Indirect Adaptive Fuzzy Output Feedback Control with Supervisory Controller for Uncertain Nonlinear Systems[1)]

TONG Shao-Cheng[1] CHAI Tian-You[2]

[1](*Department of Mathmetics and Physics, Liaoning Institute of Technology, Jinzhou* 121001)

[2](*Automation Research Center of Northeastern university, Shenyang* 110004)

(E-mail: jztsc@sohu.com)

Abstract In this paper, an indirect adaptive fuzzy output feedback controller with supervisory mode for a class of unknown nonlinear systems is developed. The proposed approach does not need the availability of the state variables, moreover, a supervisory controller is appended to the adaptive fuzzy controller to force the state to be within the constraint set. Therefore, if the adaptive fuzzy controller cannot maintain the stability, the supervisory controller starts to work to guarantee stability. On the other hand, if the adaptive fuzzy controller works well, the supervisory controller will be de-activated. The overall adaptive fuzzy control scheme guarantees the stability of the whole closed-loop systems. The simulation results confirm the effectiveness of the proposed method.

Key words Nonlinear systems, adaptive fuzzy supervisory control, observer, stability analysis

1 Introduction

Since L.A. Zadeh introduced the fuzzy set theory in 1965, it has received much attention from various fields and has also demonstrated good performance in various applications. One of those successful fuzzy applications is to model the unknown nonlinear systems by a set of fuzzy rules. One important property of fuzzy modeling approaches is that they are universal approximators[1]. In other words, fuzzy systems can be used to model virtually any nonlinear systems within a required accuracy provided that enough rules are given. Based on the universal approximation theorem and by incorporating fuzzy logic systems into adaptive control schemes, the stable direct and indirect fuzzy adaptive controllers are first proposed by Wang[1,2]. Afterwards, various adaptive fuzzy control approaches for nonlinear systems have been developed[3~6].

Generally, the direct and indirect adaptive fuzzy control approaches can have good performance. However, such approaches are based on the assumption that the state variables of the system are known, or available for feedback. If system states are not available, which could be common in practice, these adaptive fuzzy control approaches will not work and the adaptive fuzzy output feedback control using estimated states will be required. Using the state observer, an indirect adaptive fuzzy-neural controller was proposed by [7] for a class of unknown nonlinear systems, which utilized the separation property to prove the stability of the whole closed-loop systems. It is well known that in adaptive output feedback control based on observer design, a very key problem is that the stability of the whole system, with the adaptive fuzzy controller and the fuzzy observer, must be guaranteed in the case of nonlinear fuzzy model, thus, the approach of [7] lacks completeness and rigor in the stability analysis. Another type of direct adaptive fuzzy-neural control based on observer was developed by [8]. Although this control approach appended the supervisory controller to the adaptive fuzzy-neural control scheme to ensure the stability of the closed-loop system and achieved a good tracking performance, as pointed in [9], it lacks completeness and rigor in the stability analysis[8]. And utilizing the states of the system in designing the fuzzy controller contradicts the fact that the states of the system are not available in the case of output feedback control.

The main contribution of the paper is the development of the indirect adaptive fuzzy output control with supervisory controller for a class of SISO known nonlinear system. This indirect adaptive fuzzy control does not need the availability of the state variables, but only uses estimation of them.

1) Supported by National Natural Science Foundation of P. R. China (60274019) and National Key Basic Research and Development Program of P. R. China (2002CB312200)

Received February 15, 2003; in revised form June 14, 2004

Furthermore, based on the Lyapunov stability theorem, the stability of the whole closed-loop system with the fuzzy controller and the fuzzy observer is rigorously proved.

2 Description of the control systems and problem formulation

Consider an n-th order SISO nonlinear system of the form[1]:

$$x^{(n)} = f(x, \dot{x}, \ldots, x^{(n-1)}) + g(x, \dot{x}, \ldots, x^{(n-1)})u, \quad y = x \tag{1}$$

where f and g are unknown continuous functions, $\boldsymbol{x} = (x, \dot{x}, \ldots, x^{(n-1)}) \in R^n$ is the state vector of the system, $u \in R$ and $y \in R$ are the input and output of the system, respectively.

Rewrite (1) in the following form

$$\dot{\boldsymbol{x}} = A\boldsymbol{x} + \boldsymbol{B}(f(\boldsymbol{x}) + g(\boldsymbol{x})u), \quad y = \boldsymbol{C}^{\mathrm{T}}\boldsymbol{x} \tag{2}$$

where

$$A = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & \cdots & 1 \\ 0 & 0 & 0 & \cdots & 0 \end{bmatrix}, \ \boldsymbol{B} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}, \ \boldsymbol{C} = \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \\ 0 \end{bmatrix}$$

Let y_m be a bounded reference signal, and $e = y_m - y$ the output tracking error, and $\hat{\boldsymbol{x}}$ the estimate of $\boldsymbol{x}$. Denote

$$\boldsymbol{y}_m = [y_m, \dot{y}_m, \ldots, y_m^{(n-1)}]^{\mathrm{T}}, \ \boldsymbol{e} = \boldsymbol{y}_m - \boldsymbol{x} = [e, \dot{e}, \ldots, e^{(n-1)}]^{\mathrm{T}}$$
$$\hat{\boldsymbol{e}} = \boldsymbol{y}_m - \hat{\boldsymbol{x}} = [\hat{e}, \dot{\hat{e}}, \ldots, \hat{e}^{(n-1)}]^{\mathrm{T}}, \ \tilde{\boldsymbol{e}} = \boldsymbol{e} - \hat{\boldsymbol{e}}$$

Control objectives: Utilizing fuzzy systems, reference signal y_m and the output of the system to determine a fuzzy controller and an update law for adjusting the parameter vectors such that the following conditions are satisfied:

i) The closed-loop system is globally stable in the sense that all the variables involved must be uniformly bounded;

ii) The tracking error $\boldsymbol{e}$ and observation error $\tilde{\boldsymbol{e}}$ are as small as possible.

3 Observer-based adaptive fuzzy control with supervisory controller

Since the state $\boldsymbol{x}$ of system (1) is unavailable, design the control law as

$$u = u_c(\hat{\boldsymbol{x}}) + u_s(\hat{\boldsymbol{x}}) \tag{3}$$

where

$$u_c = \frac{1}{\hat{g}(\hat{\boldsymbol{x}}|\boldsymbol{\theta}_g)}[-\hat{f}(\hat{\boldsymbol{x}}|\boldsymbol{\theta}_f) + y_m^{(n)} + \boldsymbol{K}_c^{\mathrm{T}}\hat{\boldsymbol{e}}] \tag{4}$$

is the equivalent controller, and u_s is the supervisory controller which will be designed later. Substituting (4) into (2), after some manipulations, we obtain

$$\begin{aligned} x^{(n)} =& f(\boldsymbol{x}) + g(\boldsymbol{x})(u_c(\hat{\boldsymbol{x}})) + u_s(\hat{\boldsymbol{x}}) - \hat{g}(\hat{\boldsymbol{x}}|\boldsymbol{\theta}_g)u_c(\hat{\boldsymbol{x}}) + \hat{g}(\hat{\boldsymbol{x}}|\boldsymbol{\theta}_g)u_c(\hat{\boldsymbol{x}}) = \\ & y_m^{(n)} + \boldsymbol{K}_c^{\mathrm{T}}\hat{\boldsymbol{e}} + [f(\boldsymbol{x}) - \hat{f}(\hat{\boldsymbol{x}}|\boldsymbol{\theta}_f)] + [g(\boldsymbol{x}) - \hat{g}(\hat{\boldsymbol{x}}|\boldsymbol{\theta}_g)]u_c(\hat{\boldsymbol{x}}) + g(\boldsymbol{x})u_s(\hat{\boldsymbol{x}}) \end{aligned} \tag{5}$$

or equivalently

$$\begin{aligned} &\dot{\boldsymbol{e}} = A\boldsymbol{e} - B\boldsymbol{K}_c^{\mathrm{T}}\hat{\boldsymbol{e}} + \boldsymbol{B}[\hat{f}(\hat{\boldsymbol{x}}|\boldsymbol{\theta}_f) - f(\boldsymbol{x})] + [\hat{g}(\hat{\boldsymbol{x}}|\boldsymbol{\theta}_g) - g(\boldsymbol{x})]u_c(\hat{\boldsymbol{x}}) - \boldsymbol{B}g(\boldsymbol{x})u_s(\hat{\boldsymbol{x}}) \\ &\boldsymbol{e} = \boldsymbol{C}^{\mathrm{T}}\boldsymbol{e} \end{aligned} \tag{6}$$

Design the error observer as follows.

$$\dot{\hat{\boldsymbol{e}}} = A\hat{\boldsymbol{e}} - \boldsymbol{B}\boldsymbol{K}_c^{\mathrm{T}}\hat{\boldsymbol{e}} + \boldsymbol{K}_0(e - \hat{e}), \quad \hat{e} = \boldsymbol{C}^{\mathrm{T}}\hat{\boldsymbol{e}} \tag{7}$$

where $\boldsymbol{K}_0^{\mathrm{T}} = [k_1^0, k_2^0, \dots, k_n^0]$ is the observer gain vector to make sure that the characteristic polynomial of $A - \boldsymbol{K}_0\boldsymbol{C}^{\mathrm{T}}$ is a Hurwitz.

Define the observation error $\tilde{\boldsymbol{e}} = \boldsymbol{e} - \hat{\boldsymbol{e}}$. Subtracting (6) from (7) yields

$$\begin{aligned}\dot{\tilde{\boldsymbol{e}}} &= (A - \boldsymbol{K}_0\boldsymbol{C}^{\mathrm{T}})\tilde{\boldsymbol{e}} + \boldsymbol{B}[\hat{f}(\hat{\boldsymbol{x}}|\boldsymbol{\theta}_f) - f(\boldsymbol{x})] + [\hat{g}(\hat{\boldsymbol{x}}|\boldsymbol{\theta}_g) - g(\boldsymbol{x})]u_c(\hat{\boldsymbol{x}}) - \boldsymbol{B}g(\boldsymbol{x})u_s(\hat{\boldsymbol{x}}) \\ \tilde{e} &= \boldsymbol{C}^{\mathrm{T}}\tilde{\boldsymbol{e}}\end{aligned} \tag{8}$$

Consider the Lyapunov function

$$V_1 = \frac{1}{2}\tilde{\boldsymbol{e}}^{\mathrm{T}}P\tilde{\boldsymbol{e}} \tag{9}$$

where P is a positive definite solution of the following matrix equation

$$\begin{cases}(A - \boldsymbol{K}_0\boldsymbol{C}^{\mathrm{T}})^{\mathrm{T}}P + P(A - \boldsymbol{K}_0\boldsymbol{C}^{\mathrm{T}}) = -Q \\ P\boldsymbol{B} = \boldsymbol{C}\end{cases}$$

The time derivative of V_1 in (9) is

$$\begin{aligned}&V_1 = \frac{1}{2}\dot{\tilde{\boldsymbol{e}}}^{\mathrm{T}}P\tilde{\boldsymbol{e}} + \frac{1}{2}\tilde{\boldsymbol{e}}^{\mathrm{T}}P\dot{\tilde{\boldsymbol{e}}} = \\ &-\frac{1}{2}\tilde{\boldsymbol{e}}^{\mathrm{T}}Q\tilde{\boldsymbol{e}} + \tilde{\boldsymbol{e}}^{\mathrm{T}}P\boldsymbol{B}[\hat{f}(\hat{\boldsymbol{x}}|\boldsymbol{\theta}_f) - f(\boldsymbol{x}) + (\hat{g}(\hat{\boldsymbol{x}}|\boldsymbol{\theta}_g) - g(\boldsymbol{x}))u_c(\hat{\boldsymbol{x}})] - \tilde{\boldsymbol{e}}^{\mathrm{T}}P\boldsymbol{B}g(\boldsymbol{x})u_s(\hat{\boldsymbol{x}}) \leqslant \\ &-\frac{1}{2}\tilde{\boldsymbol{e}}^{\mathrm{T}}Q\tilde{\boldsymbol{e}} + |\tilde{\boldsymbol{e}}^{\mathrm{T}}P\boldsymbol{B}|\Big[|\hat{f}(\hat{\boldsymbol{x}}|\boldsymbol{\theta}_f)| + |f(\boldsymbol{x})| + |\hat{g}(\hat{\boldsymbol{x}}|\boldsymbol{\theta}_g)u_c(\hat{\boldsymbol{x}})| + |g(\boldsymbol{x})u_c(\hat{\boldsymbol{x}})|\Big] - \tilde{\boldsymbol{e}}^{\mathrm{T}}PBg(\boldsymbol{x})u_s(\hat{\boldsymbol{x}})\end{aligned} \tag{10}$$

In order to design the supervisory controller u_s and ensure $\dot{V}_1 \leqslant 0$, we make the following assumption.

Assumption 1. There exist functions $f^U(\boldsymbol{x})$, $g_L(\boldsymbol{x})$ and $g^U(\boldsymbol{x})$ such that

$$\begin{aligned}&|f(\boldsymbol{x})| \leqslant f^U(\boldsymbol{x}) \approx f^U(\hat{\boldsymbol{x}}) \\ &g_L(\hat{\boldsymbol{x}}) \approx g_L(\boldsymbol{x}) \leqslant g(\boldsymbol{x}) \leqslant g^U(\boldsymbol{x}) \approx g^U(\hat{\boldsymbol{x}})\end{aligned}$$

By observing (10), based on $f^U(\boldsymbol{x})$, $g_L(\boldsymbol{x})$ and $g^U(\boldsymbol{x})$, the supervisory controller u_s is chosen as

$$u_s(\hat{\boldsymbol{x}}) = I_1^*\mathrm{sgn}(\tilde{\boldsymbol{e}}^{\mathrm{T}}P\boldsymbol{B})\frac{1}{g_L(\hat{\boldsymbol{x}})}\Big[|\hat{f}(\hat{\boldsymbol{x}}|\boldsymbol{\theta}_f)| + f^U(\hat{\boldsymbol{x}}) + |\hat{g}(\hat{\boldsymbol{x}}|\boldsymbol{\theta}_g) + |g^U(\hat{\boldsymbol{x}})u_c(\hat{\boldsymbol{x}})|\Big] \tag{11}$$

where $I_1^* = 1$ if $V_1 \geqslant \bar{V}$ (which is a constant chosen by designer), $I_1^* = 0$ if $V_1 < \bar{V}$.

Considering the case $V_1 \geqslant \bar{V}$, and substituting (11) into (10) yields

$$\begin{aligned}&V_1 \leqslant -\frac{1}{2}\tilde{\boldsymbol{e}}^{\mathrm{T}}Q\tilde{\boldsymbol{e}} + |\tilde{\boldsymbol{e}}^{\mathrm{T}}PB|[|\hat{f}(\hat{\boldsymbol{x}}|\boldsymbol{\theta}_f)| + f^U(\hat{\boldsymbol{x}}) + |\hat{g}(\hat{\boldsymbol{x}}|\boldsymbol{\theta}_g)u_c(\hat{\boldsymbol{x}})| + |g^U(\hat{\boldsymbol{x}})u_c(\hat{\boldsymbol{x}})] - \\ &|\tilde{\boldsymbol{e}}^{\mathrm{T}}PB|[|\hat{f}(\hat{\boldsymbol{x}}|\boldsymbol{\theta}_f)| + f^U(\hat{\boldsymbol{x}}) + |\hat{g}(\hat{\boldsymbol{x}}|\boldsymbol{\theta}_g)u_c(\hat{\boldsymbol{x}})| + |g^U(\hat{\boldsymbol{x}})u_c(\hat{\boldsymbol{x}})|] \leqslant -\frac{1}{2}\tilde{\boldsymbol{e}}^{\mathrm{T}}Q\tilde{\boldsymbol{e}} \leqslant 0\end{aligned} \tag{12}$$

Therefore, we always have $V_1 < \bar{V}$ by using the supervisory controller $u_s(\hat{\boldsymbol{x}})$ (11). Because $P > 0$, the boundedness of V_1 implies the boundedness of $\tilde{\boldsymbol{e}}$, which in turn implies the boundedness of $\hat{\boldsymbol{e}}$. Moreover, it implies the boundedness of $\hat{\boldsymbol{x}}$. It is obvious that the supervisory controller $u_s(\hat{\boldsymbol{x}})$ is nonzero when V_1 is greater than a positive value $\bar{V}$. Hence, if the closed loop system with the fuzzy controller $u_c(\hat{\boldsymbol{x}})$ works well in the sense that the error is not large, *i.e.*, $V_1 < \bar{V}$, then the supervisory controller $u_s(\hat{\boldsymbol{x}})$ is zero. On the other hand, if the system tends to diverge, *i.e.*, $V_1 \geqslant \bar{V}$, then the supervisory controller $u_s(\hat{\boldsymbol{x}})$ begins to operate to force $V_1 < \bar{V}$.

Define the optimal parameter vector $\boldsymbol{\theta}_f^*$ and $\boldsymbol{\theta}_g^*$ as follows.

$$\boldsymbol{\theta}_f^* = \arg\min_{\boldsymbol{\theta}_f \in \Omega_1}\{\sup_{\boldsymbol{x} \in U_1,\, \hat{\boldsymbol{x}} \in U_2}|f(\boldsymbol{x}) - \hat{f}(\hat{\boldsymbol{x}}|\boldsymbol{\theta}_f)|\},\ \boldsymbol{\theta}_g^* = \arg\min_{\boldsymbol{\theta}_g \in \Omega_2}\{\sup_{\boldsymbol{x} \in U_1,\, \hat{\boldsymbol{x}} \in U_2}|g(\boldsymbol{x}) - \hat{g}(\hat{\boldsymbol{x}}|\boldsymbol{\theta}_g)|\}$$

where Ω_1, Ω_2, U_1 and U_2 denote the sets of suitable bounds on $\boldsymbol{\theta}_f$, $\boldsymbol{\theta}_g$, $\boldsymbol{x}$ and $\hat{\boldsymbol{x}}$, respectively. We also assume that $\boldsymbol{\theta}_f$, $\boldsymbol{\theta}_g$, $\boldsymbol{x}$ and $\hat{\boldsymbol{x}}$ never reach the boundaries of Ω_1, Ω_2, U_1 and U_2, respectively. Also the minimum approximation error is defined as

$$w = (\hat{f}(\hat{\boldsymbol{x}}|\boldsymbol{\theta}_f^*) - f(\boldsymbol{x})) + (\hat{g}(\hat{\boldsymbol{x}}|\boldsymbol{\theta}_g^*) - g(\boldsymbol{x}))u_c \tag{13}$$

Substituting (13) and (8) yields

$$\begin{aligned}&\dot{\tilde{\boldsymbol{e}}} = (A - \boldsymbol{K}_0\boldsymbol{C}^{\mathrm{T}})\tilde{\boldsymbol{e}} + \boldsymbol{B}[(\hat{f}(\hat{\boldsymbol{x}}|\boldsymbol{\theta}_f) - \hat{f}(\hat{\boldsymbol{x}}|\boldsymbol{\theta}_f^*)) + (\hat{g}(\hat{\boldsymbol{x}}|\boldsymbol{\theta}_g) - \hat{g}(\hat{\boldsymbol{x}}|\boldsymbol{\theta}_g^*))u_c + w] - \boldsymbol{B}g(\boldsymbol{x})u_s\\ &\tilde{e} = \boldsymbol{C}^{\mathrm{T}}\tilde{\boldsymbol{e}}\end{aligned} \tag{14}$$

Inserting $\hat{f}(\boldsymbol{x}|\boldsymbol{\theta}) = \boldsymbol{\theta}_f^{\mathrm{T}}\psi(\boldsymbol{x})$ and $\hat{g}(\boldsymbol{x}|\boldsymbol{\theta}) = \boldsymbol{\theta}_g^{\mathrm{T}}\psi(\boldsymbol{x})$ into (8), we obtain

$$\begin{aligned}&\dot{\tilde{\boldsymbol{e}}} = (A - \boldsymbol{K}_0\boldsymbol{C}^{\mathrm{T}})\tilde{\boldsymbol{e}} + B[\tilde{\boldsymbol{\theta}}_f^{\mathrm{T}}\psi(\hat{\boldsymbol{x}}) + \tilde{\boldsymbol{\theta}}_g^{\mathrm{T}}\psi(\hat{\boldsymbol{x}})u_c + w] - \boldsymbol{B}g(\boldsymbol{x})u_s\\ &\tilde{e} = \boldsymbol{C}^{\mathrm{T}}\tilde{\boldsymbol{e}}\end{aligned} \tag{15}$$

where $\tilde{\boldsymbol{\theta}_f} = \boldsymbol{\theta}_f - \boldsymbol{\theta}_f^*$ and $\tilde{\boldsymbol{x}}_g = \boldsymbol{\theta}_g - \boldsymbol{\theta}_g^*$ are the parameter error vectors.

It is assumed that positive definite matrices $\hat{P}$ and P are the solutions of the following equations

$$(A - \boldsymbol{B}\boldsymbol{K}_C^{\mathrm{T}})^{\mathrm{T}}\hat{P} + \hat{P}(A - \boldsymbol{B}\boldsymbol{K}_C^{\mathrm{T}}) + 2\hat{P}\boldsymbol{K}_c\boldsymbol{K}_c^{\mathrm{T}}\hat{P} = -Q \tag{16}$$

$$\begin{cases}(A - \boldsymbol{K}_0\boldsymbol{C}^{\mathrm{T}})^{\mathrm{T}}P + P(A - \boldsymbol{K}_0\boldsymbol{C}^{\mathrm{T}}) + 2\boldsymbol{C}\boldsymbol{C}^{\mathrm{T}} = -Q_1\\ P\boldsymbol{B} = \boldsymbol{C}\end{cases} \tag{17}$$

By noting that $\tilde{\boldsymbol{e}}^{\mathrm{T}}P\boldsymbol{B} = \boldsymbol{C}^{\mathrm{T}}\tilde{\boldsymbol{e}} = \tilde{e}$, and $\tilde{e} = y_m - y - \hat{e}$ are available for feedback control, the adaptive laws of parameter vectors $\boldsymbol{\theta}_f$ and $\boldsymbol{\theta}_g$ are chosen as

$$\dot{\boldsymbol{\theta}}_f = -\gamma_1\tilde{\boldsymbol{e}}^{\mathrm{T}}P\boldsymbol{B}\psi(\hat{\boldsymbol{x}}) = -\gamma_1\tilde{e}\psi(\hat{\boldsymbol{x}}) \tag{18}$$

$$\dot{\boldsymbol{\theta}}_g = -\gamma_2\tilde{\boldsymbol{e}}^{\mathrm{T}}P\boldsymbol{B}\psi(\hat{\boldsymbol{x}})u_c = -\gamma_2\tilde{e}\psi(\hat{\boldsymbol{x}})u_c \tag{19}$$

The main result of the observer-based indirect adaptive fuzzy control scheme is summarized in the following theorem.

Theorem. For the nonlinear system (1) if $\int_0^\infty |w|^2\mathrm{d}t \leqslant \infty$, and of the adaptive fuzzy control scheme is chosen as (4), (18) and (19), then the whole adaptive fuzzy control scheme guarantees the following properties:

i) $\boldsymbol{x}, \hat{\boldsymbol{x}}, \boldsymbol{e}, \hat{\boldsymbol{e}}, u \in L_\infty$. ii) $\lim_{t\to\infty}\tilde{\boldsymbol{e}} = 0, \lim_{t\to\infty}\boldsymbol{e} = 0$.

Proof. We choose the Lyapunov function as

$$V = \frac{1}{2}\hat{\mathbf{e}}^{\mathrm{T}}\hat{P}\hat{\mathrm{e}} + \frac{1}{2}\tilde{\boldsymbol{e}}^{\mathrm{T}}P\tilde{\mathrm{e}} + \frac{1}{2\gamma_1}\tilde{\boldsymbol{\theta}}_f^{\mathrm{T}}\tilde{\boldsymbol{\theta}}_f + \frac{1}{2\gamma_2}\tilde{\boldsymbol{\theta}}_g^{\mathrm{T}}\tilde{\boldsymbol{\theta}}_g \tag{20}$$

Accoding to (10) and (18), the time derivative of V is

$$\begin{aligned}\dot{V} =& \frac{1}{2}\hat{\mathbf{e}}^{\mathrm{T}}[(A - \boldsymbol{B}\boldsymbol{K}_C^{\mathrm{T}})^{\mathrm{T}}\hat{P} + \hat{P}(A - \boldsymbol{B}\boldsymbol{K}_C^{\mathrm{T}})]\hat{\mathrm{e}} + \hat{\mathbf{e}}^{\mathrm{T}}\hat{P}\boldsymbol{K}_0\boldsymbol{C}^{\mathrm{T}}\tilde{\boldsymbol{e}} +\\ &\frac{1}{2}\tilde{\boldsymbol{e}}^{\mathrm{T}}[(A - \boldsymbol{K}_0\boldsymbol{C}^{\mathrm{T}})^{\mathrm{T}}P + P(A - \boldsymbol{K}_0\boldsymbol{C}^{\mathrm{T}})]\tilde{\mathrm{e}} - \tilde{\boldsymbol{e}}^{\mathrm{T}}P\boldsymbol{B}g(\boldsymbol{x})u_s + \tilde{\boldsymbol{e}}^{\mathrm{T}}P\boldsymbol{B}w +\\ &(\tilde{\boldsymbol{e}}^{\mathrm{T}}P_1\boldsymbol{B}\tilde{\theta}_f^{\mathrm{T}}\psi(\hat{\boldsymbol{x}}) + \frac{1}{\gamma_1}\dot{\boldsymbol{\theta}}_f^{\mathrm{T}}\tilde{\boldsymbol{\theta}}_f) + (\tilde{\boldsymbol{e}}^{\mathrm{T}}P_1\boldsymbol{B}\tilde{\boldsymbol{\theta}}_f^{\mathrm{T}}\psi(\hat{\boldsymbol{x}})u + \frac{1}{\gamma_2}\dot{\boldsymbol{\theta}}_g^{\mathrm{T}}\tilde{\boldsymbol{\theta}}_g)\end{aligned} \tag{21}$$

From (16)∼(19) and based on the fact that $\tilde{\boldsymbol{e}}^{\mathrm{T}}P\boldsymbol{B}g(\boldsymbol{x})u_s \geqslant 0$, (21) becomes

$$\begin{aligned}\dot{V} \leqslant& \frac{1}{2}\hat{\mathbf{e}}^{\mathrm{T}}[(A - \boldsymbol{B}\boldsymbol{K}_C^{\mathrm{T}})^{\mathrm{T}}\hat{P} + \hat{P}(A - \boldsymbol{B}\boldsymbol{K}_C^{\mathrm{T}}) + 2\hat{P}\boldsymbol{K}_0\boldsymbol{K}_0^{\mathrm{T}}\hat{P}]\hat{\mathrm{e}} +\\ &\frac{1}{2}\tilde{\boldsymbol{e}}^{\mathrm{T}}[(A - \boldsymbol{K}_0\boldsymbol{C}^{\mathrm{T}})^{\mathrm{T}}P + P(A - \boldsymbol{K}_0\boldsymbol{C}^{\mathrm{T}}) + \boldsymbol{C}\boldsymbol{C}^{\mathrm{T}}]\tilde{\mathrm{e}} + \tilde{\boldsymbol{e}}^{\mathrm{T}}P\boldsymbol{B}w \leqslant\\ &-\frac{1}{2}\hat{\mathbf{e}}^{\mathrm{T}}Q\hat{\mathbf{e}} - \frac{1}{2}\tilde{\mathbf{e}}^{\mathrm{T}}Q_1\tilde{\boldsymbol{e}}^{\mathrm{T}}P\boldsymbol{B}w\end{aligned} \tag{22}$$

After some manipulations for (22), we obtain

$$\begin{aligned}\dot{V} \leqslant& -\frac{1}{2}\hat{\mathbf{e}}^{\mathrm{T}}Q\hat{\mathbf{e}} - \frac{1}{2}\tilde{\mathbf{e}}^{\mathrm{T}}Q_1\tilde{\mathbf{e}} + \frac{1}{2}\tilde{\mathbf{e}}^{\mathrm{T}}\tilde{\mathbf{e}} - \frac{1}{2}[\tilde{\mathbf{e}}^{\mathrm{T}}\tilde{\mathbf{e}} - 2\tilde{\mathbf{e}}^{\mathrm{T}}P\boldsymbol{B}w + w^{\mathrm{T}}\boldsymbol{B}^{\mathrm{T}}PP\boldsymbol{B}w] + \frac{1}{2}w^{\mathrm{T}}\boldsymbol{B}^{\mathrm{T}}PP\boldsymbol{B}w \leqslant\\ &-\frac{\lambda_{\min}(Q)}{2}\|\hat{\mathbf{e}}\|^2 - \frac{\lambda_{\min}(Q_1) - 1}{2}\|\tilde{\mathbf{e}}\|^2 + \frac{1}{2}\|P\boldsymbol{B}w\|^2\end{aligned} \tag{23}$$

Let $\alpha = \min\left\{\frac{\lambda_{\min}(Q)}{2}, \frac{\lambda_{\min}(Q_1)-1}{2}\right\}$, $\beta = \|P\boldsymbol{B}\|$, and $\boldsymbol{E}^{\mathrm{T}} = [\hat{\boldsymbol{e}}^{\mathrm{T}}, \tilde{\boldsymbol{e}}^{\mathrm{T}}]$. Then (23) can be written as

$$\dot{V} \leqslant -\alpha\|\boldsymbol{E}\|^2 + \beta\|w\|^2 \tag{24}$$

Integrating the above equation from 0 to T yields

$$\int_0^t \|\boldsymbol{E}\|^2 \mathrm{d}t \leqslant \frac{1}{\alpha}V(0) + \frac{\alpha}{\beta}\int_0^t \|w\|^2 \mathrm{d}t \tag{25}$$

Since $w \in L_2$ and via the same argument as [2], we can establish that $\boldsymbol{e}, \hat{\boldsymbol{e}}, \boldsymbol{x}, \hat{\boldsymbol{x}}, u \in L_\infty$, $\lim_{t\to\infty} \hat{\boldsymbol{e}} = 0$ and $\lim_{t\to\infty} \tilde{\boldsymbol{e}} = 0$.

Therefore, the theorem holds. □

4 Simulation

Consider the following dynamic equations of an inverted pendulum system

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix} = \begin{bmatrix}0 & 1\\ 0 & 0\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix} + \begin{bmatrix}0\\ 1\end{bmatrix}(f+gu)$$
$$y = [1 \quad 0]\begin{bmatrix}x_1\\ x_2\end{bmatrix} \tag{26}$$

where

$$f = \frac{mlx_2\sin x_1\cos x_1 - (M+m)g\sin x_1}{ml\cos^2 x_1 - \frac{4}{3}l(M+m)}, \quad g = \frac{-\cos x_1}{ml\cos^2 x_1 - \frac{4}{3}l(M+m)}$$

and $g = 9.8m/s^2$, $m = 0.1kg$, $M = 1kg$, $l = 0.5m$. It is obvious that $0 < g < \infty$. We first determine the bounds f^U, g^U and g_L as follows.

$$|f(x_1,x_2)| \leqslant \frac{9.8 + 0.025x_2^2}{2/3 - 0.05/1.1} = 15.78 + 0.0366x_2^2 \approx 15.78 + 0.0366\hat{x}_2^2 = f^U(\hat{x}_1,\hat{x}_2)$$
$$|g(x_1,x_2)| \leqslant 1.46 = g^U(x_1,x_2) \approx g^U(\hat{x}_1,\hat{x}_2)$$

If we require that $|x_1| \leqslant \pi/6$, then

$$|g(x_1,x_2)| \geqslant 1.12 = g_L(x_1,x_2) \approx g_L(\hat{x}_1,\hat{x}_2)$$

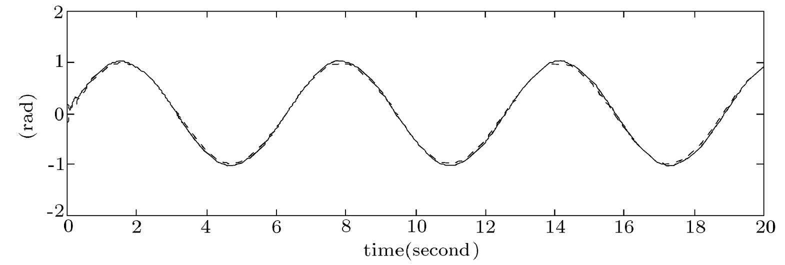
The reference signal is assumed to be $y_m = \sin t$. In the implementation, five fuzzy sets are defined over interval $\left[-\frac{\pi}{6}, \frac{\pi}{6}\right]$ for both $\hat{x}_1$ and $\hat{x}_2$, and their membership functions are

$$\mu_{N_1}(\hat{x}_i) = \exp\Big[-\Big(\frac{\hat{x}_i + \pi/6}{\pi/24}\Big)^2\Big],\ \mu_{N_2}(\hat{x}_i) = \exp\Big[-\Big(\frac{\hat{x}_i + \pi/12}{\pi/24}\Big)^2\Big]$$
$$\mu_O(\hat{x}_i) = \exp\Big[-\Big(\frac{\hat{x}_i}{\pi/24}\Big)^2\Big],\ \mu_{P_1}(\hat{x}_i) = \exp\Big[-\Big(\frac{\hat{x}_i 1\pi/12}{\pi/24}\Big)^2\Big]$$
$$\mu_{P_2}(\hat{x}_i) = \exp\Big[-\Big(\frac{\hat{x}_i - \pi/6}{\pi/24}\Big)^2\Big]$$

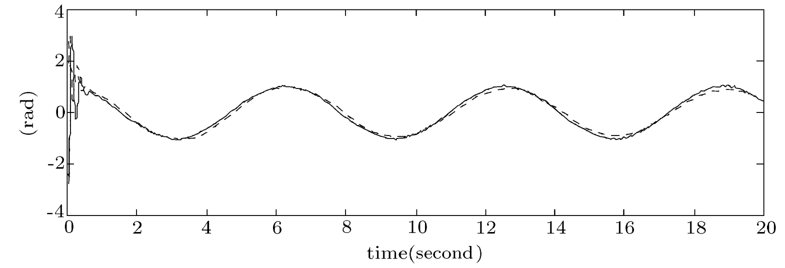
The observer and feedback gain vectors are chosen as $\boldsymbol{K}_0^{\mathrm{T}} = [40, 700]$, $\boldsymbol{K}_c^{\mathrm{T}} = [100, 10]$, respectively. Select positive definite matrices $Q = \begin{bmatrix}10 & 13\\ 13 & 28\end{bmatrix}$, $Q_1 = \begin{bmatrix}10 & 0\\ 0 & 10\end{bmatrix}$. Then after solving the first equations of (16) and (17), respectively, we obtain the positive definite matrices

$$\hat{P} = \begin{bmatrix}51 & 0.05\\ 0.05 & 0.504\end{bmatrix},\ P = \begin{bmatrix}74 & -5\\ -5 & 0.46\end{bmatrix}$$

The initial condition is assumed to be $x_1 = (0) = x_2(x) = 2$, $\hat{x}_1(0) = \hat{x}_2(0) = 1.5$, $\boldsymbol{\theta}_f(0) = \boldsymbol{0}$ and $\boldsymbol{\theta}_g(0) = 0.2 \times I$. Adaptation gains are chosen as $\gamma_1 = 200$ and $\gamma_2 = 0.5$, $\bar{V} = 1.5$. The simulation results are shown in Fig. 1∼Fig. 3



(a) State x_1 and its estimate $\hat{x}_1$



(b) State x_2 and its estimate $\hat{x}_2$

Fig. 1 Trajectories of the states and their estimates of the system

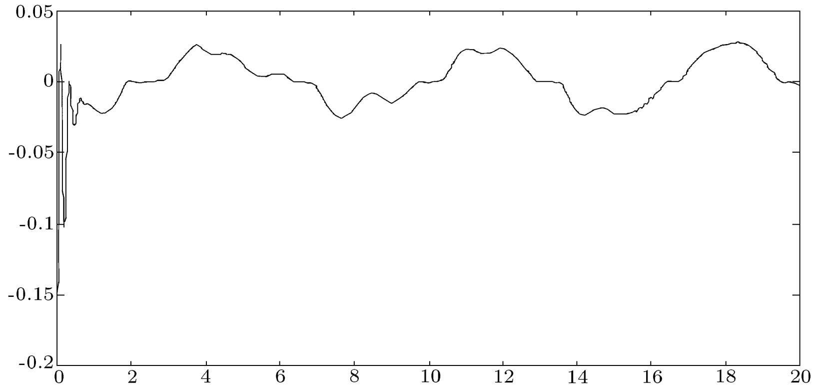


Fig. 2 The tracking error $e_1 = y_m - x_1$

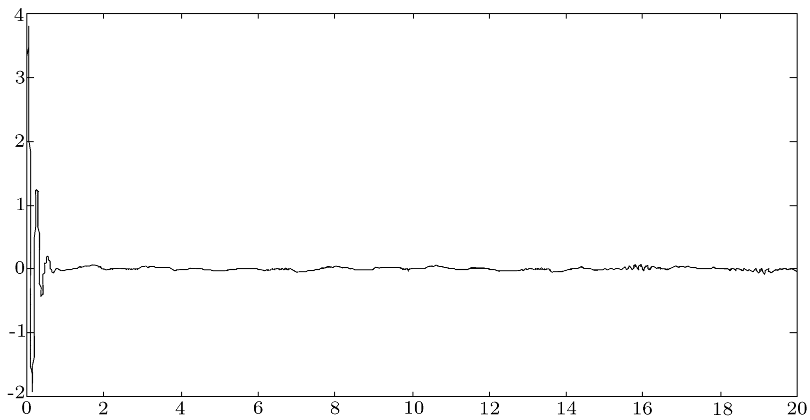


Fig. 3 The tracking error $e_2 = \dot{y}_m - x_2$

5 Conclusion

The adaptive fuzzy controllers, called observer-based indirect adaptive fuzzy output control with supervisory controller is proposed in this paper. Since the state variables of nonlinear systems are assumed to be unavailable, the state observer is first designed to estimate state variables, and fuzzy control schemes are then formulated. Based on the Lyapunov stability theorem, it is rigorously proved that the stability of the whole closed-loop system is assured and the tracking performance is achieved.

References

1 Wang L X. Adaptive fuzzy systems and control: Design and stability analysis. Englewood Cliffs, NJ: Prence-Hall, 1994, 171∼179

2 Wang L X. Stable adaptive fuzzy control of nonlinear systems. *IEEE Transactions on Fuzzy Systems*, 1993, **1**(2): 146∼155

3 Chen B S, Lee C H, Chang Y C. Tracking design of uncertain nonlinear SISO systems: adaptive fuzzy approach, *IEEE Transactions on Fuzzy Systems*, 1996, **4**(2): 32∼43

4 Spooner J T, Passino K M. Stable adaptive control of a class of nonlinear systems and neural network. *IEEE Transactions on Fuzzy Systems*, 1996, **4**(4): 339∼359

5 Zhang Nai-rao, Jin Hui. Research and improvement for the scheme of adaptive fuzzy control, *Acta Automatica Sinica*, 1997, **23**(2): 160∼166

6 Zhang Tian Ping, Feng Chun-bo. Fuzzy adaptive sliding-mode control for a class of nonlinear systems, *Acta Automatica Sinica*, 1997, **23**(3): 361∼369

7 Leu Y G, Wang W Y. Observer-based adaptive fuzzy-neural control for unknown nonlinear dynamical systems, *IEEE Transactions on Systems*, Man, and Cybernetics, 1999, **29**(3): 583∼591

8 Wang Chi-Hsu, Liu Han-Leih, Lin Tsung-Chih. Direct adaptive fuzzy-neural control with state observer and supervisory controller for unknown nonlinear dynamical systems, *IEEE Transactions on Fuzzy Systems*, 2002, **10**(1): 39∼49

9 Tong S C, Li H X, Wang W. Comment on Direct adaptive fuzzy-neural control with state observer and supervisory controller for unknown nonlinear dynamical systems, *IEEE Transactions on Fuzzy Systems*, 2003, **11**(5): 703∼705

TONG Shao-Cheng Received his master and Ph. D. degrees from Dalian Marine University in 1988 and Northeastern University in 1997, respectively. His research interests include fuzzy control theory, nonlinear adaptive control and intelligent control.

CHAI Tian-You Received his Ph. D. degrees from Northeastern University in 1984. His research interests include modeling, optimization and control for complex industrial processes.