail: mgwang@mail.neu.edu.cn)

Abstract Consistency and the weights estimation model of the interval number comparison matrix (INCM) in the analytical hierarchy process is studied under uncertainty decision-making case. Based on the weights feasible region, the local consistency definition and the local satisfactory consistency definition are given. Then, a computational model set up to test whether the INCM has the local satisfactory consistency or not. Moreover, the consistency degree based on the random crisp comparison matrix is defined as an effective index to test the consistency. Next, the upper range model, the lower range model, and the possible value model are put forward which can solve the problem that some existing approaches do not consider the consistency and its effect on the weights. According to the property of these models, a genetic algorithm is developed.

Key words Analytical hierarchy process, interval number comparison matrix, weights feasible region, integration weights model, genetic algorithm

1 Introduction

The analytical hierarchy process (AHP) is widely used in the multiple criteria decision-making fields[1~3]. When using the AHP, a decision maker often makes imprecise judgments or inconsistent judgments owing to the complexity of the decision-making problem, and adopts the interval numbers for imprecise judgments. And then, the INCM is obtained. The INCM has been studied by many researchers[4~12], and the main content and contribution of this paper are as follows.

First, consistency definition and consistency test approach. To our knowledge, few researchers focus on the consistency definition. Wei[4] and Bryson[5] gave a perfect consistency definition. However, the perfectly consistent comparison matrix cannot be obtained easily. Moreover, the consistency has its point and has many effects on the weights estimation. Therefore, the satisfactory consistency definition and its property should be studied. In addition, there are not any practicable approaches to test whether the INCM is consistent or not.

Secondly, weight estimation analysis. There are many available approaches to derive the weights. However, they have many limitations. For example, the approach from Wei[4] is not obvious and it cannot estimate the exact range. Leung[6] and Haines[8] put forward a linear model based on the weights feasible region. However, it cannot guarantee all vertices of feasible region are found when the rank of the INCM is big enough. Wang[7] did not test whether the matrix was consistent or not, and the computational work is comparatively heavy. Lipovetsky[9] derived the weights from the matrix comprised of the stochastic variables, while it did not have reciprocal property. Byeong[10] derived the weights *via* the simulation approach. However, it needed comparative computations, and it could not guarantee the exact range was found in limited iterations. Mikhailov[11] and Buckley[12] derived the weights from the fuzzy number comparison matrix, while they did not consider the consistency effects on the weights estimation. In this case, the decision-making reliability based on the weights cannot be guaranteed. In order to overcome these limitations, a new weight model solved by the genetic algorithm is put forward. The aim of this paper is to make the AHP theory perfect under uncertainty case.

2 Consistency analysis

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When studying the weights estimation approach for the fuzzy number comparison matrix, Leung[6] defined the weights feasible region as $S = \{w_i | a_{ij}^L \leqslant \frac{w_i}{w_j} \leqslant a_{ij}^U, w_i > 0,\ i, j = 1, \cdots, n\}$. Based on the feasible region, the local consistency and the local satisfactory consistency are defined.

Definition 1. The INCM $\bar{A}$ has the local consistency, if one set of weight w_i, $i = 1, \cdots, n$ meets

$$a_{ij}^L \leqslant \frac{w_i}{w_j} \leqslant a_{ij}^U, \quad \forall i, j \in J \tag{1}$$

It should be noted that the reason that the INCM satisfying (1) is called the local consistency instead of consistency is that though there is one set of weight w_i, $i = 1, \cdots, n$ derived from matrix A $(a_{ij}^L \leqslant a_{ij} \leqslant a_{ij}^U)$, the weights derived from any matrix A $(a_{ij}^L \leqslant a_{ij} \leqslant a_{ij}^U)$ may not satisfy (1). Therefore, formula (1) can only represent the local consistency.

Owing to the complexity of decision-making problem, formula (1) cannot be easily satisfied. In this case, the local satisfactory consistency is defined.

Definition 2. INCM $\bar{A}$ has the local satisfactory consistency, if one set of weight w_i, $i = 1, \cdots, n$ meets

$$(1 - \delta_{ij}) a_{ij}^L \leqslant \frac{w_i}{w_j} \leqslant a_{ij}^U (1 + \delta_{ij}), \quad \forall i, j \in J \tag{2}$$

In (2), δ_{ij} is the tolerance deviation. Let $\delta = \max\limits_{i,j} \delta_{ij}$. From Xu[1], $\lambda_{\max} - n \leqslant \frac{(n-1)}{2}\delta^2$ can be obtained, which can be transformed into $\frac{\lambda_{\max} - n}{(n-1)RI} \leqslant \frac{\delta^2}{2RI}$. If it has the satisfactory consistency, the formula $\frac{\lambda_{\max} - n}{(n-1)RI} \leqslant 0.1$ can be obtained. Then, one can obtain $\frac{\delta^2}{2RI} \leqslant 0.1$. Therefore, we can obtain $\delta = \sqrt{0.2RI}$.

The weights set from formula (2) are also considered as the feasible region which, if not confusion, will define all the weights feasible regions in the rest of the paper. Hence, the problem of testing whether the INCM $\bar{A}$ is consistent or not can be transformed into testing whether S is empty or not. Based on this idea, the model P_1 is put forward.

$$\min \beta = \beta_1 + \beta_2 \tag{3}$$

$$\text{s.t. } \ln(1-\delta) a_{ij}^L \leqslant \ln w_i - \ln w_j + \beta_{1ij}, \quad 1 \leqslant i < j \leqslant n \tag{4}$$

$$\ln w_i - \ln w_j \leqslant \ln(1+\delta) a_{ij}^U + \beta_{2ij}, \quad 1 \leqslant i < j \leqslant n \tag{5}$$

$$\beta_1 \geqslant \beta_{1ij}, \quad \beta_2 \geqslant \beta_{2ij} \tag{6}$$

$$\beta_{1ij}, \quad \beta_{2ij} \geqslant 0, \quad w_i > 0 \tag{7}$$

(3) denotes minimizing all tolerance deviations; (4) and (5) are obtained from natural logarithm of (2); (6) denotes β_1, β_2 to be the maximums of the tolerance deviations; (7) denotes all tolerance deviations and the weights are not negative.

Let $w_i' = \ln w_i$. The model P_1 can be transformed into a linear program. If $\min \beta = 0$, it means the weights feasible region S is not empty with δ, and the INCM $\bar{A}$ has the local satisfactory consistency; or else ($\min \beta > 0$), it means the INCM $\bar{A}$ has not the local satisfactory consistency.

How to test the whole consistency when the INCM has the local satisfactory consistency? When using the AHP, there should be one satisfactory consistent matrix $A = (a_{ij})_{n \times n}, a_{ij} \in \bar{a}_{ij}$, which can reflect the decision maker's actual preference. However, it cannot be obtained easily since the decision-making problem is complex. To overcome the difficulty, one can generate a satisfactory consistent comparison matrix to denote his preference approximately. Therefore, the random crisp comparison matrix is introduced.

Definition 3. The comparison matrix $A = (a_{ij})_{n \times n}$ is called the random crisp comparison matrix of the INCM $\bar{A}$, which is constructed as follows: for $1 \leqslant i < j \leqslant n$, a_{ij} is generated by the uniform distribution in $[a_{ij}^L, a_{ij}^U]$. Let $a_{ji} = 1/a_{ij}$ for $1 \leqslant i < j \leqslant n$, and let $a_{ii} = 1$ for all $i = 1, \cdots, n$.

The relationship between the random crisp comparison matrix based on Definition 3 and the weights feasible region S from the P_1 can referred to Theorem 1.

Theorem 1. The solution in the weights feasible region is corresponding to the satisfactory random crisp comparison matrix uniquely.

Proof. 1) The comparison matrix $A = (a_{ij})_{n\times n}$ can be constructed *via* the comparison based on w_i if it satisfies $a_{ij}^L \leqslant \frac{w_i}{w_j} \leqslant a_{ij}^U$. Then, one can obtain $a_{ij} = \frac{w_i}{w_j}, a_{ik} = \frac{w_i}{w_k}, a_{kj} = \frac{w_k}{w_j}$. That is, $a_{ij} = a_{ik}a_{kj}$, which means the comparison matrix A is perfectly consistent.

2) If there is one random crisp comparison matrix having the perfectly consistency, one can obtain $a_{ij}^L \leqslant a_{ij} \leqslant a_{ij}^U$ based on Definition 3. In addition, the formula $a_{ij} = \frac{w_i}{w_j}$ can be obtained. Therefore, one can obtain $a_{ij}^L \leqslant \frac{w_i}{w_j} \leqslant a_{ij}^U$.

If the random crisp comparison matrix A meets $a_{ij}^L(1-\delta) \leqslant \frac{w_i}{w_j} \leqslant a_{ij}^L, a_{ij}^U \leqslant \frac{w_i}{w_j} \leqslant a_{ij}^U(1+\delta)$. From 1), one can conclude that it has not the perfectly consistency since matrix A does not meet $a_{ij}^L \leqslant \frac{w_i}{w_j} \leqslant a_{ij}^U$. However, it has the satisfactory consistency according to the value of δ.

To a random crisp comparison matrix A, if it has the satisfactory consistency, from 2), one can conclude $a_{ij}^L(1-\delta) \leqslant \frac{w_i}{w_j} \leqslant a_{ij}^L$ or $a_{ij}^U(1+\delta) \geqslant \frac{w_i}{w_j} \geqslant a_{ij}^U$, which meets $a_{ij}^L(1-\delta) \leqslant \frac{w_i}{w_j} \leqslant a_{ij}^U(1+\delta)$.

Via the AHP theory, the weights $w_i,\ i = 1, \cdots, n$ exist uniquely. Therefore, we can conclude that the solution in the feasible region is corresponding to the satisfactory random crisp matrix uniquely. □

If the weights feasible region S is empty, there is not the satisfactory crisp comparison matrix of the INCM. If S has one set of weights, it must be corresponding to one satisfactory random crisp comparison matrix A uniquely. Moreover, the larger numbers of the solutions in the feasible region, the larger numbers of the satisfactory crisp comparison matrixes. In other words, the numbers of the satisfactory random crisp comparison matrixes denoting the decision maker′s preference are larger. In this case, the decision maker′s judgment is right and the logical harmonization is high. Therefore, the numbers of the solutions in the feasible region can test the whole consistency. However, it is difficult to solve the weights feasible region from the P_1 as the rank of the INCM is large. To avoid the difficulty, an equivalent approach is put forward.

Definition 4. N random crisp comparison matrixes denoted as $A^1, \cdots, A^N$ are generated based on Definition 3. If there are m comparison matrixes having the satisfactory consistency ($CR \leqslant 0.1$), the consistency degree η of the INCM can be defined as $\frac{m}{N} \times 100\%$.

Since the satisfactory random crisp comparison matrix is corresponding to the solutions in the weights feasible region uniquely, the value of η can reflect the consistency. In addition, the better consistency is, the larger value of η is, *vice versa*. In the following, two examples from previous literature are given to show the existence on consistency degree.

Example 1. The example is from [1,4]. $\bar{A} = \begin{vmatrix} [1,1] & [2,4] & [3,5] & [3,5] \\ [1/4.1/2] & [1,1] & [1/2,1] & [2,5] \\ [1/5,1/3] & [1,2] & [1,1] & [1/3,1] \\ [1/5,1/3] & [1/5,1/2] & [1,3] & [1,1] \end{vmatrix}$. Generate 50 random crisp comparison matrixes, and obtain $\eta = 33\%$. It shows the consistency degree is bad.

Example 2. The example is from [7]. $\bar{A} = \begin{vmatrix} [1,1] & [2,5] & [2,4] & [1,3] \\ [1/5,1/2] & [1,1] & [1,3] & [1,2] \\ [1/4,1/2] & [1/3,1] & [1,1] & [1/2,1] \\ [1/3,1] & [1/2,1] & [1,2] & [1,1] \end{vmatrix}$. The value of η equals 88% and it shows the consistency degree is high.

Therefore, one can calculate the consistency degree instead of solving the weights feasible region when analyzing the consistency of the INCM.

3 Weight model

3.1 Weight range model

Let $\mathcal{IN} = \{1, \cdots, N\}$. Generate N random crisp comparison matrixes based on Definition 3, denoted as $A^I, a_{ij}^I \in \bar{a}_{ij}, I \in \mathcal{IN}$. Solve the eigenvalue problem of A^I if $CR(A^I) \leqslant 0.1$, and obtain the standardization weights denoted as $w^I = (w_1^I, \cdots, w_n^I)^{\mathrm{T}}$. If the value of N is large enough, then the lower weight range of the INCM $\bar{A}$ can be written as

$$P_2 \qquad w_i^L = \min_{I\in IN} w_i^I \tag{8}$$

And the upper weight range can be written as

$$P_3 \qquad w_i^U = \max_{I\in IN} w_i^I \tag{9}$$

From formulas (8) and (9), the weights can be expressed as

$$\bar{w}_i = [w_i^L, w_i^U] \tag{10}$$

Then, the weights estimation problem can be transformed into estimating values of w_i^L and w_i^U. For convenience, (8) can be rewritten into (11), where G is a constant.

$$P_2 \qquad w_i^L = \max_{I \in IN}(G - w_i^I) \tag{11}$$

The optimization solution of P_2 and P_3 has the following properties (see Theorem 2).

Theorem 2. The P_2 and P_3 have the feasible solution if the INCM has the local satisfactory consistency. In addition, the optimization solution satisfies $w_i^L \geqslant \dfrac{1}{(n-1)a+1}$ and $w_i^U \leqslant \dfrac{1}{\dfrac{n-1}{a}+1}$, where n denotes the rank of the INCM and a denotes the maximal scale value.

Proof. There will be one random crisp comparison matrix satisfying $CR(A^I) \leqslant 0.1$ if the INCM has the local satisfactory consistency. Hence, the P_2 and P_3 have the feasible solution.

Let n alternatives be compared. Construct the matrix $A' = (a'_{ij})_{n\times n} = \begin{vmatrix} 1 & 1/a & 1/a & \cdots & 1/a \\ a & 1 & 1 & \cdots & 1 \\ a & 1 & 1 & \cdots & 1 \\ \cdots & \cdots & \cdots & \cdots & 1 \\ a & 1 & 1 & 1 & 1 \end{vmatrix}$.

It shows w'_1 is a minimum among n alternatives. That is, w_1^F from any one consistent comparison matrix A^F isn't less than w'_1. The weighs from the eigenvector method are close to the geometric mean. Hence, one can obtain $w'_1 = \dfrac{\sqrt[n]{\dfrac{1}{a^{n-1}}}}{(n-1)\sqrt[n]{a} + \sqrt[n]{\dfrac{1}{a^{n-1}}}} = \dfrac{1}{(n-1)a+1}$. Therefore, we can obtain $w_i^L \geqslant \dfrac{1}{(n-1)a+1}$.

Similarly, construct the reciprocal matrix $A'' = (a''_{ij})_{n\times n} = \begin{vmatrix} 1 & a & a & \cdots & a \\ 1/a & 1 & 1 & \cdots & 1 \\ 1/a & 1 & 1 & \cdots & 1 \\ \cdots & \cdots & \cdots & \cdots & 1 \\ 1/a & 1 & 1 & 1 & 1 \end{vmatrix}$. It shows w''_1 is a maximum among n alternatives. That is, w_1^F from any one consistent comparison matrix A^F isn't more than w'_1. One can obtain $w''_1 = \dfrac{\sqrt[n]{a^{n-1}}}{(n-1)\sqrt[n]{1/a} + \sqrt[n]{a^{n-1}}} = \dfrac{1}{\dfrac{n-1}{a}+1}$ as $CR(A'') \leqslant 0.1$. Therefore, we can obtain $w_i^U \leqslant \dfrac{1}{\dfrac{n-1}{a}+1}$.

In a word, the P_2 and P_3 have the feasible solution as $CR(A^I) \leqslant 0.1$. Moreover, we can obtain $\dfrac{1}{\dfrac{n-1}{a}+1} \geqslant w_i^I \geqslant \dfrac{1}{(n-1)a+1}$. □

In particular, $a = 9$ as one adopts 1∼9 scale, with $n = 4$. Then, one can obtain $w_i^L \geqslant 0.036$ and $w_i^U \leqslant 0.75$.

3.2 Possible value model

Owing to the consistency level of the comparison matrix can reflect the logical harmonization of the decision maker, one could conclude that the less CR, the better reliability on the weights if the decision maker did not have a prejudice for any particular alternative[13]. Based on this conclusion, the other model is developed. If there are many solutions in the feasible region, they must be corresponding to satisfactory random crisp comparison matrixes uniquely. Among these matrixes, there will be a comparison matrix A^g of which the CR is a minimum. Moreover, A^g is unique as long as the INCM is given reasonably. Since the decision-making reliability is higher based on the weights derived from A^g, one can take the weights from A^g as the most possible weights. Then, the weights estimation problem can be transformed into finding out A^g. Generate N random crisp comparison matrixes based on Definition 3, denoted as $A^I, I \in IN$. Solve the eigenvalue problem of A^I. Let $CR(A^g) = \min\{CR(A^I)|I \in IN\}$. And then, one can obtain A^g through solving the model P_4.

$$P_4 \qquad \min\{CR(A^I)|\ I \in IN\} \tag{12}$$

For convenience, (12) can be transformed into (13).

$$\arg\ \max G - CR(A^I) \tag{13}$$

The property of the P_4 can be referred to Theorem 3.

Theorem 3. The optimization solution is not less than zero. Moreover, if the INCM has the satisfactory consistency, one can conclude that $\min CR \leqslant 0.1$.

Proof. Omitted. □

If the optimization solution of the model P_4 is not less than 0.1, it shows the INCM has not the local satisfactory consistency. In this case, the decision maker should adjust judgments.

3.3 Integration with models P_2, P_3 and P_4

The weights of the INCM $\bar{A}$ can be expressed as $\bar{w}_i = [w_i^L, w_i^U]$, which depicts the exact weights range. However, the interval range is too large to guarantee the weights range integrality, which increases the uncertainty. Via model P_4, the decision maker can know the most possible weights, but this result does not consider the uncertainty. In this case, if we integrate these results, and express the weights as $\bar{w} = [w_i^L, w_i^g, w_i^U]$, then the integration result will have merits of models P_2, P_3 and P_4.

Owing to the comparability of $[w_i^L, w_i^g, w_i^U]$ and a triangle fuzzy number, one can adopt its arithmetic approach to obtain the synthesis weights result. Then, rank these interval numbers according to the approach from [14], simplified as

$$y(w_i) = \frac{w_i^L + 4w_i^g + w_i^U}{6}, \quad i = 1, \cdots, n \tag{14}$$

Based on the value of $y(w_i)$, one can obtain the rank of n alternatives.

4 Algorithm research

Since the relationship between $CR(A^I)$ and A^I is non-linear, the models P_2, P_3 and P_4 are non-linear programs. If the general approach is adopted, then the arithmetic is complex. In this paper, the genetic algorithm is designed. Define parameters as follows: N denotes the individual number of population size, k denotes the counter of generation, X denotes the chromosome, $f(X)$ denotes the fitness. Let $\bar{A} = ([a_{ij}^L, a_{ij}^U])_{n\times n}$. The genetic algorithm is designed as follows.

1) Code chromosome: Let $X = (a_{12}, a_{13}, \cdots, a_{ij}, \cdots, a_{n-1n})$, $1 \leqslant i < j \leqslant n$. The value of a_{ij} are randomly generated by the uniform distribution in $[a_{ij}^L, a_{ij}^U]$. In addition, one gene is corresponding to one entry of the upper triangular matrix and one chromosome is corresponding to one random crisp comparison matrix uniquely.

2) Fitness function: Let $f(X^I) = G - w_i^I - b$ denote the fitness of chromosome I as $CR(A^I) > 0.1$, where b denotes the non-feasible punishment factor. Generally, it is close to 1. Let $f(X^I) = G - w_i^I$ as $CR(A^I) \leqslant 0.1$. To the model P_4, the fitness is $f(X^I) = G - CR(A^I)$.

3) Crossover operator: The arithmetic crossover is adopted.

4) Mutation operator: The uniform mutation operator is adopted.

5) Selection operator: Models P_2, P_3 are to solve the standardization weights and P_4 is to solve the consistency ratio. Finesses of models P_2, P_3, and P_4 are not large. Hence, the wheel selection cannot differentiate these chromosomes. In this case, one can rank finesses of N chromosomes from small to large, and then, let them be the value of 1 to N accordingly. Then, the selection probability can be calculated as $p(I) = \dfrac{2I}{N(N+1)}$, for all $I = 1, \cdots, N$.

6) Stop rule: The iteration continues until a given number of generations is reached.

The algorithm can be described as follows.

Step 1. Generate the initialization population;

Step 2. Solve $f(X)$ and select N populations based on the selection operator. If the stop rule is satisfied then Stop; or go to Step 3;

Step 3. Generate N new populations *via* the crossover and mutation operators, then go to Step 2.

5 Example analysis

Example 1. Select one office director from four candidates (see [1]) .Suppose the following criteria are considered: capability, wisdom, relations, and constitution. The decision maker gives the INCM based on the capability criteria as example 1 in section 2.

Using the proposed GA, the result is: $\bar{w}_1 = [0.452, 0.5324, 0.5653]$, $\bar{w}_2 = [0.1721, 0.1975, 0.256]$, $\bar{w} = [0.115, 0.1425, 0.1975]$ and $\bar{w}_4 = [0.086, 0.1276, 0.1629]$. Rank these interval numbers, and obtain $y(w_1) = 0.524$, $y(w_2) = 0.203$, $y(w_3) = 0.147$ and $y(w_4) = 0.1266$. Then, the ranks of these candidates are $1 > 2 > 3 > 4$.

Example 2. The problem is the same as example 1, and the INCM is given as example 2 in section2. The weights are as follows: $\bar{w}_1 = [0.3687, 0.4843, 0.5370]$, $\bar{w}_2 = [0.1658, 0.2087, 0.2723]$, $\bar{w}_3 = [0.1016, 0.1372, 0.1780]$ and $\bar{w}_4 = [0.1433, 0.1698, 0.2452]$. Rank these interval numbers, and obtain $y(w_1) = 0.474$, $y(w_2) = 0.21254$, $y(w_3) = 0.138$ and $y(w_4) = 0.178$. Then, the ranks of these candidates are $1 > 2 > 4 > 3$.

6 Conclusion

The logical consistency level of the INCM can be tested according to the local satisfactory consistency and consistency degree suggested in this paper. Moreover, the weights model considers the consistency effects on the weights estimation, which is suitable for deriving the weights from the INCM with the low consistency degree specially. Most important, the model gives the weights upper range, the lower range and most possible value, which can provide the decision maker with more information.

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ZHU Jian-Jun Ph. D. candidate in School of Information Science and Engineering at Northeastern University. His research interests include multi-criteria analysis, system modelling and optimization.

LIU Shi-Xin Received his Ph. D. degree in School of Information Science and Engineering from Northeastern University. His research interests include multi-criteria analysis and decision support system.

WANG Meng-Guang Professor at Northeastern University. Her research interests include multi-criteria analysis, decision support system, and system modelling and optimization.