Multivariable Adaptive Decoupling Controller Using Hierarchical Multiple Models[1)]

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Abstract To solve the problem such as too many models, long computing time and so on, a hierarchical multiple models direct adaptive decoupling controller is designed. It consists of multiple levels. In the upper level, the best model is chosen according to the switching index. Then multiple fixed models are constructed on line to cover the region which the above chosen fixed model lies in. In the last level, one free-running and one re-initialized adaptive model are added to guarantee the stability and improve the transient response. By selection of the weighting polynomial matrix, it not only eliminates the steady output error and places the poles of the closed loop system arbitrarily, but also decouples the system dynamically. At last, for this multiple models switching system, global convergence is obtained under common assumptions. Compared with the conventional multiple models adaptive controller, it reduces the number of the fixed models greatly. If the same number of the fixed models is used, the system transient response and decoupling result are improved. The simulation example illustrates the power of the derived controller.

Key words Multiple models, hierarchical, direct adaptive control, decoupling, pole placement

1 Introduction

It is known that a system with unknown time invariant or slowly time varying parameters can get good performance by using an adaptive controller. But when the system boundary condition changes, the subsystem fails, or large external disturbance presents, the parameters of the system will change abruptly and large parameter errors will generally result in poor transient response[1]. To solve these problems, multiple models adaptive controller (MMAC) was proposed. For a continuous time system, Narendra *et al*. used multiple adaptive models to identify the unknown system parameters simultaneously[2]. These models were of different initial values and were used to cover the region where the system parameters changed. At any instant one best model was chosen according to the switching index and the corresponding controller was designed. However, after some instants multiple adaptive models would converge to a neighborhood and lose the power when the system parameters changed abruptly again. Thus, multiple fixed models with two adaptive models were used to overcome this problem[3]. In [4] the above results were extended to the discrete-time system. But all of these were dealt with the single input single output (SISO) system and adopted the indirect adaptive algorithm, which not only needed to solve equations on line but also degraded the robustness of the algorithm[1]. To solve this problem, multiple models direct adaptive decoupling controller (MMDADC) was proposed[5]. The interactions of the system variables were viewed as measurable disturbance and were eliminated using a feedforward strategy[6].

In an MMAC, to improve the transient response, a large number of fixed models are needed to cover the region where the system parameters change. In the simulation example in [7], when only one parameter changed, hundreds of fixed models must be used. This increased the computation, raised the system expense, even affected the choice of the sampling period of the discrete-time system. So how to reduce the number of the models is an important issue in MMAC, which blocks its practical use in the industrial process[8]. Zhivoglyadov *et al*. presented a localization method. At each instant, incorrect models were discarded on line to guarantee the global stability[8]. In [9] a method called Moving Bank was proposed. It tuned the center of the parameter set dynamically to cover the optimal estimation

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parameter. But all of these methods can only reduce the fixed models partially, and can not decrease the computation essentially.

In this paper a hierarchical multiple models adaptive decoupling controller (HMMADC), which adopts the direct adaptive algorithm, is presented. The hierarchical structure is utilized to cover the region where the parameters change dynamically and reduce the number of the fixed models greatly. The direct adaptive algorithm allows the controller to be chosen directly according to the switching index to decrease the computation. By selection of the polynomial matrix, it not only places the poles of the closed loop system arbitrarily but also decouples the system dynamically. The proof of the global convergence is obtained. Several simulation examples illustrate the effectiveness of the controller proposed.

2 Description of the System

The system to be controlled is a linear MIMO discrete-time system, which admits DARMA representation as

$$A(t,z^{-1})\boldsymbol{y}(t) = B(t,z^{-1})\boldsymbol{u}(t-k) + B_2(t,z^{-1})\boldsymbol{v}(t-k_2) + \boldsymbol{d}(t) \tag{1}$$

where $\boldsymbol{u}(t), \boldsymbol{v}(t), \boldsymbol{y}(t)$ are $n \times 1$ input, measurable disturbance, output vectors respectively, and $\boldsymbol{d}(t)$ is an $n \times 1$ vector denoting steady state output response for a zero input signal. k, k_2 are time delays respectively. $A(t,z^{-1}), B(t,z^{-1}), B_2(t,z^{-1})$ are polynomial matrixes in the unit delay operator z^{-1} and $B_0(t)$ is nonsingular, for any t.

The system satisfies assumptions as follows.

1) System parameters are time variant with infrequent large abrupt jumps. The period between two adjacent jumps is large enough to keep jumping parameters constant.

2) $\Phi(t) = [-A_1(t), \cdots; B_0(t), \cdots; B_{20}(t), \cdots; \boldsymbol{b}(t)]$ is the system model, which changes in a compact set Σ.

3) The upper bounds of the orders of $A(t,z^{-1}), B(t,z^{-1}), B_2(t,z^{-1})$ and the time delays k, k_2 are known a prior.

4) The system is minimum-phase.

From assumption 1), $A_i(t), B_j(t), B_{21}(t), \boldsymbol{d}(t)$ are piecewise constants. During the period when no jumps happen, (1) can be rewritten as follows without loss of generality

$$A(z^{-1})\boldsymbol{y}(t) = \boldsymbol{z}^{-1}\boldsymbol{u}(t-k) + B_2(z^{-1})\boldsymbol{v}(t-k_2) + d \tag{2}$$

3 Hierarchical multiple models adaptive controller design

To reduce the number of the fixed models, a novel MMADC using hierarchical structure is presented. Firstly, the region where the system parameters change is partitioned into several sub-regions. In each sub-region, one fixed model is designed and all these fixed models construct a multiple model set. At any instant, one best fixed model in this level is chosen according to the switching index and the corresponding sub-region is determined too. Then in the next level, partition the chosen sub-region into several sub-regions online and construct the fixed model set dynamically. Furthermore, in the last level, the best fixed model can be got. Finally to improve the transient response and guarantee the global stability, one free-running adaptive model and one re-initialized adaptive model are added. According to the switching index, the best model can be chosen. To realize the direct adaptive algorithm, all the above models are adopted such that the best controller can be chosen from these controller models directly.

3.1 Foundation of fixed system models

Definition 1. Matrix Φ, which is composed of the coefficient matrices of the matrix polynomials $A(z^{-1}), B(z^{-1}), B_2(z^{-1})$ and $\boldsymbol{d}$ is called the system model. As t changes, all $\Phi(t)$ compose the system parameter set Σ.

3.1.1 Foundation of the first level fixed system models

Utilizing the prior information, in the first level, Σ is partitioned into m_1 subsets $\Sigma_{1,s}, (s = 1, \cdots, m_1)$ and each has the following properties

1) $\bigcup_{s=1}^{m_1} \Sigma_{1,s} \supseteq \Sigma; \Sigma_{1,s}$ is not empty, $s = 1, \cdots, m_1$

2) For any $\Phi \in \Sigma_{1,s}, s = 1, \cdots, m_1$, there exists $\Phi_{1,s} \in \Sigma_{1,s}$ and $0 \leqslant r_{1,s} < \infty$ satisfying $\| \Phi - \Phi_{1,s} \| \leqslant r_{1,s}$. Φ is called the center of subset $\Sigma_{1,s}$ and $r_{1,s}$ is called its radius. The existence of $r_{1,s}$ is guaranteed by assumption 2).

From 1) and 2), it is known that in the first level, system parameter set Σ is covered by m_1 subsets $\Sigma_{1,s}$ and every subset is covered by the center $\Phi_{1,s}$ with its neighbor entirely. So centers of all subsets $\Phi_{1,s}(s = 1, \cdots, m_1)$ are set up as fixed system models in the first level to cover the system parameter set entirely.

3.1.2 Foundation of the i+1$^{\text{th}}$ level fixed system models

According to the switching index, in the i^{th} level, the j_i model Φ_{i,j_i} is chosen to be the best model as follows

$$\Phi_{i,j_i} = f(p_1, p_2, \cdots, p_{n_i}) \tag{3}$$

where $p_1, \cdots p_k, \cdots, p_{n_i}$ are system parameters. The foundation of the $i+1^{\text{th}}$ level fixed models is introduced as follows.

1) For the chosen model Φ_{i,j_i} in the i^{th} level, determine the interval where each system parameter p_k changes, *i.e.*, $p_k \in [p_k\_low,\ p_k\_high],\ k \in [1, n_i]$.

2) Partition the above interval into m_{i+1} sub-intervals. Then the h^{th} interval of the parameter p_k is as follows.

$$p_{k,h} = p_k\_low + h \times \frac{p_k\_high - p_k\_low}{\boldsymbol{m}_{i+1}}, \quad h = 1, 2 \cdots, m_{i+1} \tag{4}$$

3) Compose the h^{th} interval of all system parameters into one fixed model named the h^{th} fixed model $\Phi_{i+1,h}$ in the $i+1^{\text{th}}$ level, which covers the sub-region $\Sigma_{i+1,h}$ in the $i+1^{\text{th}}$ level with its neighborhood, *i.e.*, $\Phi_{i+1,h} = f(p_{1,h}, \cdots, p_{k,h}, \cdots, p_{n_i,h}), h = 1, 2, \cdots, m_{i+1}$.

4) Adopt all these m_{i+1} fixed models $\Phi_{i+1,1}, \cdots, \Phi_{i+1,m_i+1}$ to form the fixed model set in the $i+1^{\text{th}}$ level to cover the sub-region Σ_{i,j_i} in the i^{th} level.

Obviously, the subset has the following properties.

1) $\bigcup_{s=1}^{m_i+1} \Sigma_{i+1,s} \supseteq \Sigma_{i,j_i}, \Sigma_{i+1,s}$ is not empty, $s = 1, \cdots, m_{i+1}$;

2) For any $\Phi \in \Sigma_{i+1,s}, s = 1, \cdots, m_{i+1}$ there exists $\Phi_{i+1,s} \in \Sigma_{i+1,s}$ and $\exists 0 \leqslant r_{i+1,s} \leqslant r_{i,s}$ satisfying $\| \Phi - \Phi_{i+1,s} \| \leqslant r_{i+1,s}$. $\Phi_{i+1,s}$ is called the center of the subset $\Sigma_{i+1,s}$ and $r_{i+1,s}$ is called its radius. The existence of $r_{i+1,s}$ is guaranteed by assumption 2).

Similarly, the fixed models can be set up until the last level, *i.e.*, the l^{th} level, so the fixed system models using the hierarchical structure are founded.

3.2 Foundation of fixed controller models

Like the conventional optimal controller design[5], fixed controller models are derived from the fixed system models. The cost function to be considered is of the form

$$\boldsymbol{J}_c = \| P(z^{-1})\boldsymbol{y}(t+k) - R(z^{-1})\boldsymbol{w}(t) + Q(z^{-1})\boldsymbol{u}(t) + S(z^{-1})\boldsymbol{v}(t-k-k_2) + \boldsymbol{r} \|^2 \tag{5}$$

where $\boldsymbol{w}(t)$ is the known reference signal, and $P(z^{-1}), Q(z^{-1}), R(z^{-1}), S(z^{-1})$ are weighting polynomial matrixes, respectively. Introduce the identity as follows

$$P(z^{-1}) = F(z^{-1})A(z^{-1}) + z^{-k}G(z^{-1}) \tag{6}$$

In order to get unique polynomial matrixes $F(z^{-1}), G(z^{-1})$, orders of $F(z^{-1}), G(z^{-1})$ are chosen as

$$n_f = k - 1, \quad n_g = n_a - 1 \tag{7}$$

Multiplying (2) by $F(z^{-1})$ from left and using (6), the optimal control law can be derived as follows.

$$G(z^{-1})\boldsymbol{y}(t) + H(z^{-1})\boldsymbol{u}(t) + H_2(z^{-1})\boldsymbol{v}(t+k-k_2) + \bar{\boldsymbol{r}} = R(z^{-1})\boldsymbol{w}(t) \tag{8}$$

where

$$H(z^{-1}) = F(z^{-1})B(z^{-1}) + Q(z^{-1}) \tag{9}$$

$$H_2(z^{-1}) = F(z^{-1})B_2(z^{-1}) + S(z^{-1}) \tag{10}$$

$$\bar{\boldsymbol{r}} = F(1)\boldsymbol{d} + \boldsymbol{r} \tag{11}$$

Using (8) and (2), the closed loop system equation is as follows.

$$\begin{aligned}&[P(z^{-1})+Q(z^{-1})B^{-1}(z^{-1})A(z^{-1})]\boldsymbol{y}(t+k)=\\&[Q(z^{-1})B(z^{-1})^{-1}B_2(z^{-1})-S(z^{-1})]\boldsymbol{v}(t+k-k_2)+Q(1)B^{-1}(1)\boldsymbol{d}-\boldsymbol{r}\end{aligned}\tag{12}$$

For a minimum phase system, let

$$Q(z^{-1})=R_1B(z^{-1})\tag{13}$$

where R_1 is a constant matrix. To place poles arbitrarily, eliminate the steady state error and the effect of $\boldsymbol{d}$ exactly, let

$$P(z^{-1})+R_1A(z^{-1})=T(z^{-1})\tag{14}$$

$$R(z^{-1})=T(1)\tag{15}$$

$$S(z^{-1})=R_1B_2(z^{-1})\tag{16}$$

$$\boldsymbol{r}=R_1\boldsymbol{d}\tag{17}$$

The polynomial matrix $T(z^{-1})$ is assumed to be stable and has the form

$$T(z^{-1})=I+T_1z^{-1}+\cdots+T_{n_t}z^{-n_t}\tag{18}$$

where T_i is a diagonal matrix which is decided by the designer. Then the closed loop system is derived as

$$T(z^{-1})\boldsymbol{y}(t+k)=T(1)\boldsymbol{w}(t)\tag{19}$$

By selection of weighting polynomial matrixes, it not only decouples the system dynamically but also places those poles arbitrarily.

Definition 2. Matrix Θ is composed of the coefficient matrixes of $G(z^{-1}), H(z^{-1}), H_2(z^{-1}), \bar{\boldsymbol{r}}$ which is derived from system model Φ by using (13)∼(17), (6), (8)∼(11), is called the controller model. All $\Theta(t)$ derived from $\Phi(t)$ compose the controller parameter set Ω. The set $\Omega_{i,s}$, which is composed of the $\Theta(t)$ derived from the $\Phi(t)\in\Sigma_{i,s}(s=1,\cdots,m_i)$ in the i^{th} level, is called the controller parameter subset in the i^{th} level and $\Theta_{i,s}(s=1,\cdots,m_i)$ derived from $\Phi_{i,s}\in\Sigma_{i,s}(s=1,\cdots,m_i)$ is called the center of subset $\Omega_{i,s}$ in the i^{th} level.

3.4 Multiple models directly adaptive controller

Definition 3. Multiple controller models in the i^{th} level are composed of m_i fixed controller models $\Theta_{i,s}, s=1,\cdots,m_i, i=1,2,\cdots,l$. In the last level, *i.e.*, the $l+1^{\text{th}}$ level, multiple controller models are composed of the best fixed controller model chosen from the l^{th} level named $\Theta_{l+1,1}$, one free-running adaptive controller model $\Theta_{l+1,2}$ and one re-initialized adaptive controller model $\Theta_{l+1,3}$. The free-running adaptive controller model $\Theta_{l+1,2}$ is used to guarantee the stability of the system and the re-initialized adaptive controller model $\Theta_{l+1,3}$ is used to improve the transient response.

For the adaptive model, $\hat{\Theta}_{l+1,2}(t)$ and $\hat{\Theta}_{l+1,3}$, multiplying (2) by $F(z^{-1})+R_1$, and utilizing(6), (13)∼(14), (8)∼(11), (16)∼(17), the recursive estimation algorithm is described as follows

$$T(z^{-1})\boldsymbol{y}(t+k)=G(z^{-1})\boldsymbol{y}(t)+H(z^{-1})\boldsymbol{u}(t)+H_2(z^{-1})\boldsymbol{v}(t+k,-k_2)+\bar{\boldsymbol{r}}\tag{20}$$

$$\hat{\boldsymbol{\theta}}_i(t)=\hat{\boldsymbol{\theta}}_i(t-1)+\frac{a(t)\boldsymbol{X}(t-k)}{1+\boldsymbol{X}(t-k)^{\mathrm{T}}\boldsymbol{X}(t-k)}[y_{fi}(t)-\boldsymbol{X}(t-k)^{\mathrm{T}}\hat{\boldsymbol{\theta}}_i(t-1)]\tag{21}$$

where $y_{fi}=t_{ii}(z^{-1})y_i(t)$ is the auxiliary system output, $\boldsymbol{X}(t)=[\boldsymbol{y}(t)^{\mathrm{T}},\cdots;\boldsymbol{u}(t)^{\mathrm{T}},\cdots,\boldsymbol{v}(t+k-k_2)^{\mathrm{T}},\cdots,1]^{\mathrm{T}}$ is the data vector, $\Theta=[\theta_1,\cdots,\theta_n]$ is the controller parameter matrix and $\boldsymbol{\theta}_i=[g_{i1}^0,\cdots,g_{in}^0;g_{in}^1,\cdots;h_{i1}^0,\cdots,h_{in}^0;h_{i1}^1,\cdots,h_{in}^1,\cdots]^{\mathrm{T}}, i=1,2,\cdots,n$. The scalar $a(t)$ is designed to avoid the singularity problem when estimating $\hat{H}(0)$. If $\hat{H}(0)$ is singular, let $a(t)$ equal another constant value in the interval $\sigma<a(t)<2-\sigma, 0<\sigma<1$ to estimate $\hat{H}(0)$ again[10].

In the i^{th} level, $(i=1,2,l+1)$, the switching index is as follows

$$J_{i,s}=\frac{\left\|\underset{i,s}{\boldsymbol{e}_f}(t)\right\|^2}{1+\boldsymbol{X}(t-k)^{\mathrm{T}}\boldsymbol{X}(t-k)}=\frac{\left\|\boldsymbol{y}_f(t)-\underset{i,s}{\boldsymbol{y}_f}(t)\right\|^2}{1+\boldsymbol{X}(t-k)^{\mathrm{T}}\boldsymbol{X}(t-k)},\quad s=1,\cdots,m_i\tag{22}$$

where $\underset{i,s}{\boldsymbol{e}_f}(t)$ is the auxiliary output error in the i^{th} level between the real system and the model s. $\boldsymbol{y}_f(t) = T(z^{-1})\boldsymbol{y}(t) = \Theta_0^{\mathrm{T}}\boldsymbol{X}(t-k)$ is the auxiliary output of the system and Θ_0 is the real value of the system. $\underset{i,s}{\boldsymbol{y}_f}(t) = T(z^{-1})\boldsymbol{y}_{i,s}(t) = \Theta_{i,s}^{\mathrm{T}}\boldsymbol{X}(t-k)$ is the auxiliary output of the model s in the i^{th} level.

In the i^{th} level, let $j_i = \arg\min(J_{i,s})\ s = 1,\cdots,m_i, i = 1,2,\cdots,l$, correspond to the model Θ_{j_i} whose auxiliary output error is minimum; then it is chosen to be the best fixed controller model. In the last level, *i.e.*, $l+1^{\mathrm{th}}$ level, let $j_{t+1} = \arg\min(J_{l+1,s})\ s = 1,2,3$ correspond to the model $\Theta_{j_{l+1}}$ whose auxiliary output error is minimum; then it is chosen to be the controller.

1) If $j_{j+1} \neq 3$, which means $\hat{\Theta}_{l+1,3}(t)$ is not the minimum output error controller, then re-initialize $\hat{\Theta}_{l+1,3}(t)$ as the optimal controller parameter to improve the transient response, *i.e.*, $\hat{\Theta}_{l+1,3}(t) = \Theta_{l+1,j_{l+1}}$, $\hat{\Theta}_{l+1,2}(t)$ are estimated using (21) respectively and the controller is set as $\hat{\Theta}(t) = \Theta_{l+1,j_{l+1}}$.

2) If $j_{l+1} = 3$, then $\hat{\Theta}_{l+1,2}(t)$, $\hat{\Theta}_{l+1,3}(t)$ are estimated using (21) respectively and the controller is set as $\hat{\Theta}(t) = \hat{\Theta}_{l+1,3}(t)$.

The optimal control law can be obtained as

$$\hat{G}(z^{-1})\boldsymbol{y}(t) + \hat{H}(z^{-1})\boldsymbol{u}(t) + \hat{H}_2(z^{-1})\boldsymbol{v}(t+k-k_2) + \hat{\bar{\boldsymbol{r}}} = R(z^{-1})\boldsymbol{w}(t) \tag{23}$$

4 Global convergence analysis

Theorem 1. Subject to assumptions 1)-4), if algorithm (23) is applied to system (2), $\{\boldsymbol{y}(t)\}, \{\boldsymbol{u}(t)\}$ are bounded and $\lim_{t\to\infty} \| \boldsymbol{e}(t) \| = 0$.

Proof. a) If $\underset{l+1,1}{\boldsymbol{e}_f}(t) \neq 0$, which it means that when the controller with fixed parameters is adopted, there must exist an error such that the convergence property can not be guaranteed. Let $\varepsilon_i(t) = \min\left|\underset{l+1,1}{\boldsymbol{e}_f}(t)\right|$.

For the free running adaptive controller 2, the recursive estimation algorithm (21) has the property[10]

$$\lim_{t\to\infty} \frac{\underset{l+1,2i}{\boldsymbol{e}_f}(t)^2}{1+\boldsymbol{X}(t-k)^{\mathrm{T}}\boldsymbol{X}(t-k)} = 0 \tag{24}$$

Then there must exist an instant t_s, when $t > t_s$

$$\frac{\underset{l+1,2i}{\boldsymbol{e}_f}(t)^2}{1+\boldsymbol{X}(t-k)^{\mathrm{T}}\boldsymbol{X}(t-k)} \leqslant \frac{\varepsilon_i(t)^2}{1+\boldsymbol{X}(t-k)^{\mathrm{T}}\boldsymbol{X}(t-k)}\quad i = 1,\cdots,n \tag{25}$$

which means that after instant t_s, no fixed controller models can be chosen as the controller. The controller is selected between the free-running adaptive controller model 2 and the re-initialized adaptive controller model 3. According to the switching index (22), it follows that

$$0 \leqslant \frac{\boldsymbol{e}_{fi}(t)^2}{1+\boldsymbol{X}(t-k)^{\mathrm{T}}\boldsymbol{X}(t-k)} \leqslant \frac{\underset{l+1,2i}{\boldsymbol{e}_f}(t)^2}{1+\boldsymbol{X}(t-k)^{\mathrm{T}}\boldsymbol{X}(t-k)} \tag{26}$$

So

$$\lim_{t\to\infty} \frac{\boldsymbol{e}_i(t)^2}{1+\boldsymbol{X}(t-k)^{\mathrm{T}}\boldsymbol{X}(t-k)} = 0, i = 1,\cdots,n \tag{27}$$

Subject to the stability of $T(z^{-1})$, the minimum phase system and bounded $\boldsymbol{w}_i(t), v_i(t+k-k_2), d_i$, it is obtained that[10]

$$|u_i(t-k)| \leqslant K_1 + K_2 \max_{\substack{1\leqslant\tau\leqslant t\\1\leqslant j\leqslant n}} |y_j(\tau)|, 0 < K_1 < \infty, 0 < K_2 < \infty, 1 \leqslant t \leqslant N, i = 1,\cdots,n \tag{28}$$

$$|y_i(t)| \leqslant K_3 + K_4 \max_{\substack{1\leqslant\tau\leqslant t\\1\leqslant j\leqslant n}} |e_{fj}(\tau)|, 0 < K_3 < \infty, 0 < K_4 < \infty, 1 \leqslant t \leqslant N, i = 1,\cdots,n \tag{29}$$

$$|u_i(t-k)| \leqslant K_5 + K_6 \max_{\substack{1\leqslant\tau\leqslant t\\1\leqslant j\leqslant n}} |e_{fj}(\tau)|, 0 < K_5 < \infty, 0 < K_6 < \infty, 1 \leqslant t \leqslant N, i = 1,\cdots,n \tag{30}$$

According to lemma 3.1 in [10], it follows that $\{\boldsymbol{y}(t)\}, \{\boldsymbol{u}(t)\}$ is bounded and $\lim_{t\to\infty} \| \boldsymbol{e}_f(t) \| = 0$.

b) If $\boldsymbol{e}_{f\,l+1,1}(t) = 0$, that is, the parameter matrix $\Theta_{l+1,1}$ of the fixed controller equals the real value of system Θ_0 or $\Theta_{l+i,1} - \Theta_0$ is orthogonal to the data vector $\boldsymbol{X}(t-k)$. then $J_{l+1,1} = 0$ and Θ is chosen to be the controller. Then the case reduces to the single model case and switching stops.

Using a similar argument, subject to the stability of $T(z^{-1})$, the minimum phase system and bounded $w_i(t), v_i(t+k_2), d_i$, it is concluded that[10]

$$|y_i(t)| \leqslant K_7 + K_8 \max_{\substack{1\leqslant\tau\leqslant t \\ 1\leqslant j\leqslant n}} |e_{fj}(\tau)|, 0 < K_7 < \infty, 0 < K_8 < \infty, 1 \leqslant t \leqslant N, i = 1, \cdots, n \tag{31}$$

$$|u_i(t-k)| \leqslant K_9 + K_{10} \max_{\substack{1\leqslant\tau\leqslant t \\ 1\leqslant j\leqslant n}} |e_{fj}(\tau)|, 0 < K_9 < \infty, 0 < K_10 < \infty, 1 \leqslant t \leqslant N, i = 1, \cdots, n \tag{32}$$

Therefore, $\{\boldsymbol{y}(t)\}, \{\boldsymbol{u}(t)\}$ are bounded and $\| \boldsymbol{e}_f(t) \| = \left\| \boldsymbol{e}_{f\,l+1,1}(t) \right\| = 0$.

Considering the above cases together, it follows that $\{\boldsymbol{y}(t)\}, \{\boldsymbol{u}(t)\}$ is bounded and $\lim_{t\to\infty} \| \boldsymbol{e}_f(t) \| = 0$.

Considering the $\boldsymbol{e}_f(t) = T(z^{-1})\boldsymbol{e}(t)$ and $T(z^{-1})$ is stable, it is concluded that[11]

$$\lim_{t\to\infty} \| \boldsymbol{e}(t) \| = 0 \tag{33}$$

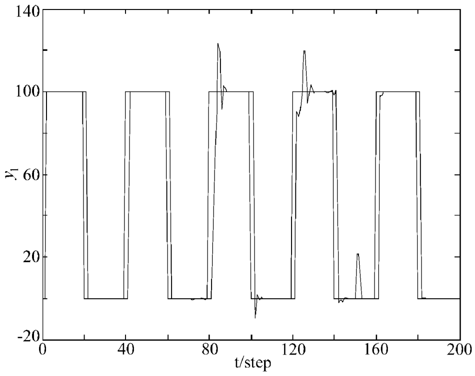
Therefore, the theorem holds. □

5 Simulation studies

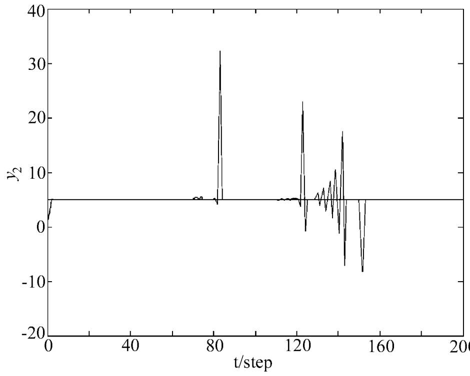
A linear multivariable system is described as follows

$$(I + A_1 z^{-1} + A_2 z^{-1})\boldsymbol{y}(t) = (B_0 + B_1 z^{-1})\boldsymbol{u}(t-2) + (B_{20} + B_{21} z^{-1})\boldsymbol{v}(t-2) + \boldsymbol{d} \tag{34}$$

where $A_1 = \begin{bmatrix} 0.2 & 0.5 \\ 0.5 & 0.2 \end{bmatrix}, A_2 = \begin{bmatrix} 0.03 & 0.04 \\ 0.04 & 0.03 \end{bmatrix}, B_0 = \begin{bmatrix} 1 & 2 \\ 2 & 10 \end{bmatrix}, B_1 = \begin{bmatrix} 0.2 & 0.3 \\ 0.3 & 0.2 \end{bmatrix}, B_{20} = \begin{bmatrix} 0.1 & 0.7 \\ 0.5 & 0.2 \end{bmatrix}$, $B_{21} = \begin{bmatrix} 0.2 & 0.3 \\ 0.5 & 0.1 \end{bmatrix}, \boldsymbol{d} = \begin{bmatrix} 0.2 \\ 0.5 \end{bmatrix}$. The measurable disturbance is a square wave with magnitude 2 and the known reference signal w_1 is set to be a square wave with magnitude 100, too. All the time w_2 is a constant value at 5. When $t = 80, b_{11}^0$ jumps to 0.125 abruptly and when $t = 150$ it jumps back to 9.998 again. Utilizing the prior information, the region where jumping parameter b_{11}^0 changes is evaluated at [0,10] and multiple fixed models are set up accordingly. Note that the real system model is not among fixed system models. Other parameters are estimated using the conventional adaptive control algorithm. In this paper, a hierarchical MMAC with three levels and 10 models in each level is proposed, which has 30 models totally and can represent 1000 models. In Fig. 1, the system response of an MMAC with 30 fixed models is shown, which has the same models as those in HMMAC in Fig. 3. In Fig. 2, the simulation result of an MMAC with 1000 fixed models is shown, which cover the same models as those in Fig. 3. The results show that when more fixed models are adopted, the transient response in Fig. 2 is much better than that in Fig. 1. For the HMMAC in Fig. 3, when the same number of the fixed models is used, the system transient response and decoupling result are improved (see Fig. 1). With same transient response and decoupling result the number of the fixed models in HMMAC is much less than that in MMAC (see Fig. 2).

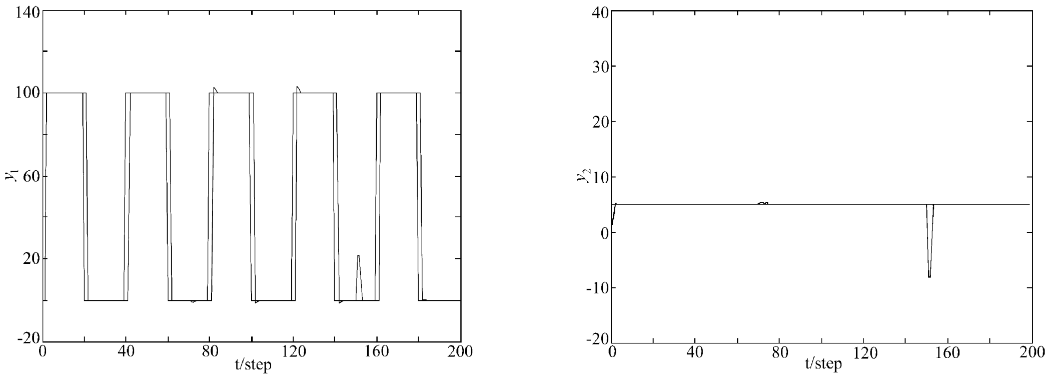


(a) The output $y_1(t)$ of the system



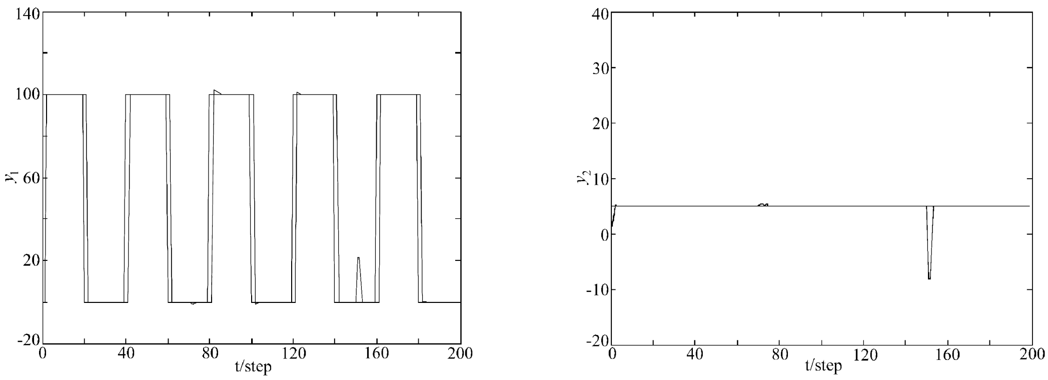
(b) The output $y_2(t)$ of the system

Fig. 1 Simulation of MMAC using 30 fixed models with 2 adaptive models



(a) The output $y_1(t)$ of the system (b) The output $y_2(t)$ of the system

Fig. 2 Simulation of MMAC using 1000 fixed models with 2 adaptive models



(a) The output $y_1(t)$ of the system (b) The output $y_2(t)$ of the system

Fig. 3 Simulation of MMAC using 30 fixed models with 2 adaptive models

6 Conclusion

In this paper, a hierarchical multiple models direct adaptive decoupling controller is designed. In each level, the best model is chosen according to the switching index. In the next level, multiple fixed models are set up to cover the range where the parameter jumps dynamically. When compared with the conventional HMMAC, it not only reduces the number of the fixed models greatly, but also improves the transient response.

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