Robust Guaranteed Cost Observer for Uncertain Descriptor Time-delay Systems with Markovian Jumping Parameters[1)]

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Abstract This paper investigates the design of robust guaranteed cost observer for a class of linear descriptor time-delay systems with jumping parameters. The system under study involves time delays, jumping parameters and uncertainties. The transition of the jumping parameters in systems is governed by a finite-state Markov process. The objective is to design linear memoryless observers such that for all uncertainties, the resulting augmented system is regular, impulse free, robust stochastically stable independent of delays and satisfies the proposed guaranteed cost performance. Based on stability theory in stochastic differential equations, a sufficient condition on the existence of robust guaranteed cost observers is derived. Robust guaranteed cost observers are designed in terms of linear matrix inequalities. A convex optimization problem with LMI constraints is formulated to design the suboptimal guaranteed cost filters.

Key words Descriptor systems, Markovian jumping parameters, guaranteed cost observers, linear matrix inequalities, time-delay systems

1 Introduction

Descriptor systems capture the dynamic behavior of many natural phenomena, and have applications in many fields, such as network theory, robotics, and so on (see for example ([1~3]) and the references therein). Descriptor systems are also referred to as singular systems, implicit systems, generalized state-space systems or semi-state systems. During the past decades, many results on descriptor systems have been proposed and various methods obtained (see for example ([1~3]) and the references therein).

It is well known that stochastic modeling has come to play an important role in many branches of science and industry. An area of particular interest has been Markovian jump systems[4,5]. On the other hand, state estimation plays an important role in systems and control theory, signal processing and information fusion. Certainly, the most widely used estimation method is the well-known Kalman filtering[6,7]. A common feature in the Kalman filtering is that an accurate model is available. In some applications, however, when the system is subject to parameter uncertainties, the accurate system model is difficult to obtain. To overcome this difficulty, the guaranteed cost filtering approach has been proposed, which was introduced for guaranteeing the upper bound of guaranteed cost function[8]. Recently, the filtering problem for time-delay systems has been the focus. Robust H_∞ filtering for uncertain Markovian jump systems with mode-dependent time delays was proposed in [9]. In [10], guaranteed cost and H_∞ filtering for time-delay systems was presented in terms of LMIs.

In this paper, we address the design problem of the robust guaranteed cost observer for a class of uncertain descriptor time-delay systems with Markovian jumping parameters based on LMI method. The design problem proposed here is to design a memoryless observer such that for all uncertainties, the resulting augmented system is regular, impulse free, robust stochastically stable independent of delay and satisfies the proposed guaranteed cost performance.

2 Problem formulation

Consider the following descriptor time-delay systems with Markovian jumping parameters:

$$\begin{cases} E(\boldsymbol{r}(t))\dot{\boldsymbol{x}}(t) = A(\boldsymbol{r}(t),t)\boldsymbol{x}(t) + A_1(\boldsymbol{r}(t),t)\boldsymbol{x}(t-d) \\ \boldsymbol{y}(t) = C(\boldsymbol{r}(t),t)\boldsymbol{x}(t) + C_1(\boldsymbol{r}(r),t)\boldsymbol{x}(t-d) \\ \boldsymbol{x}(t) = \boldsymbol{\varphi}(t),\ \forall t \in [-d,0] \end{cases} \tag{1}$$

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where $\boldsymbol{x}(t) \in R^n$, and $\boldsymbol{y}(t) \in R^r$ are the state vector and the controlled output, respectively. d represents the state time-delay. For convenience, the input terms in system (1) has been omitted. $\boldsymbol{\varphi}(t) \in L_2[-d, 0]$ is a continuous vector-valued initial function. The random parameter $\boldsymbol{r}(t)$ represents a continuous-time discrete-state Markov process taking values in a finite set $N = \{1, 2, \cdots, s\}$ and having the transition probability matrix $\prod = [\pi_{ij}]_{i,j\in N}$. The transition probability from mode i to mode j is defined by

$$P\{\boldsymbol{r}(t+\Delta = j)|\boldsymbol{r}(t) = i\} = \begin{cases} \pi_{ij}\Delta + o(\Delta), & i \neq j \\ 1 + \pi_{ii}\Delta + o(\Delta), & i \neq j \end{cases} \tag{2}$$

where $\Delta > 0$ satisfies $\lim\limits_{\Delta\to 0} \dfrac{o(\Delta)}{\Delta} = 0, \pi_{ij} > 0$ is the transition probability from mode i to mode j and satisfies $\sum\limits_{i\neq j} \pi_{ij} = -\pi_{ii}$. The matrix $E(\boldsymbol{r}(t)) \in R^{n\times n}$ may be singular with $\text{rank}E(\boldsymbol{r}(t)) = n_{E(r(t))} \leqslant n$. $A(\boldsymbol{r}(t), t), A_1(\boldsymbol{r}(t), t), C(\boldsymbol{r}(t), t)$ and $C_1(\boldsymbol{r}(t), t)$ are matrix functions of the random jumping process $\{\boldsymbol{r}(t)\}$. To simplify the notion, the notation $A_i(t)$ represents $A(\boldsymbol{r}(t), t)$ when $\boldsymbol{r}(t) = i$. For example, $A_1(\boldsymbol{r}(t), t)$ is denoted by $A_{1i}(t)$, and so on. Further, for each $\boldsymbol{r}(t) = i \in N$, it is assumed that the matrices $A_i(t), A_{1i}(t), C_i(t)$ and $C_{1i}(t)$ can be described by the following form.

$$A_i(t) = A_i + \Delta A_i(t),\ A_{1i}(t) = A_{1i} + \Delta A_{1i}(t),\ C_i(t) = C_i + \Delta C_i(t),\ C_{1i}(t) = C_{1i} + \Delta C_{1i}(t) \tag{3}$$

where A_i, A_{1i}, C_i are C_{1i} known real coefficient matrices with appropriate dimensions. Time-varying matrices $\Delta A_i(t), \Delta A_{1i}(t), \Delta C_i(t)$ and $\Delta C_{1i}(t)$ represent norm-bounded uncertainties and satisfy

$$\begin{bmatrix} \Delta A_i(t) & \Delta A_{1i}(t) \\ \Delta C_i(t) & \Delta C_{1i}(t) \end{bmatrix} = \begin{bmatrix} M_{1i} \\ M_{2i} \end{bmatrix} F_i(t)[N_{1i} \quad N_{2i}] \tag{4}$$

where M_{1i}, M_{2i}, M_{1i} and N_{2i} are known constant real matrices of appropriate dimensions, which represent the structure of uncertainties, and $F_i(t)$ is an unknown matrix function with Lebesgue measurable elements and satisfies $F_i(t)F_i^{\mathrm{T}}(t) \leqslant I$.

Further, for convenience, it is assumed that the system has the same dimension at each mode and the Markov process is irreducible.

Consider the following nominal unforced descriptor system of (1) with a state delay:

$$\begin{cases} E_i\dot{\boldsymbol{x}}(t) = A_i\boldsymbol{x}(t) + A_{1i}\boldsymbol{x}(t-d) \\ \boldsymbol{x}(t) = \boldsymbol{\varphi}(t),\ \forall t \in [-d, 0] \end{cases} \tag{5}$$

Let $\boldsymbol{x}_0, \boldsymbol{r}_0$, and $\boldsymbol{x}(t, \varphi, r_0)$ be the initial state, initial mode and the corresponding solution the system (5) at time t, respectively. We have the following definition.

Definition 1. System (5) is said to be stochastically stable if for all $\varphi(t) \in L_2[-d, 0]$ and initial mode $r_0 \in N$, there exists a matrix $M > 0$ such that

$$E\Big\{\int_0^\infty \|\boldsymbol{x}(t, \varphi, r_0)\|^2 \mathrm{d}t | r_0, \boldsymbol{x}(t) = \boldsymbol{\varphi}(t), t \in [-d, 0]\Big\} \leqslant \boldsymbol{x}_0^{\mathrm{T}} M \boldsymbol{x}_0 \tag{6}$$

The following definition can be regarded as an extension of the definition in [2].

Definition 2. 1) System (5) is said to be regular if $\det(sE_i - A_i), i = 1, 2, \cdots, s$ are not identically zero.

2) System (5) is said to be impulse free if $\deg(\det(sE_i - A_i)) = \text{rank}E_i, i = 1, 2, \cdots, s$.

3) System (5) is said to be admissible if it is regular, impulse free and stochastically stable.

In this paper, the linear memoryless observer under consideration is of the form

$$\begin{cases} E_i\dot{\hat{\boldsymbol{x}}}(t) = K_{1i}\hat{\boldsymbol{x}}(t) + K_{2i}\boldsymbol{y}(t) \\ \hat{\boldsymbol{x}}_0 = 0 \end{cases} \tag{7}$$

where $\hat{\boldsymbol{x}}(t) \in R^n$ is the observer state, and the constant matrices K_{1i} and K_{2i} are observer parameters to be designed.

Denote the error state $\boldsymbol{e}(t) = \boldsymbol{x}(t) = \hat{\boldsymbol{x}}(t)$, and the augmented state vector $\boldsymbol{x}_f = [\boldsymbol{x}^{\mathrm{T}}(t) \quad \boldsymbol{e}^{\mathrm{T}}(t)]^{\mathrm{T}}$. Let $\boldsymbol{x}(t) = \boldsymbol{L}\boldsymbol{e}(t)$ represent the output of the error states, where L is a known constant matrix. Now, by defining

$$A_{fi} = \begin{bmatrix} A_i & 0 \\ A_i - K_{1i} - K_{2i}C_i & K_{1i} \end{bmatrix},\ A_{f1i} = \begin{bmatrix} A_{1i} & 0 \\ A_{1i} - K_{2i}C_{1i} & 0 \end{bmatrix},\ E_{fi} = \begin{bmatrix} E_i & 0 \\ 0 & E_i \end{bmatrix}$$

$$M_{fi} = M_{f1i} = \begin{bmatrix} M_{1i} \\ M_{1i} - K_{2i}M_{2i} \end{bmatrix} N_{fi} = [N_{1i} \quad 0],\ \Delta A_{fi} = M_{fi}F_iN_{fi},\ N_{f1i} = [N_{2i} \quad 0]$$
$$\Delta A_{f1i} = M_{f1i}F_iN_{f1i},\ C_f = [0 \quad L] \tag{8}$$

and combining (1) and (7), we obtain the augmented systems as follows.

$$\begin{cases} E_{fi}\dot{\boldsymbol{x}}_f(t) = (A_{fi} + \Delta A_{fi})\boldsymbol{x}_f(t) + (A_{f1i} + \Delta A_{f1i})\boldsymbol{x}_f(t-d) \\ \boldsymbol{z}(t) = C_f\boldsymbol{x}_f(t) \\ \boldsymbol{x}_{f0}(t) = [\boldsymbol{\varphi}^{\mathrm{T}}(t), \boldsymbol{\varphi}^{\mathrm{T}}(t)]^{\mathrm{T}},\ \forall t \in [-d, 0] \end{cases} \tag{9}$$

Similar to [5], it is also assumed in this paper that for all $\varsigma \in [-d, 0]$, there exists a scalar $h > 0$ such that $\|\boldsymbol{x}_f(t+\varsigma)\| \leqslant h\|\boldsymbol{x}_f(t)\|$.

Associated with system (9) is the cost function

$$J = E\Big\{\int_0^{\infty} \boldsymbol{z}^{\mathrm{T}}\boldsymbol{z}(t)\mathrm{d}t\Big\} \tag{10}$$

Based on Definition 1, we have the following definition.

Definition 3. Consider the augmented system (9), if there exist the observer parameters K_{1i}, K_{2i} and a positive scalar J^*, for all uncertainties, such that the augmented system (9) is robust stochastically stable and the value of the cost function (10) satisfies $J \leqslant J^*$, then J^* is said to be a robust guaranteed cost and observer (7) is said to be a robust guaranteed cost observer for system (1).

With the above description, the problem to be solved in this paper can be stated as follows.

Problem 1. (Robust guaranteed cost observer problem) Given system (1), determine the observer parameters K_{1i} and K_{2i} such that observer (7) is a robust guaranteed cost observer for system (1).

3 Main results

Theorem 1. Consider the augmented system (9) with the cost function (10). Then there exist parameters K_{1i} and K_{2i} that solve the addressed robust guaranteed cost observer problem if there exist matrices P_i, K_{1i} and $K_{2i}, i = 1, 2, \cdots, s$, and symmetric positive definite matrix Q, such that

$$E_{fi}^{\mathrm{T}}P_i = P_i^{\mathrm{T}}E_{fi} \geqslant 0 \tag{11}$$
$$\begin{bmatrix} \Pi_i + C_f^{\mathrm{T}}C_f & P_i(A_{f1i} + \Delta A_{f1i}) \\ (A_{f1i} + \Delta A_{f1i})^{\mathrm{T}}P_i & -Q \end{bmatrix} < 0,\ i = 1, 2, \cdots, s \tag{12}$$

where $\Pi_i = (A_{fi} + \Delta A_{fi})^{\mathrm{T}}P_i + P_i(A_{fi} + \Delta A_{fi}) + \sum_{j=1}^{s}\pi_{ij}E_{fj}^{\mathrm{T}}P_j + Q$.

Proof. Based on Definition 2 and Theorem 1 in [2], it follows from (11) and (12) that system (9) is regular and impulse free. Let the mode at time t be i, and consider the following positive definite function as a stochastic Lyapunov function of the augmented system (9)

$$V(\boldsymbol{x}_f(t), \boldsymbol{r}(t) = i) = \boldsymbol{x}_f^{\mathrm{T}}(t)E_{fi}^{\mathrm{T}}P_i\boldsymbol{x}_f(t) + \int_{t-d}^{t}\boldsymbol{x}_f^{\mathrm{T}}(t)Q\boldsymbol{x}_f\mathrm{d}t \tag{13}$$

where Q is the symmetric positive definite matrix to be chosen, and P_i is a matrix satisfying (12). The weak infinitesimal operator L of the stochastic process $\{\boldsymbol{r}(t), \boldsymbol{x}_f(t)\}, t \geqslant 0$, is given by

$$\begin{aligned} LV(\boldsymbol{x}_f(t), \boldsymbol{r}(t) = i) = \lim_{\Delta\to 0}\frac{1}{\Delta}[E\{V(\boldsymbol{x}(t+\Delta), r(t+\Delta))\boldsymbol{x}(t), \boldsymbol{r}(t) = i|\} - V(\boldsymbol{x}(t), \boldsymbol{r}(t) = i)] = \\ \boldsymbol{x}_f^{\mathrm{T}}(t)\Big\{\Big[(A_{fi} + \Delta A_{fi})^{\mathrm{T}}P_i + P_i(A_{fi} + \Delta A_{fi}) + \sum_{j=1}^{s}\pi_{ij}E_{fj}^{\mathrm{T}}P_j + Q\Big]\boldsymbol{x}_f(t) + \\ 2\boldsymbol{x}_f^{\mathrm{T}}(t)P_i(A_{f1i} + \Delta A_{f1i})\boldsymbol{x}_f(t-d) - \boldsymbol{x}_f^{\mathrm{T}}(t-d)Q\boldsymbol{x}_f(t-d) \end{aligned} \tag{14}$$

It follows from (14) that

$$LV(\boldsymbol{x}_f, t) \leqslant [\boldsymbol{x}_f^{\mathrm{T}}(t) \quad \boldsymbol{x}_f^{\mathrm{T}}(t-d)]\begin{bmatrix} \Pi_i & P_i(A_{f1i} = \Delta A_{f1i}) \\ (A_{f1i} + \Delta A_{f1i})^{\mathrm{T}}P_i & -Q \end{bmatrix}\begin{bmatrix} \boldsymbol{x}_f(t) \\ \boldsymbol{x}_f(t-d) \end{bmatrix} \tag{15}$$

Similar to [5], using Dynkin′s formula, we obtain for each $i \in N$:

$$\lim_{T\to\infty} E\Big\{\int_0^{\mathrm{T}} \boldsymbol{x}_f^{\mathrm{T}}\boldsymbol{x}_f(t)\mathrm{d}t|\varphi_f, r_0 = i\Big\} \leqslant \boldsymbol{x}_{f0}^{\mathrm{T}} M \boldsymbol{x}_{f0} \tag{16}$$

From Definition 1, it results in that the augmented system (9) is stochastically stable. Furthermore, from (12), we obtain

$$LV(\boldsymbol{x}_f(t), \boldsymbol{r}(t) = i) < -\boldsymbol{x}^{\mathrm{T}}(t)C_f^{\mathrm{T}} C_f \boldsymbol{x}(t) < 0 \tag{17}$$

On the other hand, we have

$$J = E\{\boldsymbol{x}_f^{\mathrm{T}}(t)C_f^{\mathrm{T}} C_f \boldsymbol{x}_f(t)\mathrm{d}t\} < -\int_0^{\infty} LV(\boldsymbol{x}_f(t), \boldsymbol{r}(t))\mathrm{d}t = -E\{\lim_{t\to\infty} V(\boldsymbol{x}(t), \boldsymbol{r}(t))\} + V(\boldsymbol{x}_0, \boldsymbol{r}_0) \tag{18}$$

As the augmented system (9) is stochastically stable, it follows from (18) that $J < V(\boldsymbol{x}_{f0}, r_0)$. From Definition 3, it concludes that a robust guaranteed cost for the augmented system (9) can be given by $J^* = \boldsymbol{x}_{f0}^{\mathrm{T}}(t)E_{fr_0}^{\mathrm{T}} P(\boldsymbol{r}_0)\boldsymbol{x}_{f0} + \int_{-d}^{0} \boldsymbol{x}_f^{\mathrm{T}}(t)Q\boldsymbol{x}_f(t)\mathrm{d}t$. □

In the following, based on the above sufficient condition, the design of robust guaranteed cost observers can be turned into the solvability of a system of LMIs.

Theorem 2. Consider system (9) with the cost function (10). If there exist matrices Y_{1i} and Y_{2i}, $i = 1, 2, \cdots, s$ positive scalars ε_i, $i = 1, 2, \cdots, s$, symmetric positive definite matrix Q, and the full rank matrices P_{2i}, and matrices $P_i = \mathrm{diag}(P_{1i}, P_{2i})$, $i = 1, 2, \cdots, s$, such that

$$E_{fi}^{\mathrm{T}} P_i = P_i^{\mathrm{T}} E_{fi} \geqslant 0 \tag{19}$$

$$\begin{bmatrix} \phi_{1i} & \phi_{2i} & \phi_{3i} \\ \phi_{2i}^{\mathrm{T}} & -Q & 0 \\ \phi_{3i}^{\mathrm{T}} & 0 & -\varepsilon_i I \end{bmatrix} < 0,\ i = 1, 2, \cdots, s \tag{20}$$

where

$$\phi_{1i} = \begin{bmatrix} P_{1i}A_i + A_i^{\mathrm{T}} P_{1i} & A_i^{\mathrm{T}} P_{2i} - Y_{1i}^{\mathrm{T}} - C_i^{\mathrm{T}} Y_{2i}^{\mathrm{T}} \\ P_{2i}A_i - Y_{1i} - Y_{2i}C_i & Y_{1i}^{\mathrm{T}} + Y_{1i} \end{bmatrix} + Q + C_f^{\mathrm{T}} C_f + \sum_{j=1}^{s} \pi_{ij} E_{fj}^{\mathrm{T}} P_j$$

and

$$\phi_{2i} = \begin{bmatrix} P_{1i}A_i & 0 \\ P_{2i}A_i - Y_{1i} - Y_{2i}C_i & 0 \end{bmatrix}, \quad \phi_{3i} = \begin{bmatrix} P_{1i}M_{1i} \\ P_{2i}M_{1i} - Y_{2i}M_{2i} \end{bmatrix}$$

then the addressed robust guaranteed cost observer problem is solvable. In this case, a suitable robust guaranteed cost observer in the form of (7) has parameters as follows.

$$K_{1i} = P_{2i}^{-1} Y_{1i}, \quad K_{2i} = P_{2i}^{-1} Y_{2i} \tag{21}$$

and J^* is a robust guaranteed cost for system (9).

Proof. Define

$$A_i = \begin{bmatrix} A_{fi}^{\mathrm{T}} P_i + P_i A_{fi} + \sum_{j=1}^{s} \pi_{ij} P_j + Q + C_f^{\mathrm{T}} C_f & P_i A_{1fi} \\ A_{fi}^{\mathrm{T}} P_i & -Q \end{bmatrix} \tag{22}$$

Then (12) is equivalent to

$$A_i + \begin{bmatrix} P_i M_{fi} \\ 0 \end{bmatrix} F_i [N_{fi} \quad N_{f1i}] + [N_{fi} \quad F_{f1i}]^{\mathrm{T}} F_i^{\mathrm{T}} \begin{bmatrix} P_i M_{fi} \\ 0 \end{bmatrix}^{\mathrm{T}} < 0 \tag{23}$$

By applying Lemma 2.4 in [12], (23) holds for all uncertainties F_i satisfying $F_i^{\mathrm{T}} F_i < I$ if and only if there exist positive scalars $\varepsilon_i, i = 1, 2, \cdots, s$, such that

$$A_i + \varepsilon_i^{-1} \begin{bmatrix} P_i M_{fi} \\ 0 \end{bmatrix} \begin{bmatrix} P_i M_{fi} \\ 0 \end{bmatrix}^{\mathrm{T}} + \varepsilon_i [N_{fi} \quad N_{f1i}]^{\mathrm{T}} [N_{fi} \quad N_{f1i}] < 0 \tag{24}$$

Let $P_i = \mathrm{diag}(P_{1i}, P_{2i})$, and using (21), we can conclude from Schur complement results that the above matrix inequalities are equivalent to the coupled LMIs (20). It further follows from Theorem 1 that J^* is a robust guaranteed cost for system (9). □

Remark 1. The solution of LMIs (19) and (20) parametrizes the set of the proposed robust guaranteed cost observers. This parametrized representation can be used to design the guaranteed cost observer with some additional performance constraints. By applying the methods in [11], the suboptimal guaranteed cost observer can be determined by solving a certain optimization problem. This is the following theorem.

Theorem 3. Consider system (9) with the cost function (10), and suppose that the initial conditions $\boldsymbol{r}_0$ and $\boldsymbol{x}_{f0}$ are known, if the following optimization problem

$$\min_{Q,P_{1i},P_{2i},\varepsilon_i,Y_{1i} \text{ and } Y_{2i}} J^* \quad \text{s.t. LMIs (19) and (20)} \tag{25}$$

has a solution $Q, P_{1i}, P_{2i}, \varepsilon_i, Y_{1i}$, and Y_{2i}, $i = 1, 2, \cdots, s$, then the observer (7) is a suboptimal guaranteed cost observer for system (1), where $J^* = \boldsymbol{x}_{f0}^{\mathrm{T}} E_{fr_0}^{\mathrm{T}} P(\boldsymbol{r}_0)\boldsymbol{x}_{f0} + \mathrm{tr}(\int_{-d}^{0} \boldsymbol{x}_{f0}(t)\boldsymbol{x}_{f0}(t)\boldsymbol{x}_{f0}^{\mathrm{T}}\mathrm{d}tQ)$.

Proof. It follows from Theorem 2 that the observer (7) constructed in terms of the solution $Q, P_{1i}, P_{2i}, \varepsilon_i, Y_{1i}$, and Y_{2i}, $i = 1, 2, \cdots, s$, is a robust guaranteed cost observer. By noting that

$$\int_{-d}^{0} \boldsymbol{x}_{f0}^{\mathrm{T}}(t)Q\boldsymbol{x}_{f0}(t)\mathrm{d}t = \int_{-d}^{0} \mathrm{tr}(\boldsymbol{x}_{f0}^{\mathrm{T}}(t)Q\boldsymbol{x}_{f0}(t)\mathrm{d}t = \mathrm{tr}(\int_{-d}^{0} \boldsymbol{x}_{f0}(t)\boldsymbol{x}_{f0}^{\mathrm{T}}(t)\mathrm{d}tQ) \tag{26}$$

it follows that the suboptimal guaranteed cost observer problem is turned into the minimization problem (25). □

Remark 2. Theorem 3 gives the suboptimal guaranteed cost observer conditions of a class of linear time-delay systems with Markovian jumping parameters based on the convex optimization problem with LMI constraints which can be easily solved by the LMI toolbox in MATLAB.

4 Conclusions

In this paper, the robust guaranteed cost observer problem for a class of uncertain descriptor time-delay systems with Markovian jumping parameters is studied by using LMI method. The uncertainty is time-varying and is assumed to be norm-bounded. Memoryless guaranteed cost observers are designed in terms of a set of linear coupled matrix inequalities. The suboptimal guaranteed cost observer is designed by solving a certain optimization problem.

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