The Lifting Technique for Sampled-data Systems: Useful or Useless?[1)]

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Abstract The lifting technique is now a well recognized tool for H_∞ design and analysis of sampled-data systems. However, the efficiency of the method depends on the structure of the problem. The structure of the H_∞ sensitivity problem is analyzed in this paper. And the constraints on the H_∞-optimization problem and on the design parameters in lifting design are also discussed. Under such constraints the resulting performance from the design is generally low. Therefore, the lifting technique can not be recommended as a synthesis tool for the sampled-data systems. An example is also given in the paper.

Key words Sampled-data system, lifting technique, weighted sensitivity, H_∞-optimization problem

1 Introduction

Bcause the intersample behavior can be considered, the lifting technique now becomes the first option for the H_∞ design of sampled-data systems. However, the general methodology for H_∞ optimization design can not be used as usual in lifting design, and the result is generally poor. Since the system sensitivity is usually an indispensable part of the H_∞ design, the H_∞ sensitivity problem is used in this note as a typical example to demonstrate the troubles with the lifting design.

2 The lifting technique

The lifting can be viewed as breaking up the continuous-time signal $f(t)$ into an infinite number of consecutive pieces $\hat{f}_k(t)$[1].

$$\hat{f}_k(t) = f(\tau k + t), \quad 0 \leqslant t \leqslant \tau \tag{1}$$

The sequence $\{\hat{f}_k\}$ is a discrete-time signal which, for each time k, is a function in $L_2[0,\tau]$.

Let $w(t)$ be the input to the system, and $\{\hat{w}_k\}$ its lifting. The state equation under the input $\{\hat{w}_k\}$ is

$$x(k\tau + t) = e^{At}x(k\tau) + \int_0^t e^{A(t-s)}B_1\hat{w}_k(s)\mathrm{d}s, \quad 0 \leqslant t \leqslant \tau \tag{2}$$

Let $x_k := x(k\tau)$; then (2) can be rewritten in the operator form as[1]

$$x_{k+1} = e^{A\tau}x_k + \Phi_b\hat{w}_k \tag{3}$$

where Φ_b is an operator, $\Phi_b : L_2[0,\tau] \to \mathbb{R}^x$, where x stands for the dimension of the signal x.

With similar computations, the state space realization of the lifted system $\hat{G}$ can be obtained in the operator form[1]. The next step in the lifting design is to transform $\hat{G}$ to an equivalent discrete-time plant

$$G_d = \left[\begin{array}{c|cc} A_d & B_{1d} & B_{2d} \\ \hline C_{1d} & 0 & D_{12d} \\ C_2 & 0 & 0 \end{array}\right] \tag{4}$$

3 The weighted sensitivity design

Fig. 1 shows the weighted sensitivity problem in H_∞ design, where W_1 is the weighting function, P is the plant, K is the sampled-data controller, and F is the antialiasing filter.

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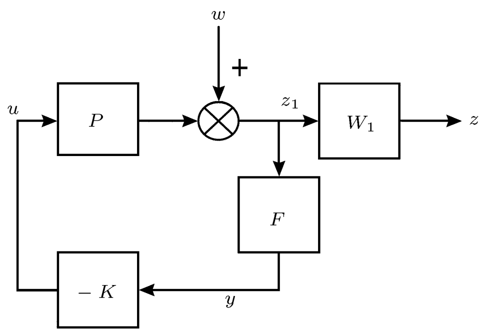

Fig. 1 The weighted sensitivity problem

The general H_∞ optimization design is to design a system such that the minimum of the weighted sensitivity is the nominal value, say 1. In other words, the objective of the design is to solve the following optimization problem

$$\min_K \|T_{zw}\|_\infty = \min_K \|W_1 S\|_\infty \leqslant 1 \tag{5}$$

where W_1 is a high-gain low-pass filter.

In the standard H_∞ problem, T_{zw} is represented by the linear fractional transformation (LFT) $F_l(G,K)$ as

$$T_{zw} = F_l(G,K) = G_{11} - G_{12}(I + KG_{22})^{-1}KG_{21} \tag{6}$$

In the sensitivity problem of Fig. 1, the LFT (6) is written as

$$T_{zw} = W_1(s) - W_1(s)T(s) \tag{7}$$

where $T(s)$ is the closed-loop transfer function of the system.

If the system is continuous-time, T_{zw} can further be reduced as follows.

$$T_{zw} = W_1(s)[I - T(s)] = W_1(s)S(s) \tag{8}$$

It means that this T_{zw} is really the weighted sensitivity of this system.

But for the sampled-data system, the lifted input $\{\hat{w}_k\}$ acts on the two different parts of (7): $W_1(s)$ and $T(s)$ (through the antialiasing filter). Because the two parts are different from each other by lifting, (7) can no longer be reduced to a weighted sensitivity by using a common factor W_1 as in (8).

Because the lifted signal $\{\hat{w}_k\}$ acts on two parts of (7), it directly affects the input matrix B_{1d} of (4). The entries of B_{1d} are related to the bandwidth of the corresponding part. If its bandwidth is wide, the integral in (2) will increase quickly, and the corresponding entries of B_{1d} become large. As for (7), the first part W_1 is of narrow bandwidth, the corresponding entries of B_{1d} are small, but for the second part, the corresponding entries increase significantly. Thus, the second part will play a dominant role, and the lifted T_{zw} shows like a weighted closed-loop transfer function W_1T, and has nothing to do with the sensitivity $S(j\omega)$. Therefore, in lifting design $W_1(s)$ must not be a low-pass function as usual, hence the optimization problem (5) can no longer be used in consequence.

The following is an alternative optimization problem which can be used in lifting design.

$$\min \|W_1 S\|_\infty \tag{9}$$

where W_1 is an ideal filter with wide bandwidth ω_0:

$$\begin{cases} |W_1(j\omega)| = 1, & \omega \leqslant \omega_0 \\ |W_1(j\omega)| = 0, & \omega > \omega_0 \end{cases} \tag{10}$$

The solution of (9) is a wide range of minimum $|S(j\omega)|$ over $0 \sim \omega_0$.

The bandwidth ω_0 of W_1 should be large, otherwise, the lifted T_{zw} can not reflect the sensitivity $S(j\omega)$ as mentioned above. It can be shown (by the following example) that when ω_0 is up to $\omega_N/5$, the lifted T_{zw} can present the property of a weighted sensitivity. Notice that the highest frequency in a sampled-data system is the Nyquist frequency ω_N(*i.e.*, $\omega_s/2$). Now because the bandwidth of W_1 is $\omega_0 = \omega_N/5$, the crossover frequency ω_c must lie between $0.2\omega_N \sim \omega_N$. That is to say, the $\ln|S(j\omega)|$ curve must intersect the 0 dB line in so narrow an interval. According to the Bode integrals[2,3], the negative area of $\ln|S(j\omega)|$ is equal to the positive area on the frequency band $0 \sim \omega_N$. Since ω_c must be large, the optimal performance $S_{\min}$ thus obtained is limited. It can not be as small as expected, *e.g.*, in reference [4] the optimal $S_{\min}$ was about 0.2∼0.6.

4 Design example

Suppose that the plant, the weighting and the filter in Fig. 1 are as follows.

$$P(s) = \frac{20-s}{(s+1)(s+20)}, \quad W_1(s) = \left(\frac{0.4\pi}{s+0.4\pi}\right)^2, \quad F(s) = \frac{2\pi}{s+2\pi}$$

Let the sampling period $\tau = 0.5$sec. According to the standard lifting approach[4], the equivalent discrete-time plant G_d [equation(4)] can be obtained as

$$G_d = \left[\begin{array}{ccccc|cccc}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1.9999 \\
0.0319 & 0.6065 & 0 & 0 & 0 & 0 & 0 & 0 & 0.3296 \\
0.0344 & 0.6736 & 0.0432 & 0.0138 & 0.0285 & 1.7683 & -0.0032 & 0.0033 & 0.2187 \\
0.0171 & 0.3611 & 0 & 0.5470 & 0.0299 & 0.6000 & 0.3041 & -0.0142 & 0.0752 \\
0.0048 & 0.1114 & 0 & 0.3411 & 0.5470 & 0.1000 & 0.1779 & 0.0265 & 0.0120 \\
\hline
0.0010 & 0.0271 & 0 & 0.1386 & 0.5439 & 0 & 0 & 0 & 0.0015 \\
0.0011 & 0.0269 & 0 & 0.0873 & -0.0240 & 0 & 0 & 0 & 0.0022 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1.0000 & 0 & 0 & 0 & 0 & 0 & 0
\end{array}\right] \tag{11}$$

The optimization problem (9) is then solved by using the MATLAB function *dhfsyn*. The resulting optimal H_∞ norm is $\gamma_{\min} = \min \|T_{zw}\|_\infty = 0.6451$. And the corresponding H_∞ controller is

$$K(z) = \frac{-3.7187(z-0.6065)(z-0.2708)(z-0.04321)(z-4.54e^{-0.05})}{(z+0.6552)(z+0.008101)(z^2-0.7998z+0.2372)} \tag{12}$$

Fig. 2 shows the hybrid simulation of the step response from w. The static error is 0.4168. It is rather large, and it is the actual minimum of the sensitivity, $S_{\min}$. This is because W_1 is an approximate ideal-filter, and the norm of W_1S is equal to the value of $S(j\omega)$ at $\omega = 0$. Therefore, the steady state value of the above mentioned step response is really the $S_{\min}$. Notice that the solution of the H_∞ optimization problem for the lifted system is $\gamma_{\min} = 0.6451$. Hence, the lifting design is also conservative.

5 Conclusion

In the H_∞ sensitivity problem, the lifting technique can only be used with the optimization problem (9), and the bandwidth of the weighting function W_1 can only be chosen as $\omega_0 = \omega_N/5$, so the bandwidth of the system is close to the Nyquist frequency ω_N with no other choices. Furthermore, according to the Bode integrals, the optimal performance obtained is quite limited. Furthermore, the resulting norm γ from the lifting design is also conservative.

Because the bandwidth of the lifting design is close to $\omega_N = \omega_s/2$, the magnitudes of the dominant poles are near $\omega_s/2$. This means that the time response may be a damped oscillation with frequency of $\omega_s/2$ (see Fig. 2), and it is also not required.

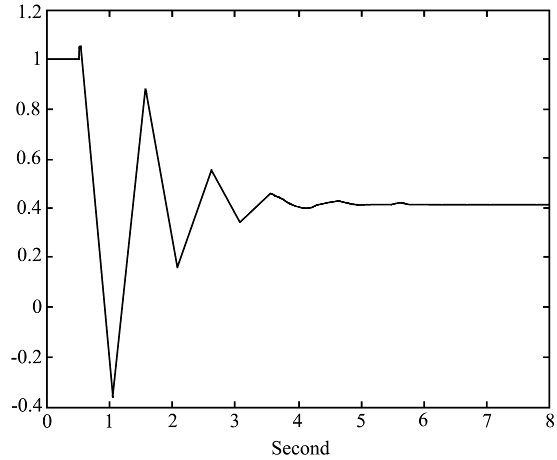

Fig. 2 Step response z_1 of the system

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