Robust and Non-fragile H_∞ Control for a Class of Uncertain Jump Linear Systems

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Abstract This paper describes the synthesis of robust and non-fragile H_∞ state feedback controllers for a class of uncertain jump linear systems with Markovian jumping parameters and state multiplicative noises. Under the assumption of a complete access to the norm-bounds of the system uncertainties and controller gain variations, sufficient conditions on the existence of robust stochastic stability and γ-disturbance attenuation H_∞ property are presented. A key feature of this scheme is that the gain matrices of controller are only based on l_t, the observed projection of the current regime r_t.

Key words Robust and non-fragile H_∞ control, Markovian jumping parameters, exponential mean-square stability

1 Introduction

A great deal of attention has recently been devoted to the uncertain jump linear systems with Markovian jumping parameters[1~4]. In this paper, we study the robust non-fragile H_∞ control problem for a class of uncertain jump linear systems with state multiplicative noises. Two observation errors – false alarms and detection delays of the jumping parameters are considered. Under the condition of exactly knowing the norm bound of the system uncertainties, sufficient conditions on the existence of robust stochastic stability and γ-disturbance attenuation H_∞ property are presented. To facilitate the solution process, the linear matrix inequality (LMI) approach will be employed in the present development by using Schur complement.

2 Model description and preliminaries

Consider the following class of uncertain jump linear systems

$$\begin{cases} d\boldsymbol{x}(t) = [A_1(r_t) + \triangle A_1(r_t,t)]\boldsymbol{x}(t)dt + B_2(r_t)\boldsymbol{w}(t)dt + [B_1(r_t) + \triangle B_1(r_t,t)]\boldsymbol{u}(t)dt + B_3(r_t)\boldsymbol{x}(t)dw_1(t) \\ \boldsymbol{z}(t) \ = [C(r_t) + \triangle C(r_t,t)]\boldsymbol{x}(t) \end{cases} \tag{1}$$

where $\boldsymbol{x}(t) \in \mathbb{R}^n$ is the state, $\boldsymbol{u}(t) \in \mathbb{R}^{m_1}$ is the control input, $\boldsymbol{z}(t) \in \mathbb{R}^{m_3}$ is the output, $\boldsymbol{w}(t) \in \mathbb{R}^{m_2}$ is the arbitrary exogenous disturbance signal in $\mathcal{L}_2[0,\infty)$, and it is assumed that the Markov process r_t is independent of $\boldsymbol{w}(t)$, the noise $w_1(t)$ is an independent Wiener process on $\mathbb{R}$ and the interpretation of this diffusion with Markovian switching coefficients is as a collection of piecewise defined Ito stochastic differential equations. The parameter r_t is continuous-time Markov process on the probability space which takes values in the finite discrete state-space $S = \{1, 2, \cdots, N\}$ with generator $\prod = (\pi_{ij})_{N\times N} \quad (i, j \in S)$ given by

$$P\{r_{t+\Delta} = j | r_t = i\} = \begin{cases} \pi_{ij}\Delta + o(\Delta), & i \neq j \\ 1 + \pi_{ii}\Delta + o(\Delta), & i = j \end{cases}$$

where $\lim_{\Delta\to 0} o(\Delta)/\Delta = 0 (\Delta > 0)$, π_{ij} is the transition rate from i to j, and

$$\pi_{ii} = -\sum_{j\neq i} \pi_{ij}, \quad (\pi_{ij} \geqslant 0, j \neq i)$$

Considering two kinds of failure: false alarms and detection delays in the observation channel, we introduce the discrete variable $l_t \in \{1, 2, \cdots, N\}$, which is thus of the same nature as the true regime. More precisely, [5] has assumed that the following model describes the detection delays:

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When r_t has jumped from i to j, l_t follows with a delay that is an independent exponentially distributed random variable with mean $1/\pi_{ij}^0$. It is written as

$$P\left\{l_{t+\Delta}=j|l_s=i,s\in\left[\ t_0,\quad t\ \right],r_{t_0}=j,r_{t_0^-}=i\right\}=\begin{cases}\pi_{ij}^0\Delta+o(\Delta), & i\neq j\\ 1+\pi_{ii}^0\Delta+o(\Delta), & i=j\end{cases} \tag{2}$$

The entries of the matrix $\Pi^0=(\pi_{ij}^0)_{i,j=1,\cdots,N}$ are evaluated from observed sample paths.

False alarms are described in similar terms:

When r_t in fact remains at i, occasional declaration of an l_t has transitioned from i to j. An independent exponential distribution with rate π_{ij}^1 is again assumed

$$P\left\{l_{t+\Delta}=j|l_s=i,s\in[\ t_0,\quad t\ ]\right\}=\begin{cases}\pi_{ij}^1\Delta+o(\Delta), & i\neq j\\ 1+\pi_{ii}^1\Delta+o(\Delta), & i=j\end{cases} \tag{3}$$

with a matrix $\Pi^1=(\pi_{ij}^1)_{i,j=1,\cdots,N}$ of false alarm rates.

The unknown matrices $\triangle A_1(r_t,t)\in\mathbb{R}^{n\times n}$, $\triangle B_1(r_t,t)\in\mathbb{R}^{n\times m_1}$, and $\triangle C(r_t,t)\in\mathbb{R}^{m_3\times n}$ represent time-varying parameter uncertainties, and are assumed to be of the form

$$[\triangle A_1(r_t,t),\ \triangle B_1(r_t,t)]=H_1(r_t)F(r_t,t)\,[E_1(r_t),\ E_2(r_t)]$$
$$\triangle C(r_t,t)=H_2(r_t)F(r_t,t)E_3(r_t)$$

where $H_1(r_t),H_2(r_t),E_1(r_t),E_2(r_t)$ and $E_3(r_t)$ are known real constant matrices for all $r_t\in S$, and $F(r_t,t)$, for all $r_t\in S$, are the uncertain time-varying matrices satisfying

$$F^{\mathrm{T}}(r_t,t)F(r_t,t)\leqslant I,\quad \forall\, r_t\in S \tag{4}$$

For sake of simplicity, we denote the current regime by an index (*e.g.* A_i stands for $A(r_t)$ when $r_t=i\in S$).

Although one may find the controller $\boldsymbol{u}(t)=K(l_t)\boldsymbol{x}(t)$, the actual controller implemented is assumed as the one in [6]

$$\boldsymbol{u}(t)=[I+\alpha(r_t)\phi(r_t,t)]K(l_t)\boldsymbol{x}(t)$$

where $K(l_t)$ is the nominal controller gain for each $l_t\in S$, $\alpha(r_t)>0$ is the positive constant for each $r_t\in S$, and the term $\alpha(r_t)\phi(r_t,t)K(l_t)$ represents controller gain variations, and $\phi(r_t,t)$ is defined as

$$\phi^{\mathrm{T}}(r_t,t)\phi(r_t,t)\leqslant I,\quad \forall\, r_t\in S$$

Now, the corresponding closed loop system is given by

$$\begin{cases} d\boldsymbol{x}(t)=[A_1(r_t)+\triangle A_1(r_t,t)]\,\boldsymbol{x}(t)dt+B_2(r_t)\boldsymbol{w}(t)dt+B_3(r_t)\boldsymbol{x}(t)dw_1(t)\\ \qquad\qquad +[B_1(r_t)+\triangle B_1(r_t,t)]\,[I+\alpha(r_t)\phi(r_t,t)]\,K(l_t)\boldsymbol{x}(t)dt\\ \boldsymbol{z}(t)=[C(r_t)+\triangle C(r_t,t)]\,\boldsymbol{x}(t)\end{cases} \tag{5}$$

Our controller design objective is described as

1) Given that the disturbance input is zero all the time, to establish the sufficient conditions for the robust and non-fragile exponential stability in mean square of the closed loop system (5).

2) Under the zero initial condition, to establish the sufficient conditions for the $r-$disturbance attenuation property of the closed loop system (5), that is,

$$J=E\left\{\int_0^{\mathrm{T}}\left[z^{\mathrm{T}}(t)\boldsymbol{z}(t)-\gamma^2w^{\mathrm{T}}(t)\boldsymbol{w}(t)\right]dt\right\}<0,\ \ (\boldsymbol{w}(t)\neq 0) \tag{6}$$

3 Robust non-fragile control for uncertain jump linear systems

Lemma 1[7]**.** Given matrices $Q=Q^{\mathrm{T}},H,E$ and $R=R^{\mathrm{T}}>0$ of appropriate dimensions,

$$Q+HFE+E^{\mathrm{T}}F^{\mathrm{T}}H^{\mathrm{T}}<0$$

for all F satisfying $F^{\mathrm{T}}F \leqslant R$, if and only if there exits some $\lambda > 0$ such that

$$Q + \lambda HH^{\mathrm{T}} + \lambda^{-1}E^{\mathrm{T}}RE < 0$$

Lemma 2[5]**.** Assume that the functions $f(\cdot)$, and $h(\cdot)$ are continuous in $t, \boldsymbol{x}(t)$ for $r_t, l_t \in S$, and satisfy the usual growth and smoothness hypothesis.

$$d\boldsymbol{x} = f(\boldsymbol{x}, \boldsymbol{u}(t), l_t, r_t, t)dt + h(\boldsymbol{x}, r_t)dw_1(t)$$

Let $g(\boldsymbol{x}(t), l_t, r_t, t)$ be a scalar function satisfying the conditions of Appendix 1 in [5]. Then the generator of the pair $(\boldsymbol{x}(t), l_t, r_t)$ under the control action $\boldsymbol{u}(t)$ is the operator $\varPsi$ such that: when $l_t = r_t = i$

$$\begin{aligned}\varPsi g(\boldsymbol{x}(t), i, i, t) = \lim_{\triangle\to 0}\frac{1}{\triangle}\left[E\left\{g(x(t+\triangle), l_{t+\triangle}, r_{t+\triangle}, t+\triangle)|\boldsymbol{x}(t)=x, l_t = i, r_t = i, t\right\} - g(\boldsymbol{x}, i, i, t)\right] = \\ g_t(\boldsymbol{x}, i, i, t) + f^{\mathrm{T}}(\boldsymbol{x}, \boldsymbol{u}(t), i, i, t)g_x(\boldsymbol{x}, i, i, t) + \sum_{j=1}^{N}\pi_{ij}g(\boldsymbol{x}, i, j, t) + \sum_{j=1}^{N}\pi_{ij}^{1}g(\boldsymbol{x}, j, i, t) + \\ \frac{1}{2}trace[h^{\mathrm{T}}(\boldsymbol{x}, i)g_{xx}(\boldsymbol{x}, i, i, t)h(\boldsymbol{x}, i)]\end{aligned} \tag{7}$$

when $l_t = j \neq r_t = i$

$$\begin{aligned}\varPsi g(\boldsymbol{x}(t), j, i, t) =& g_t(\boldsymbol{x}, j, i, t) + f^{\mathrm{T}}(\boldsymbol{x}, \boldsymbol{u}(t), j, i, t)g_x(\boldsymbol{x}, j, i, t) + \pi_{ji}^{0}g(\boldsymbol{x}, i, i, t) - \pi_{ji}^{0}g(\boldsymbol{x}, j, i, t) + \\ & \frac{1}{2}trace[h^{\mathrm{T}}(\boldsymbol{x}, i)g_{xx}(\boldsymbol{x}, j, i, t)h(\boldsymbol{x}, i)]\end{aligned} \tag{8}$$

Theorem 1. The uncertain jump linear system (5) without disturbance achieves exponential mean-square stability, and the controller $\boldsymbol{u}(t) = K(l_t)\boldsymbol{x}(t)$ is robust and non-fragiled if there exist symmetric positive-definite matrices $P_{ji} = P_{ji}^{\mathrm{T}} > 0$, controller gain matrices K_j and positive constants $\lambda_{1ji} > 0, \lambda_{2ji} > 0$ such that the following LMIs hold for $j, i \in$ S

$$\begin{bmatrix} \varOmega_{ji} + \lambda_{2ji}E_{1i}^{\mathrm{T}}E_{1i} & P_{ji}B_{1i} + \lambda_{2ji}E_{1i}^{\mathrm{T}}E_{2i} & K_j^{\mathrm{T}} & P_{ji}H_{1i} & K_j^{\mathrm{T}} \\ B_{1i}^{\mathrm{T}}P_{ji} + \lambda_{2ji}E_{2i}^{\mathrm{T}}E_{1i} & -I + \lambda_{2ji}E_{2i}^{\mathrm{T}}E_{2i} & 0 & 0 & 0 \\ K_j & 0 & -I + \lambda_{1ji}\alpha_i^2 I & 0 & 0 \\ H_{1i}^{\mathrm{T}}P_{ji} & 0 & 0 & -\lambda_{2ji}I & 0 \\ K_j & 0 & 0 & 0 & -\lambda_{1ji}I \end{bmatrix} < 0, \quad j, i \in S \tag{9}$$

where

$$\varOmega_{ji} = \begin{cases} A_{1i}^{\mathrm{T}}P_{ii} + P_{ii}A_{1i} + \displaystyle\sum_{j=1}^{N}\pi_{ij}P_{ij} + \sum_{j=1}^{N}\pi_{ij}^{1}P_{ji} + B_{3i}^{\mathrm{T}}P_{ii}B_{3i}, & if \quad j = i \\ A_{1i}^{\mathrm{T}}P_{ji} + P_{ji}A_{1i} + \pi_{ji}^{0}(P_{ii} - P_{ji}) + B_{3i}^{\mathrm{T}}P_{ji}B_{3i}, & if \quad j \neq i \end{cases}$$

Proof. For the nominal jump linear system Σ_0 without disturbance, which means $\Delta A_1(r_t, t) = 0$, $\Delta B_1(r_t, t) = 0, \Delta C(r_t, t) = 0, \phi(r_t, t) = 0$ and $\boldsymbol{w}(t) = \boldsymbol{0}$ in (5), consider the following Lyapunov function:

$$V(\boldsymbol{x}(t), l_t, r_t) = \boldsymbol{x}^{\mathrm{T}}(t)P(l_t, r_t)\boldsymbol{x}(t) \tag{10}$$

where $P(l_t, r_t)$ are positive defined and symmetrical matrices. Using the infinitesimal generator $\varPsi$, we have

Case 1. $l_t = r_t = i$

$$\varPsi V(\boldsymbol{x}(t), i, i) = \boldsymbol{x}^{\mathrm{T}}(t)M_{ii}\boldsymbol{x}(t) \tag{11}$$

where $M_{ii} = \left[(A_{1i} + B_{1i}K_i)^{\mathrm{T}}P_{ii} + P_{ii}(A_{1i} + B_{1i}K_i) + \displaystyle\sum_{j=1}^{N}\pi_{ij}P_{ij} + \sum_{j=1}^{N}\pi_{ij}^{1}P_{ji} + B_{3i}^{\mathrm{T}}P_{ii}B_{3i}\right]$.

It′s easy to see that $M_{ii} < 0$ is equivalent to

$$\left[A_{1i}^{\mathrm{T}}P_{ii} + P_{ii}A_{1i} + \sum_{j=1}^{N}\pi_{ij}P_{ij} + \sum_{j=1}^{N}\pi_{ij}^{1}P_{ji} + B_{3i}^{\mathrm{T}}P_{ii}B_{3i}\right] + P_{ii}B_{1i}IK_i + K_i^{\mathrm{T}}IB_{1i}^{\mathrm{T}}P_{ii} < 0 \tag{12}$$

By Lemma 1, a sufficient condition guaranteeing (12) is that there exists a positive number $\lambda_{ii} > 0, (i \in S)$ such that

$$\lambda_{ii}\left[A_{1i}^{\mathrm{T}}P_{ii} + P_{ii}A_{1i} + \sum_{j=1}^{N}\pi_{ij}P_{ij} + \sum_{j=1}^{N}\pi_{ij}^{1}P_{ji} + B_{3i}^{\mathrm{T}}P_{ii}B_{3i}\right] + \lambda_{ii}^{2}P_{ii}B_{1i}IB_{1i}^{\mathrm{T}}P_{ii} + K_i^{\mathrm{T}}IK_i < 0 \quad (13)$$

Replacing $\lambda_{ii}P_{ii}$ with P_{ii}, and applying the Schur complement leads to that (13) is equivalent to

$$\begin{bmatrix} \Omega_{ii} & P_{ii}B_{1i} & K_i^{\mathrm{T}} \\ B_{1i}^{\mathrm{T}}P_{ii} & -I & 0 \\ K_i & 0 & -I \end{bmatrix} < 0, \quad i \in S \quad (14)$$

where $\Omega_{ii} = A_{1i}^{\mathrm{T}}P_{ii} + P_{ii}A_{1i} + \sum_{j=1}^{N}\pi_{ij}P_{ij} + \sum_{j=1}^{N}\pi_{ij}^{1}P_{ji} + B_{3i}^{\mathrm{T}}P_{ii}B_{3i}$.

Case 2. $l_t = j, r_t = i$ and $j \neq i$

$$\Psi V(\boldsymbol{x}(t), j, i) = \boldsymbol{x}^{\mathrm{T}}(t)M_{ji}\boldsymbol{x}(t) \quad (15)$$

where $M_{ji} = \left[(A_{1i} + B_{1i}K_j)^{\mathrm{T}}P_{ji} + P_{ji}(A_{1i} + B_{1i}K_j) + \pi_{ji}^{0}(P_{ii} - P_{ji}) + B_{3i}^{\mathrm{T}}P_{ji}B_{3i}\right], \quad j \neq i$.

Following similar lines as in the proof of case 1, we canobtain that the following LMI

$$\begin{bmatrix} \Omega_{ji} & P_{ji}B_{1i} & K_j^{\mathrm{T}} \\ B_{1i}^{\mathrm{T}}P_{ji} & -I & 0 \\ K_j & 0 & -I \end{bmatrix} < 0, \quad j, i \in S, \ \textit{and}\ j \neq i \quad (16)$$

guarantees $M_{ji} < 0$, where $\Omega_{ji} = [A_{1i}^{\mathrm{T}}P_{ji} + P_{ji}A_{1i} + \pi_{ji}^{0}(P_{ii} - P_{ji}) + B_{3i}^{\mathrm{T}}P_{ji}B_{3i}]$.

Hence, (14) and (16) are sufficient to guarantee the negativity of $\Psi V(\boldsymbol{x}(t), l_t, r_t)$ for all $j, i \in S$. By Theorem 2.3 and Theorem 7.2 in [5], the nominal jump linear system Σ_0 is exponentially stable in mean square.

Then, to the uncertain jump linear system (5) without disturbance, we only need to replace A_{1i}, B_{1i} and K_j in (14) and (16) with $A_{1i} + H_{1i}F_i(t)E_{1i}, B_{1i} + H_{1i}F_i(t)E_{2i}$ and $K_j + \alpha_i\phi_i(t)K_j$ for all $j, i \in S$. By Lemma 1, a sufficient condition guaranteeing (14) and (16) is that there exist positive constants $\lambda_{1ji} > 0, \lambda_{2ji} > 0, (j, i \in S)$ such that

$$\begin{bmatrix} \Omega_{ji} & P_{ji}B_{1i} & K_j^{\mathrm{T}} \\ B_{1i}^{\mathrm{T}}P_{ji} & -I & 0 \\ K_j & 0 & -I \end{bmatrix} + \lambda_{1ji}\begin{bmatrix} 0 \\ 0 \\ \alpha_i \end{bmatrix}\begin{bmatrix} 0 \\ 0 \\ \alpha_i \end{bmatrix}^{\mathrm{T}} + \lambda_{1ji}^{-1}\begin{bmatrix} K_j^{\mathrm{T}} \\ 0 \\ 0 \end{bmatrix}\begin{bmatrix} K_j^{\mathrm{T}} \\ 0 \\ 0 \end{bmatrix}^{\mathrm{T}} +$$

$$\lambda_{2ji}^{-1}\begin{bmatrix} P_{ji}H_{1i} \\ 0 \\ 0 \end{bmatrix}\begin{bmatrix} P_{ji}H_{1i} \\ 0 \\ 0 \end{bmatrix}^{\mathrm{T}} + \lambda_{2ji}\begin{bmatrix} E_{1i}^{\mathrm{T}} \\ E_{2i}^{\mathrm{T}} \\ 0 \end{bmatrix}\begin{bmatrix} E_{1i}^{\mathrm{T}} \\ E_{2i}^{\mathrm{T}} \\ 0 \end{bmatrix}^{\mathrm{T}} < 0 \quad (17)$$

Applying the Schur complement yields that (9) is equivalent to (17) for all $j, i \in S$. This completes the proof. □

4 Robust non-fragile H_∞ control for uncertain jump linear systems

In this section, we consider robust and non-fragile H_∞ disturbance attenuation for the uncertain jump linear system (5).

Theorem 2. The uncertain jump linear system (5) is stochastically stable with γ-disturbance attenuation property (6), and the controller $\boldsymbol{u}(t) = K(l_t)\boldsymbol{x}(t)$ is robust and non-fragile if there exist symmetric positive-definite matrices $P_{ji} = P_{ji}^{\mathrm{T}} > 0$, controller gain matrices K_j and positive constants

$\lambda_{1ji} > 0$, $\lambda_{2ji} > 0, \lambda_{3ji} > 0$, such that and the following LMIs hold for $j, i \in \mathrm{S}$

$$\left[\begin{array}{cccccccc} \Omega_{ji} + \lambda_{2ji} E_{1i}^{\mathrm{T}} E_{1i} & P_{ji} B_{2i} & P_{ji} B_{1i} + \lambda_{2ji} E_{1i}^{\mathrm{T}} E_{2i} & K_j^{\mathrm{T}} & C_i^{\mathrm{T}} & P_{ji} H_{1i} & K_j^{\mathrm{T}} & E_{3i}^{\mathrm{T}} \\ B_{2i}^{\mathrm{T}} P_{ji} & -\gamma^2 I & 0 & 0 & 0 & 0 & 0 & 0 \\ B_{1i}^{\mathrm{T}} P_{ji} + \lambda_{2ji} E_{2i}^{\mathrm{T}} E_{1i} & 0 & -I + \lambda_{2ji} E_{2i}^{\mathrm{T}} E_{2i} & 0 & 0 & 0 & 0 & 0 \\ K_j & 0 & 0 & -I + \lambda_{1ji} \alpha_i^2 I & 0 & 0 & 0 & 0 \\ C_i & 0 & 0 & 0 & -I + \lambda_{3ji} H_{2i} H_{2i}^{\mathrm{T}} & 0 & 0 & 0 \\ H_{1i} P_{ji} & 0 & 0 & 0 & 0 & -\lambda_{2ji} I & 0 & 0 \\ K_j & 0 & 0 & 0 & 0 & 0 & -\lambda_{1ji} I & 0 \\ E_{3i} & 0 & 0 & 0 & 0 & 0 & 0 & -\lambda_{3ji} I \end{array}\right] < 0 \tag{18}$$

where $\Omega_{ji} = \begin{cases} A_{1i}^{\mathrm{T}} P_{ii} + P_{ii} A_{1i} + \sum\limits_{j=1}^{N} \pi_{ij} P_{ij} + \sum\limits_{j=1}^{N} \pi_{ij}^1 P_{ji} + B_{3i}^{\mathrm{T}} P_{ii} B_{3i}, if \quad j = i \\ A_{1i}^{\mathrm{T}} P_{ji} + P_{ji} A_{1i} + \pi_{ji}^0 (P_{ii} - P_{ji}) + B_{3i}^{\mathrm{T}} P_{ji} B_{3i}, \qquad if \quad j \neq i \end{cases}.$

Proof. Let us first look at the nominal jump linear system $\sum_1$, which means $\Delta A_1(r_t, t) = 0, \Delta B_1(r_t, t) = 0, \Delta C(r_t, t) = 0$ and $\phi(r_t, t) = 0$ in (5). Again taking the Lyapunov function as (10) and following a similar line as in the proof of Theorem 1, we have

$$\Psi V(\boldsymbol{x}(t), j, i) = \begin{bmatrix} \boldsymbol{x}(t) \\ \boldsymbol{w}(t) \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} M_{ji} & P_{ji} B_{2i} \\ B_{2i}^{\mathrm{T}} P_{ji} & 0 \end{bmatrix} \begin{bmatrix} \boldsymbol{x}(t) \\ \boldsymbol{w}(t) \end{bmatrix} \tag{19}$$

where

$$M_{ji} = \begin{cases} (A_{1i} + B_{1i} K_i)^{\mathrm{T}} P_{ii} + P_{ii} (A_{1i} + B_{1i} K_i) + \sum\limits_{j=1}^{N} \pi_{ij} P_{ij} + \sum\limits_{j=1}^{N} \pi_{ij}^1 P_{ji} + B_{3i}^{\mathrm{T}} P_{ii} B_{3i}, \text{ if } j = i \\ (A_{1i} + B_{1i} K_j)^{\mathrm{T}} P_{ji} + P_{ji} (A_{1i} + B_{1i} K_j) + \pi_{ji}^0 (P_{ii} - P_{ji}) + B_{3i}^{\mathrm{T}} P_{ji} B_{3i}, \qquad \text{if } j \neq i \end{cases}$$

By Dynkin′s formula, we have

$$E\left\{\int_0^{\mathrm{T}} \Psi V(\mathrm{x}(s), l_s, r_s, s) ds\right\} = E\{V(\boldsymbol{x}(T), l_T, r_T, T)\} - E\{V(\boldsymbol{x}(0), l_0, r_0, 0)\}$$

Under the zero initial condition, we have

$$J \leqslant E\left\{\int_0^{\mathrm{T}} \left[\boldsymbol{z}^{\mathrm{T}}(t) \boldsymbol{z}(t) - \gamma^2 \boldsymbol{w}^{\mathrm{T}}(t) \boldsymbol{w}(t) + \Psi V(\boldsymbol{x}(t), l_t, r_t, t)\right] dt\right\}, \forall \quad \boldsymbol{w}(t) \in \mathcal{L}_2[0 \quad \infty)$$

Taking (19) into the above inequality, we have

$$J \leqslant E\left\{\int_0^{\mathrm{T}} \left[\begin{bmatrix} \boldsymbol{x}(t) \\ \boldsymbol{w}(t) \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} M_{ji} + C_i^{\mathrm{T}} C_i & P_{ji} B_{2i} \\ B_{2i}^{\mathrm{T}} P_{ji} & -\gamma^2 I \end{bmatrix} \begin{bmatrix} \boldsymbol{x}(t) \\ \boldsymbol{w}(t) \end{bmatrix}\right] dt\right\} \tag{20}$$

By Lemma 1 and applying the Schur complement we obtain that

$$\begin{bmatrix} \Omega_{ji} + C_i^{\mathrm{T}} C_i & P_{ji} B_{2i} & P_{ji} B_{1i} & K_j^{\mathrm{T}} \\ B_{2i}^{\mathrm{T}} P_{ji} & -\gamma^2 I & 0 & 0 \\ B_{1i}^{\mathrm{T}} P_{ji} & 0 & -I & 0 \\ K_j & 0 & 0 & -I \end{bmatrix} < 0, \quad j, i \in \mathrm{S} \tag{21}$$

where

$$\Omega_{ji}=\begin{cases} A_{1i}^{\mathrm{T}}P_{ii}+P_{ii}A_{1i}+\sum\limits_{j=1}^{N}\pi_{ij}P_{ij}+\sum\limits_{j=1}^{N}\pi_{ij}^{1}P_{ji}+B_{3i}^{\mathrm{T}}P_{ii}B_{3i}, if \quad j=i \\ A_{1i}^{\mathrm{T}}P_{ji}+P_{ji}A_{1i}+\pi_{ji}^{0}(P_{ii}-P_{ji})+B_{3i}^{\mathrm{T}}P_{ji}B_{3i}, \qquad if \quad j\neq i \end{cases}$$

guarantees

$$\begin{bmatrix} M_{ji}+C_i^{\mathrm{T}}C_i & P_{ji}B_{2i} \\ B_{2i}^{\mathrm{T}}P_{ji} & -\gamma^2 I \end{bmatrix}<0$$

Then for the uncertain system (5), replacing A_{1i}, B_{1i}, C_i and K_j in (21) with $A_{1i}+H_{1i}F_i(t)E_{1i}$, $B_{1i}+H_{1i}F_i(t)E_{2i}$, $C_i+H_{2i}F_i(t)E_{3i}$ and $K_j+\alpha_i\phi_i(t)K_j$, and using Lemma 1, we can complete the proof.□

5 Conclusion

In this paper, we study the robust non-fragile H_∞ control problem for a class of uncertain jump linear systems. Sufficient conditions on the existence of robust stochastic stability and γ-disturbance attenuation H_∞ property are presented based on coupled LMI′s. All of these results established are dependent of the priori knowledge of the norm bounds of the system′s uncertainties. A possible direction for future work is to obtain the robust and adaptive H_∞ control laws of the unknown norm bounds.

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