Symmetric Workpiece Localization Algorithms: Convergence and Improvements

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Abstract Symmetric workpiece localization algorithms combine alternating optimization and linearization. The iterative variables are partitioned into two groups. Then simple optimization approaches can be employed for each subset of variables, where optimization of configuration variables is simplified as a linear least-squares problem (LSP). Convergence of current symmetric localization algorithms is discussed firstly. It is shown that simply taking the solution of the LSP as start of the next iteration may result in divergence or incorrect convergence. Therefore in our enhanced algorithms, line search is performed along the solution of the LSP in order to find a better point reducing the value of objective function. We choose this point as start of the next iteration. Better convergence is verified by numerical simulation. Besides, imposing boundary constraints on the LSP proves to be another efficient way.

Key words Symmetric workpiece localization, algorithm, improvement, line search

1 Introduction

Localization refers to the problem of workpiece alignment with respect to the machine frame. Traditionally, workpiece is physically aligned by use of special tools and fixtures based on the 3-2-1 approach. Workpiece localization problem in the context of this paper is to automatically determine the location of the workpiece with respect to the machine frame, where the workpiece is allowed to be arbitrarily fixed on the machine table. A set of discrete points are sampled on the surface and best fitted to the nominal model in the least-squares (LS) sense. It is fundamental to fixtureless manufacturing, which reduces cost while increase productivity considerably. On the other hand, as there are often no explicit datum references on complex surfaces, precision localization is hardly completed physically. Localization is then very important and even critical. Another motivation lies in the similar mathematics as with the geometrical tolerance assessment problem[1]. Localization is a special case of LS evaluation of profile tolerances.

Based on the theory of configuration space of geometrical features, the localization problem can be grouped into three types: regular localization, symmetric localization, and hybrid localization/envelopment[2∼4]. Generally, the configuration space of a rigid body is identical with the special Euclidean group $SE(3)$. While for symmetric features, the configuration space is identified with the quotient group $SE(3)/G_0$, where G_0 is the symmetry subgroup of features. Taking the identity element I as symmetry subgroup of asymmetric features, the regular localization can be considered as trivial case of the symmetric localization. And the hybrid localization/envelopment is divided into symmetric localization and envelopment problems. Therefore symmetric localization algorithms are fundamental.

2 Symmetric localization problem and algorithms

Let C_M and C_W denote the CAD model frame and the machine reference frame, respectively. $g = (P, R) \in SE(3)/G_0$ represents the configuration of C_M with respect to C_W. G_0 is the symmetry subgroup of the feature. A base of the Lie algebra of $SE(3)/G_0$ is $\{\hat{\eta}_1, \cdots, \hat{\eta}_r\}$, where $\hat{\eta}_j$ is called the twist[2]. Suppose the feature is sampled and a set of measurements $y = \{\boldsymbol{y}_i \in R^3, i = 1, 2, \cdots, n\}$ is obtained. The symmetric localization problem can be formulated as follows.

Problem 1. (Symmetric localization)[2] Find the optimal configuration $g \in SE(3)/G_0$ of C_M with respect to C_W, such that the function defined in the following.

$$\varepsilon(\boldsymbol{g}, \boldsymbol{x}) = \sum_{i=1}^{n} \|\boldsymbol{y}_i - \boldsymbol{g}\boldsymbol{x}_i\|^2 \tag{1}$$

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$$\varepsilon(\boldsymbol{g},\boldsymbol{x})=\sum_{i=1}^{n}\langle\boldsymbol{g}^{-1}\boldsymbol{y}_i-\boldsymbol{x}_i,\boldsymbol{n}_i\rangle^2 \tag{2}$$

is minimized, where $\boldsymbol{n}_i$ is the unit normal vector at point $\boldsymbol{x}_i$ on the home surface.

Problem 1 is a bi-level (configuration $\boldsymbol{g}$ and home surface point $\boldsymbol{x}$) optimization problem. There are usually two kinds of methods for the problem: the simultaneous optimization (SO) and the alternating optimization (AO). The basic idea of AO is optimizing the individual (set of) variable(s) separately, with the other (set of) variable(s) fixed. Repeat the individual optimization alternatingly and the iterations will finally converge to a solution. Generally speaking, converging speed of AO-type algorithms is lower than that of SO-type algorithms. Moreover, oscillating is possible to occur as a result of ignoring the interaction of variables[5]. But in some cases AO-type algorithms seem much more efficient. Particularly, when there is a natural division of variables into several subsets for which explicit partial minimizer formulae exist, AO-type algorithms exhibit better convergence performance[6].

Combining the AO method and linearization, the symmetric localization algorithms are given as follows.

Algorithm 1. (Symmetric localization algorithm)[2]

1) Initialization: Set $k=0$. Choose an initial configuration $\boldsymbol{g}^0$.
2) Find home surface points $\boldsymbol{x}^k$ with $\boldsymbol{g}^k$ fixed. Calculate the function value $\varepsilon(\boldsymbol{g}^k,\boldsymbol{x}^k)$.
3) Solve the configuration optimization subproblem to get $\boldsymbol{g}^{k+1}$ with $\boldsymbol{x}^k$ fixed.
4) Find $\boldsymbol{x}^{k+1}$ with $\boldsymbol{g}^{k+1}$ fixed. Calculate the function value $\varepsilon(\boldsymbol{g}^{k+1},\boldsymbol{x}^{k+1})$.
5) If $|1-\varepsilon(\boldsymbol{g}^{k+1},\boldsymbol{x}^{k+1})/\varepsilon(\boldsymbol{g}^k,\boldsymbol{x}^k)|>\delta$, set $k=k+1$ and go to 3); otherwise quit.

In step 3) using the linearization

$$\boldsymbol{g}^{k+1}=\boldsymbol{g}^k e^{\hat{m}}\approx\boldsymbol{g}^k(I+\hat{\boldsymbol{m}}) \tag{3}$$

where

$$\hat{\boldsymbol{m}}=\sum_{j=1}^{r}m_j\hat{\boldsymbol{\eta}}_j \tag{4}$$

the configuration optimization subproblem can be simplified as a sequence of linear LS problems (LSPs). Taking (1) as objective function, the LSP is as follows.

$$\min_{m}\sum_i\|\boldsymbol{y}_i-\boldsymbol{g}^k(I+\hat{\boldsymbol{m}})\boldsymbol{x}_i^k\|^2 \tag{5}$$

According to the necessary first order optimality conditions , the solution m to the LSP is obtained by solving linear equation

$$G\boldsymbol{m}=\boldsymbol{h} \tag{6}$$

where

$$G=\left(\sum_i(\boldsymbol{x}_i^k)^{\mathrm{T}}\hat{\eta}_l^{\mathrm{T}}\hat{\eta}_j\boldsymbol{x}_i^k\right),\quad \boldsymbol{h}=\left(\sum_i((\boldsymbol{g}^k)^{-1}\boldsymbol{y}_i-\boldsymbol{x}_i^k)^{\mathrm{T}}\hat{\eta}_j\boldsymbol{x}_i^k\right) \tag{7}$$

It is referred to as the tangent symmetric localization (TSL) algorithm[2].

When (2) is adopted as objective function, the LSP is as follows

$$\min_{m}\sum_i\langle(I-\hat{\boldsymbol{m}})(\boldsymbol{g}^k)^{-1}\boldsymbol{y}_i-\boldsymbol{x}_i^k,\boldsymbol{n}_i^k\rangle^2 \tag{8}$$

It can be written in the standard form of linear LSP

$$\min\|\boldsymbol{S}^{\mathrm{T}}\boldsymbol{m}-\boldsymbol{t}\|^2 \tag{9}$$

where

$$\boldsymbol{S}=(s_{j,i})=((\boldsymbol{n}_i^k)^{\mathrm{T}}\hat{\boldsymbol{\eta}}_j\boldsymbol{x}_i^k)=((\boldsymbol{n}_i^k)^{\mathrm{T}}\hat{\boldsymbol{\eta}}_j(\boldsymbol{g}^k)^{-1}\boldsymbol{y}_i),\ \boldsymbol{t}=(\boldsymbol{t}_i)=((\boldsymbol{n}_i^k)^{\mathrm{T}}((\boldsymbol{g}^k)^{-1}\boldsymbol{y}_i-\boldsymbol{x}_i^k)) \tag{10}$$

It is called the Fast symmetric localization (FSL) algorithm[2].

It was proved that iterations of the TSL algorithm will converge to a local minimum $(\boldsymbol{g}^*,\boldsymbol{x}^*)$, if an initial configuration $\boldsymbol{g}^0$ is properly chosen such that a unique minimum exists in the area $U=\{(\boldsymbol{g},\boldsymbol{x})\in$

$SE(3) \times f|\varepsilon(\boldsymbol{g}, \boldsymbol{x}) \leqslant (\boldsymbol{g}^0, \boldsymbol{x}^0)\}$[3]. One of the necessary conditions is the monotone decreasing property of the iteration sequence

$$\varepsilon(\boldsymbol{g}^k, \boldsymbol{x}^{k-1}) \geqslant \varepsilon(\boldsymbol{g}^k, \boldsymbol{x}^k) \geqslant \varepsilon(\boldsymbol{g}^{k-1}, \boldsymbol{x}^k) \tag{11}$$

where the two inequalities are guaranteed by optimization of $\boldsymbol{x}$ and $\boldsymbol{g}$ respectively. As (8) is adopted in the FSL algorithm, where $\boldsymbol{n}_i^k$ is the normal at point $(\boldsymbol{g}^k)^{-1}\boldsymbol{y}$ but not necessarily the normal at point $(\boldsymbol{g}^{k+1})^{-1}\boldsymbol{y}_i$, inequality (11) is not guaranteed (see Fig. 1). However, if the FSL algorithm converges, the speed is higher than the TSL algorithm.

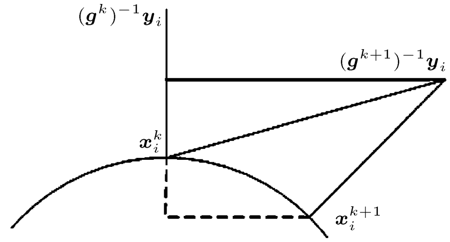


Fig. 1 False iteration in the FSL algorithm

Actually, due to local validity of linearization, the solution to LSP is not always the minimum of the original configuration optimization subproblem. It may result in an increasing objective and violate inequality (11) even for the TSL algorithm, which leads to incorrect convergence or divergence. Such problems arose in simulations in [4]. It was advised that the step of variables be sufficiently small. Two strategies are adopted here to improve the performance of the TSL and FSL algorithms.

3 Improvements to the symmetric localization algorithms

Solution $\boldsymbol{m}^k$ to the LSP decreases the function value of the linearized problem and hence indicates a descent direction for the original configuration optimization subproblem. Searching along this direction will probably reach a better point, at which the objective will absolutely not increase. Suppose the better step length is λ^k. Substituting $\lambda^k\boldsymbol{m}^k$ for $\boldsymbol{m}^k$ in the TSL or FSL algorithms can improve the convergence performance. For the TSL algorithm, apply the line search

$$\min_{0<\lambda<1} \sum_i \|\boldsymbol{y}_i - \boldsymbol{g}^k e^{\lambda\hat{m}} \boldsymbol{x}_i^k\|^2 \tag{12}$$

While for the FSL algorithm, line search is as follows

$$\min_{0<\lambda<1} \sum_i \langle e^{-\lambda\hat{m}} (\boldsymbol{g}^k)^{-1}\boldsymbol{y}_i - \boldsymbol{x}_i^k, \boldsymbol{n}_i^k\rangle^2 \tag{13}$$

An interval containing the local minimum need to be determined before the line search. Experience from the gradient-type algorithms for unconstrained optimization tells us that a locally good direction is not necessarily globally good. Therefore we need not find the minimum in the line search, for the sake of saving time.

Equation (8) adopted in the FSL algorithm is linearization of the following.

$$\min_m \sum_i \|(I - \hat{\boldsymbol{m}})(\boldsymbol{g}^k)^{-1}\boldsymbol{y}_i - \boldsymbol{x}_i^k\|^2 \tag{14}$$

The solution to the LSP in the FSL algorithm is not necessarily a good descent direction for (12). Even when (13) is utilized, convergence is not guaranteed. However, imposing boundary constraints on the LSP proves to improve the performance. Thus we obtain the Ball-constrained LSP (BCLSP).

Problem 2. (Ball constrained LSP in the FSL algorithm) Imposing boundary constraints on the LSP (8) yields the following BCLSP

$$\begin{aligned} &\min_m \sum_i \langle (I - \hat{\boldsymbol{m}})(\boldsymbol{g}^k)^{-1}\boldsymbol{y}_i - \boldsymbol{x}_i^k, \boldsymbol{n}_i^k\rangle^2 \\ &\text{s.t. } \|\boldsymbol{m}\| \leqslant \alpha \end{aligned} \tag{15}$$

Readers are referred to [7] for the algorithm for the BCLSP. Major computation involves solving a nonlinear equation and singular value decomposition.

The line search and BCLSP are proposed to obtain stable convergence for the TSL and FSL algorithms. We call them stabilized TSL (STSL) and stabilized FSL (SFSL) algorithms, respectively.

Algorithm 2. (STSL/SFSL algorithms)

Input: Measurements $\boldsymbol{y} = \{y_i \in R^3, i = 1, 2, \cdots, n\}$; CAD model and the base $\{\hat{\eta}_1, \cdots, \hat{\eta}_r\}$.

Output: Optimal configuration $\boldsymbol{g}$.

1) Initialization : Set $k = 0$. Choose an initial configuration $\boldsymbol{g}^0$.

2) Find $\boldsymbol{x}^k$ with $\boldsymbol{g}^k$ fixed. Calculate the function value $\varepsilon(\boldsymbol{g}^k, \boldsymbol{x}^k)$.

3) Solve the configuration optimization subproblem to get $\boldsymbol{m}^k$ with $\boldsymbol{x}^k$ fixed.

4) Apply the line search along the direction $\boldsymbol{m}^k$ and get the step length λ^k.

5) Fix $\boldsymbol{g}^{k+1} = \boldsymbol{g}^k e^{\lambda^k \hat{\boldsymbol{m}}}$ and calculate $\boldsymbol{x}^{k+1}$. Calculate the function value $\varepsilon(\boldsymbol{g}^{k+1}, \boldsymbol{x}^{k+1})$.

6) If $|1 - \varepsilon(\boldsymbol{g}^{k+1}, \boldsymbol{x}^{k+1})/\varepsilon(\boldsymbol{g}^k, \boldsymbol{x}^k)| > \delta$, set $k = k + 1$ and go to 3); otherwise quit.

Remarks

1) For the SFSL algorithm, solve the BCLSP in step 3);

2) The line search is defined in (12) and (13) respectively for the STSL and SFSL algorithms;

3) For features such as circle and sphere, whose configuration spaces are 2D or 3D translation groups, the line search is skipped, since the two sides of (3) are strictly equal.

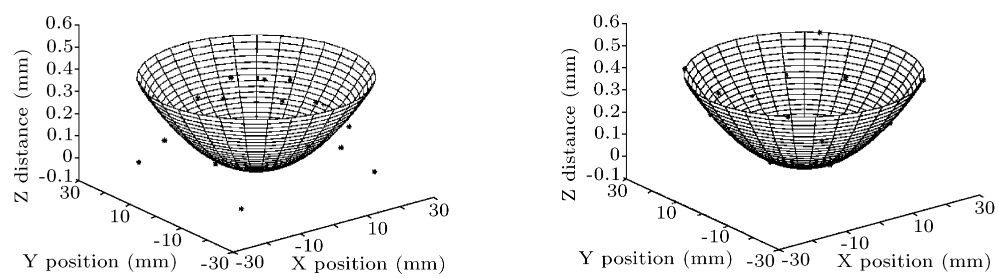
4 Simulations

We take example of an axisymmetric paraboloid. Localization of it is equivalent to assessment of the LS profile tolerance. The nominal surface is described by equation $x^2 + y^2 = 2pz$, where $p = 1000$ is twice focal length. In the area $U = \{(x, y)|x \in [-20, 20], y \in [-20, 20]\}$, 25 points are sampled once every 10 units. They are transformed with given configuration to simulate the real measurements. The given configuration is rotating around the vector $(1\ 1\ 0)^{\mathrm{T}}$ by 80 degrees. Initial configuration is set to be the identity element. The results are tabulated in Table 1, where the TSL, FSL, STSL, and SFSL algorithms are applied respectively with the same terminating condition $\delta = 10^{-6}$.

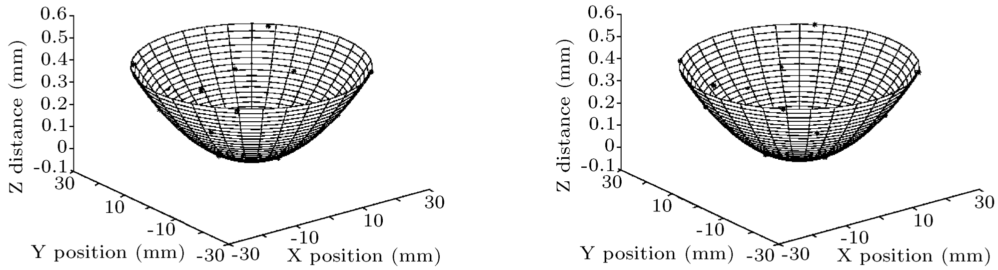
Table 1 Comparison of results of different algorithms

Algorithm	Resultant configuration	Iterations	Algorithm	Resultant configuration	Iterations
TSL	$\begin{bmatrix} 0.4133 & 0.5867 & -0.6964 & 0.2990 \\ 0.5867 & 0.4133 & 0.6964 & -0.2990 \\ 0.6964 & -0.6964 & -0.1735 & -0.0952 \\ 0 & 0 & 0 & 1 \end{bmatrix}$	6	FSL with bound constraints	$\begin{bmatrix} 0.5868 & 0.4132 & 0.6964 & 0 \\ 0.4132 & 0.5868 & -0.6964 & 0 \\ -0.6964 & 0.6964 & 0.1736 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$	4
STSL	$\begin{bmatrix} 0.5867 & 0.4133 & 0.6964 & -0.0270 \\ 0.4133 & 0.5867 & -0.6964 & 0.0270 \\ -0.6964 & 0.6964 & 0.1734 & 0.2166 \\ 0 & 0 & 0 & 1 \end{bmatrix}$	2	SFSL	$\begin{bmatrix} 0.5868 & 0.4132 & 0.6964 & 0 \\ 0.4132 & 0.5868 & -0.6964 & 0 \\ -0.6964 & 0.6964 & 0.1736 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$	4
FSL	Diverge	–			

The measurements are again transformed back into the model frame with the resultant configurations (see Fig. 2, where "∗" marks the measurements). It shown that the workpiece is correctly located using the SFSL, STSL and FSL algorithms with boundary constraints. In contrast, the TSL algorithm converges falsely. The improved algorithms are robust against the initial conditions. They also show better convergence performance in our machining practice of aspheric surface, where localization of the surface involves a large data set.



(a) TSL (b) FSL with boundary constraints ($\alpha = 1$)



(c) STSL (d) SFSL ($\alpha = 1$)

Fig. 2 Workpiece localization in the model frame using different algorithms

5 Conclusions

Local convergence is not guaranteed in the current symmetric localization algorithms. Combining the line search technique, the STSL algorithm yields a monotone decreasing sequence, which ensures convergence. Performance of the FSL algorithm is also improved by imposing boundary constraints to the LSP and then applying the line search. Such improvements are applicable to other successive linearization algorithms, such as the envelopment algorithm for the hybrid localization/envelopment problem, LS algorithms and the symmetric minimum zone (SMZ) algorithm for form error evaluation[1].

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