Fast Rate Fault Detection Filter for Multirate Sampled-data Systems

ZHONG Mai-Ying MA Chuan-Feng LIU Yun-Xia

(*School of Control Science and Engineering, Shandong University, Jinan* 250061)

(E-mail: myzhong@sdu.edu.cn)

Abstract This paper focuses on the fast rate fault detection filter (FDF) problem for a class of multirate sampled-data (MSD) systems. A lifting technique is used to convert such an MSD system into a linear time-invariant discrete-time one and an unknown input observer (UIO) is considered as FDF to generate residual. The design of FDF is formulated as an H_∞ optimization problem and a solvable condition as well as an optimal solution are derived. The causality of the residual generator can be guaranteed so that the fast rate residual can be implemented via inverse lifting. A numerical example is included to demonstrate the feasibility of the obtained results.

Key words Fault detection filter, fast rate, residual generator, multirate sampled-data

1 Introduction

In many computer control systems, data sampling is required, and the modelling and control of sampled-data (SD) systems has received great interest. Especially in recent years, research on fault detection and isolation (FDI) of SD systems has become a more active topic. In [1∼3] fault detection problems for single rate SD systems have been intensively investigated. By using lifting technique, [4] converted an MSD system into a linear time invariant (LTI) one and the problem of FDI can be solved by using the FDI methods of single rate SD system, while the generated residual signals can only be updated at the end of the repetition period, namely the slow rate residual generator. To detect a fault as soon as possible after its occurrence, the timely generation of residual plays an important role. Study on fast rate FDI for MSD system is not only theoretically interesting but also very important in practical applications. To the best of authors′ knowledge, however, only few researches have done on this topic[5∼7]. The problem of fast rate observer-based FDI for MSD systems has not yet been fully investigated.

The fast rate observer-based fault detection problem will be studied in this paper. The lifting technique will be introduced to convert such an MSD system into an LTI one with slow sampling period and a UIO will be used as FDF. The existence condition for the solvability as well as an optimal solution to the considered problem will be derived. The fast rate residual generator will be implemented by employing inverse lifting technique. Throughout this paper, all the notations are as general.

2 Problem formulation

The plant of the considered MSD system is a continuous-time LTI process, which can be expressed by

$$\begin{cases} \dot{\boldsymbol{x}}(t) = A_c\boldsymbol{x}(t) + B_c\boldsymbol{u}(t) + B_{cd}\boldsymbol{d}(t) + B_{cf}\boldsymbol{f}(t) \\ \boldsymbol{y}(t) = C\boldsymbol{x}(t) + D\boldsymbol{u}(t) + D_d\boldsymbol{d}(t) + D_f\boldsymbol{f}(t) \end{cases} \tag{1}$$

where $\boldsymbol{x}(t) \in \mathbb{R}^n$, $\boldsymbol{u}(t) \in \mathbb{R}^p$, $\boldsymbol{y}(t) \in \mathbb{R}^q$, $\boldsymbol{d}(t) \in \mathbb{R}^m$, and $\boldsymbol{f}(t) \in \mathbb{R}^{q_f}$ denote the state, control input, measurement output, L_2-norm bounded unknown input, and fault, respectively. $A_c, B_c, B_{cd}, B_{cf}, C, D, D_d$ and D_f are known matrices with appropriate dimensions. The D/A converter in the input channel is supposed to be fast rate with sampling period h. The A/D converters in the output channel may have different sampling rates. Suppose $\boldsymbol{y}_l \in \mathbb{R}^{q_l \times 1}$ is the lth component of the output vector $\boldsymbol{y}$, which is measured every $T_l h$ time, and $\boldsymbol{\Psi}_l(k) = \boldsymbol{y}_l(kT_l h)$ for $l = 1, \ldots, N$ and $k = 0, 1, \ldots$, where T_l are positive integers, $\boldsymbol{\Psi}_l(k)$ denote the sampled process output sequences with the same sampling period $T_l h$ and $\sum\limits_{l=1}^{N} q_l = q$.

1) Supported by National Natural Science Foundation of P. R. China (60374021), the Natural Science Foundation of Shandong Province (Y2002G05), the Youth Scientists Foundation of Shandong Province (03BS091, 05BS01007), and Education Ministry Foundation of P. R. China (20050422036)

Received December 3, 2004; in revised form January 9, 2006

Define T_M as the least common multiple of $T_l\,(l=1,\ldots,N)$, and let $n_l = \dfrac{T_M}{T_l}$, $Q = \sum_{l=1}^{N} n_l$. Introduce lifting operator $\mathcal{L}$ and inverse lifting operator $\mathcal{L}^{-1}$ as

$$\begin{aligned}\boldsymbol{\xi} =&[\xi_1(0),\cdots,\xi_1(n_1-1),\cdots,\xi_N(0),\cdots,\xi_N(n_N-1),\xi_1(n_1),\cdots,\xi_1(2n_1-1),\\ &\cdots,\xi_N(n_N),\cdots,\xi_N(2n_N-1),\cdots] = \mathcal{L}^{-1}\underline{\boldsymbol{\xi}}\end{aligned} \tag{2}$$

$$\underline{\boldsymbol{\xi}} =\mathcal{L}\boldsymbol{\xi} = \left\{\left[\xi_1^T(0)\ \cdots\ \xi_1^T(n_1-1)\ \cdots\ \xi_N^T(0)\ \cdots\ \xi_N^T(n_N-1)\right]^T,\cdots\right\} \tag{3}$$

and let

$$\begin{aligned}
\boldsymbol{x}(k_s) :=&\boldsymbol{x}(k_sT_Mh),\quad \boldsymbol{x}(k_s+1) := \boldsymbol{x}((k_s+1)T_Mh),\quad \underline{\boldsymbol{\Psi}} = \mathcal{L}\boldsymbol{\Psi}\\
\underline{\boldsymbol{v}}(k_s) :=&[v^{\mathrm{T}}(k_sT_Mh)\quad v^{\mathrm{T}}(k_sT_Mh+h)\quad \cdots\quad v^{\mathrm{T}}((k_s+1)\,T_Mh-h)]^{\mathrm{T}}\\
\underline{\boldsymbol{f}}(k_s) :=&[f^{\mathrm{T}}(k_sT_Mh)\quad f^{\mathrm{T}}(k_sT_Mh+h)\quad \cdots\quad f^{\mathrm{T}}((k_s+1)\,T_Mh-h)]^{\mathrm{T}}\\
\underline{\boldsymbol{d}}(k_s) :=&[d^{\mathrm{T}}(k_sT_Mh)\quad d^{\mathrm{T}}(k_sT_Mh+h)\quad \cdots\quad d^{\mathrm{T}}((k_s+1)T_Mh-h)]^{\mathrm{T}}\\
A_l =&A^{T_l},\quad B_l = [A^{T_l-1}B\quad \cdots\quad AB\quad B]\\
B_{dl} =&[A^{T_l-1}B_d\quad \cdots\quad AB_d\quad B_d],\quad B_{fl} = [A^{T_l-1}B_f\quad \cdots\quad AB_f\quad B_f]\\
\underline{A} =&A^{T_M} = A_l^{n_l},\quad \underline{B}_v = [A^{T_M-1}B\quad \cdots\quad AB\quad B]\\
\underline{B}_d =&[A^{T_M-1}B_d\quad \cdots\quad AB_d\quad B_d],\quad \underline{B}_f = [A^{T_M-1}B_f\quad \cdots\quad AB_f\quad B_f]\\
\underline{C} =& \begin{bmatrix}\underline{C}_1\\ \vdots\\ \underline{C}_N\end{bmatrix},\quad \underline{C}_l = \begin{bmatrix}C_l\\ C_lA_l\\ \vdots\\ C_lA_l^{n_l-1}\end{bmatrix},\quad \underline{D}_v = \begin{bmatrix}\underline{D}_1\\ \vdots\\ \underline{D}_N\end{bmatrix},\quad \underline{D}_d = \begin{bmatrix}\underline{D}_{d1}\\ \vdots\\ \underline{D}_{dN}\end{bmatrix}\\
\underline{D}_f =& \begin{bmatrix}\underline{D}_{f1}\\ \vdots\\ \underline{D}_{fN}\end{bmatrix}],\quad \underline{D}_l = \begin{bmatrix}0 & 0 & \cdots & 0\\ C_lB_l & 0 & \ddots & \vdots\\ \vdots & \ddots & \ddots & 0\\ C_lA_l^{n_l-2}B_l & \cdots & C_lB_l & 0\end{bmatrix}\\
\underline{D}_{dl} =& \begin{bmatrix}0 & 0 & \cdots & 0\\ C_lB_{dl} & 0 & \ddots & \vdots\\ \vdots & \ddots & \ddots & 0\\ C_lA_l^{n_l-2}B_{dl} & \cdots & C_lB_{dl} & 0\end{bmatrix},\quad \underline{D}_{fl} = \begin{bmatrix}0 & 0 & \cdots & 0\\ C_lB_{fl} & 0 & \ddots & \vdots\\ \vdots & \ddots & \ddots & 0\\ C_lA_l^{n_l-2}B_{fl} & \cdots & C_lB_{fl} & 0\end{bmatrix}\\
l =&1,2,\ldots,N
\end{aligned}$$

The dynamics of the MSD system can be rewritten into

$$\begin{cases}\boldsymbol{x}\,(k_s+1) = \underline{A}\boldsymbol{x}\,(k_s) + \underline{B}_v\underline{\boldsymbol{v}}(k_s) + \underline{B}_d\underline{\boldsymbol{d}}(k_s) + \underline{B}_f\underline{\boldsymbol{f}}(k_s)\\ \underline{\boldsymbol{\Psi}}(k_s) = \underline{C}\boldsymbol{x}\,(k_s) + \underline{D}_v\underline{\boldsymbol{v}}(k_s) + \underline{D}_d\underline{\boldsymbol{d}}(k_s) + \underline{D}_f\underline{\boldsymbol{f}}(k_s)\end{cases} \tag{4}$$

where k_s denotes the slow rate time index. (4) is the so called lifted model of MSD system, which is in fact an LTI discrete time system with sampling period T_Mh.

We consider the following unknown input observer (UIO) as an FDF

$$\begin{cases}\boldsymbol{\eta}(k_s+1) = G\boldsymbol{\eta}(k_s) + H_1\underline{\boldsymbol{v}}(k_s) + H_2\underline{\boldsymbol{\Psi}}(k_s)\\ \underline{\boldsymbol{r}}(k_s) = L_1\boldsymbol{\eta}(k_s) + L_2\underline{\boldsymbol{v}}(k_s) + L_3\underline{\boldsymbol{\Psi}}(k_s)\end{cases} \tag{5}$$

where $\underline{\boldsymbol{r}}(k_s) \in \mathbb{R}^{(\sum_{i=1}^{N} q_i)}$ denotes the generated residual with sampling period T_Mh, and

$$\begin{aligned}
\underline{\boldsymbol{r}}(k_s) =&[\underline{r}_1^{\mathrm{T}}(k_s)\quad \underline{r}_2^{\mathrm{T}}(k_s)\quad \cdots\quad \underline{r}_N^{\mathrm{T}}(k_s)]^{\mathrm{T}}\\
\underline{\boldsymbol{r}}_l(k_s) =&[r_l^{\mathrm{T}}(k_sn_l)\quad r_l^{\mathrm{T}}(k_sn_l+1)\quad \cdots\quad r_l^{\mathrm{T}}(k_sn_l+n_l-1)]\\
\boldsymbol{r}_l(k_l) =&:r_l(k_lT_lh) \in \mathbb{R}^{q_l},\quad l=1,2,\ldots N,\quad k_l = 0,1,\ldots
\end{aligned}$$

Let $\boldsymbol{e}(k_s) = \boldsymbol{\eta}(k_s) - W\boldsymbol{x}(k_s)$, W be invertible. The matrices of a UIO satisfy

$$\begin{cases} G \text{ has stable eigenvalues} \\ W\underline{A} - GW = H_2\underline{C}, \quad H_1 = W\underline{B}_v - H_2\underline{D}_v, \quad L_1W + L_3\underline{C} = 0, \quad L_2 + L_3\underline{D}_v = 0 \end{cases} \tag{6}$$

Then, we have

$$\begin{cases} \boldsymbol{e}(k_s+1) = G\boldsymbol{e}(k_s) + (H_2\underline{D}_d - W\underline{B}_d)\underline{\boldsymbol{d}}(k_s) + (H_2\underline{D}_f - W\underline{B}_f)\boldsymbol{f}(k_s) \\ \underline{\boldsymbol{r}}(k_s) = L_1\boldsymbol{e}(k_s) + L_3\underline{D}_d\underline{\boldsymbol{d}}(k_s) + L_3\underline{D}_f\underline{\boldsymbol{f}}(k_s) \end{cases} \tag{7}$$

The main purpose of this paper is to design $G, H_1, H_2, L_1, L_2, L_3$ and W such that

$$\min_{G,H_1,H_2,L_1,L_2,L_3,W} J_{\infty/\infty} = \min_{G,H_1,H_2,L_1,L_2,L_3,W} \frac{\|T_{\underline{r}\underline{d}}(z)\|_\infty}{\|T_{\underline{r}\underline{f}}(z)\|_\infty} \tag{8}$$

and the residual generator is fast rate, where

$$\begin{cases} T_{\underline{r}\underline{d}}(z) = L_1(zI-G)^{-1}(H_2\underline{D}_d - W\underline{B}_d) + L_3\underline{D}_d \\ T_{\underline{r}\underline{f}}(z) = L_1(zI-G)^{-1}(H_2\underline{D}_f - W\underline{B}_f) + L_3\underline{D}_f \end{cases} \tag{9}$$

3 Main results

Lemma[2]**.** Considers a discrete-time residual generator $\boldsymbol{r}(z) = T_{rd}(z)\boldsymbol{d}(z) + T_{rf}(z)\boldsymbol{f}(z)$, in which

$$\begin{aligned} T_{rd}(z) =& V(C(zI-A+HC)^{-1}(B_d - HD_d) + D_d) \\ T_{rf}(z) =& V(C(zI-A+HC)^{-1}(B_f - HD_f) + D_f) \end{aligned}$$

Assume that (A, B_d, C, D_d) has no zeros, no unreachable null modes or and no unobservable modes on the unit circle, and (C, A) is detectable. Then V, $H = -L_0^{\mathrm{T}}$ solves optimization problem

$$\min_{V,H} J = \min_{V,H} \frac{\|T_{rd}(z)\|_\infty}{\|T_{rf}(z)\|_\infty}$$

where V is left inverse of Γ which satisfies $\Gamma\Gamma^{\mathrm{T}} = CXC^{\mathrm{T}} + D_dD_d^{\mathrm{T}}$, (X, L_0) is the stabilizing solution of discrete-time algebraic Riccati equation (DARE)

$$\begin{bmatrix} AXA^{\mathrm{T}} - X + B_dB_d^{\mathrm{T}} & AXC^{\mathrm{T}} + B_dD_d^{\mathrm{T}} \\ CXA^{\mathrm{T}} + D_dB_d^{\mathrm{T}} & CXC^{\mathrm{T}} + D_dD_d^{\mathrm{T}} \end{bmatrix} \times \begin{bmatrix} I \\ L_0 \end{bmatrix} = 0$$

By applying Lemma, one optimal solution of the FDF can be derived as in the following theorem. Since the limitation of space, its proof is omitted.

Theorem. Assume that $(\underline{A}, \underline{B}_d, \underline{C}, \underline{D}_d)$ is detectable, has no zeros, no unreachable null modes or no unobservable modes on the unit circle. Then, an optimal solution of the problem (8) is

$$G = W\left(\underline{A} - \hat{H}_2\underline{C}\right)W^{-1}, \quad H_2 = W\hat{H}_2, \quad H_1 = W\underline{B}_v - H_2\underline{D}_v \tag{10}$$

$$L_1 = -L_3\underline{C}W^{-1}, \quad L_2 = -L_3\underline{D}_v \tag{11}$$

where $\hat{H}_2 = -L_0^{\mathrm{T}}$, L_3 is left inverse of Θ which satisfies $\Theta\Theta^{\mathrm{T}} = \underline{C}X\underline{C}^{\mathrm{T}} + \underline{D}_d\underline{D}_d^{\mathrm{T}}$, (X, L_0) is the stabilizing solution of discrete-time algebraic Riccati equation (DARE)

$$\begin{bmatrix} \underline{A}X\underline{A}^{\mathrm{T}} - X + \underline{B}_d\underline{B}_d^{\mathrm{T}} & \underline{A}X\underline{C}^{\mathrm{T}} + \underline{B}_d\underline{D}_d^{\mathrm{T}} \\ \underline{C}X\underline{A}^{\mathrm{T}} + \underline{D}_d\underline{B}_d^{\mathrm{T}} & \underline{C}X\underline{C}^{\mathrm{T}} + \underline{D}_d\underline{D}_d^{\mathrm{T}} \end{bmatrix} \times \begin{bmatrix} I \\ L_0 \end{bmatrix} = 0 \tag{12}$$

and W is any n-dimensional invertible matrix.

Suppose $(G, H_1, H_2, L_1, L_2, L_3)$ is a solution of the UIO parameter matrices. From (6) we have $L_1 = L_3\underline{C}W^{-1}$ and $L_2 = L_3\underline{D}_v$ with any given invertible W. For any orthogonal matrix U with appropriate dimensions, $\left(G, H_1, H_2, \tilde{L}_1, \tilde{L}_2, \tilde{L}_3\right)$ with $\tilde{L}_1 = UL_3\underline{C}W^{-1}$, $\tilde{L}_2 = UL_3\underline{D}_v$, and $\tilde{L}_3 = UL_3$ remains a solution of the UIO parameter matrices. To implement the fast rate residual by inverse lifting $r = \mathcal{L}^{-\infty}\underline{\nabla}$, U should be chosen to ensure the causality of residual generator so that the dependency of $r_l(k_s n_l + j)$ $(l = 1, 2, \ldots, N;\ j = 0, 1, \ldots, n_i - 1)$ on the future values in $\underline{\boldsymbol{\Psi}}(k_s)$, $\hat{\underline{\boldsymbol{\Psi}}}(k_s)$ and $\underline{v}(k_s)$ are removed. Because of the lower triangular structure of $\underline{D}_l$ $(l = 1, 2, \ldots, N)$ in $\underline{D}_v$, the causality issue enforces some of the entries of $\tilde{L}_3 = UL_3$ to be zero, *i.e.* $\tilde{L}_3(i, j) = 0$ if

$$\begin{cases} 1 \leqslant i \leqslant n_1, \quad 1 \leqslant j \leqslant n_1, \quad j > i \\ \text{or,} \begin{cases} 1 + \sum\limits_{k=1}^{m_j-1} n_k \leqslant j \leqslant \sum\limits_{k=1}^{m_j} n_k \leqslant Q, \quad N \geqslant m_j > 1 \\ \left(j - \sum\limits_{k=1}^{m_j-1} n_k - 1\right) T_{m_j} > (i-1)\, T_1, \quad 1 \leqslant i \leqslant n_1 \end{cases} \\ \text{or,} \begin{cases} 1 + \sum\limits_{k=1}^{m_i-1} n_k \leqslant i \leqslant \sum\limits_{k=1}^{m_i} n_k \leqslant Q, \quad N \geqslant m_i > 1 \\ (j-1)\, T_1 > \left(i - \sum\limits_{k=1}^{m_i-1} n_k - 1\right) T_{m_i}, \quad 1 \leqslant j \leqslant n_1 \end{cases} \\ \text{or,} \begin{cases} 1 + \sum\limits_{k=1}^{m_i-1} n_k \leqslant i \leqslant \sum\limits_{k=1}^{m_i} n_k \leqslant Q, \quad N \geqslant m_i > 1 \\ 1 + \sum\limits_{k=1}^{m_j-1} n_k \leqslant j \leqslant \sum\limits_{k=1}^{m_j} n_k \leqslant Q, \quad N \geqslant m_i > 1 \\ \left(j - \sum\limits_{k=1}^{m_j-1} n_k - 1\right) T_{m_j} > \left(i - \sum\limits_{k=1}^{m_i-1} n_k - 1\right) T_{m_i} \end{cases} \end{cases}$$

Then orthogonal matrix U can be determined based on the above constraints.

4 Numerical example

Consider an MSD system with continuous-time process

$$\begin{cases} \dot{\boldsymbol{x}}(t) = \begin{bmatrix} -1 & 5 \\ 0 & -2 \end{bmatrix} \boldsymbol{x}(t) + \begin{bmatrix} 1 \\ 1 \end{bmatrix} \boldsymbol{u}(t) + \begin{bmatrix} 0.1 & 0 \\ 1 & 1 \end{bmatrix} \boldsymbol{d}(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} \boldsymbol{f}(t) \\ \boldsymbol{y}(t) = \boldsymbol{x}(t) \end{cases}$$

The sampling time of D/A converter is $h = 0.5$ sec, the sampling times of A/D converters are $T_{y_1} = 1$ sec and $T_{y_2} = 1.5$ sec respectively. For this system we have

$$T_1 = 2, \quad T_2 = 3, \quad T_M = 6, \quad n_1 = 3, \quad n_2 = 2, \quad h = 0.5 \text{ sec}$$
$$A = \begin{bmatrix} 0.61 & 1.19 \\ 0 & 0.37 \end{bmatrix}, \quad B = \begin{bmatrix} 0.39 \\ 0.32 \end{bmatrix}, \quad B_d = \begin{bmatrix} 0.42 & 0.39 \\ 0.32 & 0.32 \end{bmatrix}, \quad B_f = \begin{bmatrix} 0.39 \\ 0.32 \end{bmatrix}$$

One solution of the optimal FDF parameter matrices is

$$G = \begin{bmatrix} -2.224 & 0 \\ 0 & -0.005 \end{bmatrix} \times 10^{-15}, \quad H_1 = \begin{bmatrix} 0 & 0 & 0 & -0.02 & -9.76 & -14.77 \\ 0 & 0 & 0 & 0 & 0.03 & -40.07 \end{bmatrix} \times 10^{-2}$$
$$H_2 = \begin{bmatrix} -5.18 & 0.01 & -1.31 & -24.19 & 0.32 \\ 0.04 & 0 & 0.01 & -0.14 & 0 \end{bmatrix} \times 10^{-2}$$
$$L_1 = \begin{bmatrix} 5.1208 & -0.8557 \\ -0.0018 & 0.0108 \\ 0 & 0 \\ -4.9388 & 3.2775 \\ 0 & 0 \end{bmatrix}, \quad L_2 = \begin{bmatrix} 0.06 & 0.25 & 0 & 0 & 0 & 0 \\ -16.58 & -66.12 & 0.11 & 0.41 & 0 & 0 \\ 0.06 & -0.47 & -9.39 & -67.42 & 0 & 0 \\ 0.07 & 0.28 & 0 & 0 & 0 & 0 \\ 0.01 & -0.06 & -70.09 & 9.16 & 0 & 0 \end{bmatrix} \times 10^{-2}$$

$$L_3 = \begin{bmatrix} 5.1501 & -0.0064 & 0 & -4.9852 & 0 \\ -0.0064 & 1.6954 & -0.0105 & -0.0073 & -0.0002 \\ 0 & -0.0105 & 1.7288 & 0 & -0.2349 \\ -4.9852 & -0.0073 & 0 & 7.9011 & 0 \\ 0 & -0.0002 & -0.2349 & 0 & 2.2619 \end{bmatrix}$$

$$U = \begin{bmatrix} -0.1842 & 0.2576 & 0.0604 & 0.9466 & -0.0070 \\ -0.2731 & 0.8378 & 0.3614 & -0.3041 & 0.0207 \\ 0.0361 & -0.3707 & 0.9264 & 0.0490 & 0.0261 \\ -0.9434 & -0.3071 & -0.0814 & -0.0948 & 0.0091 \\ 0.0120 & -0.0031 & -0.0305 & 0.0125 & 0.9994 \end{bmatrix}$$

That shows the feasibility of the proposed method.

5 Conclusion

In this paper, the fast rate FDF problem for a class of MSD systems has been investigated. The lifting technique has been used to convert an MSD system into LTI one and a UIO has been considered as residual generator. The FDF problem has been formulated as an H_∞ optimization problem and the existence condition for the solvability as well as an optimal solution have been obtained in terms of discrete-time algebraic Riccati equation. The causality of the designed residual generator has been guaranteed by utilizing the design freedom of the UIO parameters. Furthermore, the fast rate residual generator has been implemented by employing an inverse lifting operation. A numerical example has been given to demonstrate the validity of the proposed scheme.

References

1 Zhang P, Ding S X, Wang G, Zhou D. An FDI approach for sampled-data systems. In: Proceedings of American Control Conference, Arlington, USA: IEEE Press, 2001. 2702~2707

2 Zhang P, Ding S X, Wang G, Zhou D. An H_∞ approach to fault detection for sampled-data systems. In: Proceedings of American Control Conference, Anchorage, AK: IEEE Press, 2002. 2196~2201

3 Zhang P, Ding S X, Wang G, Zhou D. A frequency domain approach to fault detection in sampled-data system. *Automatica*, 2003, **39**(7): 1303~1307

4 Zhang P, Ding S X, Wang G, Zhou D. Fault detection for multirate sampled-data systems with time delays. *International Journal of Control*, 2002, **75**(18): 1457~1471

5 Fadali M S. Observer-based robust fault detection of multirate linear system using a lift reformulation. *Computers and Electrical Engineering*, 2003, **29**(1): 235~243

6 Fadali M S, Emara-Shabaik H J. Timely robust fault detection for multirate linear systems. *International Journal of Control*, 2002, **75**(5): 305~313

7 Izadi I, Zhao Q, Chen T. An optimal scheme for fast rate fault detection based on multirate sampled data. *Journal of Process Control*, 2005, **15**(3): 307~319

ZHONG Mai-Ying Received her Ph. D. degree in control theory and control engineering from Northeastern University, China, in 1999. She is now a professor at Shandong University. Her research interests include robust control, model based fault diagnosis, and fault tolerant systems and their application in industrial systems.

MA Chuan-Feng Ph. D. candidate. His research interests include model based fault diagnosis, fault tolerant systems and their application in industrial systems.

LIU Yun-Xia Ph. D. candidate. Her research interests include model based fault detection and isolation.