Application of MSPCA to Sensor Fault Diagnosis

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Abstract A multiscale principal component analysis method is proposed for sensor fault detection and identification. After decomposition of sensor signal by wavelet transform, the coarse-scale coefficients from the sensors with strong correlation are employed to establish the principal component analysis model. A moving window is designed to monitor data from each sensor using the model. For the purpose of sensor fault detection and identification, the data in the window is decomposed with wavelet transform to acquire the coarse-scale coefficients firstly, and the square prediction error is used to detect the failure. Then the sensor validity index is introduced to identify faulty sensor, which provides a quantitative identifying index rather than qualitative contrast given by the approach with contribution. Finally, the applicability and effectiveness of the proposed method is illustrated by sensors of industrial boiler.

Key words Principal component analysis, wavelet transform, multiscale, square prediction error, sensor validity index

1 Introduction

Analytical redundancy as an effective method for sensor fault diagnosis includes analytical redundancy based on the sensor′s mathematical model and analytical redundancy based on multi-sensors[1]. As an effective tool for modeling multi-sensors, the principal component analysis is often applied into sensor fault diagnosis and data reconstruction[2,3]. In 1998, Bakshi proposed the multiscale principal component analysis (MSPCA) approach, which integrates PCA with wavelet analysis[4]. MSPCA simultaneously extracts both cross correlation across sensors (the PCA approach) and auto-correlation within a sensor (the wavelet approach), and establishes models with coefficients of each scale. In [5] the authors firstly detected sensor fault with MSPCA model, in which the statistical variable T^2 of the principal component space was monitored. The square prediction error (SPE) was employed to detect sensor drifting error and identify faulty sensor by contribution to SPE of each sensor[6].

In this paper, multi-scale is used to eliminate noise in the sensor data, and the PCA model is established at the appropriate scale for sensor fault diagnosis based upon the principle of MSPCA. After detecting the sensor fault with SPE, we employ sensor validity index (SVI) for fault sensor identification. While the identifying approach using contribution just provides a qualitative standard, the identifying approach using SVI can not only give a quantitative standard but also discriminate abnormal operation from sensor fault.

2 Multiscale analysis

Conventional approach with FFT just decomposes signal into frequency domain without time discrimination. Yet, wavelet transform can well localize the characteristic of a signal both in time and frequency domains. So the key information can be extracted from signal by multi-scale to eliminate noise effectively.

Suppose the sensor signal is $f(x)$, whose wavelet transformation is defined as below:

$$W_f(a,b)=\int_R f(x)\bar{\psi}_{(a,b)}(x)\mathrm{d}x=\frac{1}{\sqrt{|a|}}\int_R f(x)\bar{\psi}\left(\frac{x-b}{a}\right)\mathrm{d}x \tag{1}$$

where $\psi(x)$ is the wavelet base function.

Mallat proposed the theory about multi-resolution analysis for designing the orthogonal wavelet base, and presented the fast algorithm for the design of orthogonal wavelet base and the orthogonal

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wavelet transform, that is, the Mallat algorithm[7]. Based on multi-resolution analysis, the sub-space V_0 of multi-resolution analysis can be approximated with finite number of sub-spaces, *i.e.*,

$$V_0 = V_1 \oplus W_1 = V_2 \oplus W_2 \oplus W_1 = V_3 \oplus W_3 \oplus W_3 \oplus W_2 \oplus W_1 = \cdots \tag{2}$$

Therefore, sensor signal $f(x) \in V_0$ can be decomposed into fine scale W_1 and coarse scale V_1 . After that, the approximation of coarse scale can be decomposed further. With repetition of decomposition, approximation at any scale can be achieved.

3 Diagnosis model with PCA

PCA is a modeling method that is independent of the knowledge about the systematic mathematical model. The output of sensors can establish a statistical model for sensor fault diagnosis sensors[2].

3.1 Sensor fault detection with PCA

In normal condition the sensor database $X_{m\times n}$ is acquvied, where m is the number of samples, n is the number of sensors. Then $X_{m\times n}$ will be standardized to eliminate the effects of different unitages of variables, namly,

$$\bar{X} = D_\sigma^{-1}[\boldsymbol{X} - E(x)] \tag{3}$$

where $\boldsymbol{X} = [x_1, x_2, \cdots, x_n]^{\mathrm{T}} \in \Re^n$ is the data vector at the specified point in $X_{m\times n}$, $E(x) = [\mu_1, \mu_2, \cdots, \mu_n]^{\mathrm{T}}$ is the mean vector of $X_{m\times n}$. $D_\sigma = \mathrm{diag}(\sigma_1, \sigma_2, \cdots, \sigma_n)$, where $\sigma_i = \sqrt{E(x_i - \mu_i)^2}$ is the ith standard variance of $X_{m\times n}$. So the standard database $\bar{X}_{m\times n}$ is achieved. With $\bar{X}_{m\times n}$, the correlation matrix is calculated and then is singularly decomposed. At last, $\bar{X}_{m\times n}$ is projected into the principal component space $\hat{X}$ and residual space $\tilde{X}$, *i.e.*,

$$\bar{X}_{m\times n} = \hat{X} + \tilde{X} = \hat{C}\bar{X}_{m\times n} + \tilde{C}\bar{X}_{m\times n} \tag{4}$$

where the projection matrix $\hat{C} = PP^{\mathrm{T}}$ and $\tilde{C} = \tilde{P}\tilde{P}^{\mathrm{T}} = I - \hat{C}$.

The statistical variable in the sub-space should be specified to detect sensor fault. [8] presented the contrast between some statistical variables, and proved that SPE in the residual space was more appropriate to reflect the change of correlation between sensors′ output than Hotelling T^2 in the principal component space. So, SPE is monitored in real time to detect sensor fault. The definition of SPE is as below:

$$SPE = \|\tilde{C}\bar{x}\|^2 \leqslant \delta_{SPE}^2 \tag{5}$$

where δ_{SPE} is the threshold of SPE, which can be calculated with the samples distribution of SPE[9].

3.2 Faulty sensor identification with SVI

Firstly, E residual is defined as follows

$$E_i = SPE_i - \frac{(\hat{x}_i - x_i)^2}{1 - c_{ii}} \tag{6}$$

Based upon (5), E residual presents the result that the model estimation using the vector achieved above is subtracted from the adjusted measurement vector. It could describe the effect of the faulty sensor clearly.

Now an index will be defined to truly describe the characteristic of sensor. With accurate measurement range, this index should have nothing to do with the number of principal components, noise, variable variance, and the mode of fault. Indeed, the ratio between E residual and SPE is such a kind of index[10], which is defined as:

$$\eta_i^2 = \frac{E_i}{SPE_i} = 1 - \frac{(\hat{x}_i - x_i)^2}{(1 - c_{ii})\|\tilde{C}\bar{x}_i\|} \tag{7}$$

Based upon (6), we know $\eta_i \in (0, 1]$, which is called the sensor validity index–SVI.

If SVI approaches 1, then the alteration of this sensor is consistent with other ones. If the ith sensor is faulty, the corresponding η_i will approach 0. When multiple sensors′ alteration destroys the correlation from the principal component model, the expression is just like

$$\frac{(\hat{x}_i - x_i)^2}{(1 - c_{ii})\|\tilde{C}\bar{x}_i\|} \to 0 \tag{8}$$

Because the ith measurement value is not used when calculating E_i, the index can discriminate the abnormal operation from sensor fault. Meanwhile, this method for faulty sensor identification is different from the method with contribution to SPE that gives just the qualitative identification.

4 Model for sensor fault diagnosis with MSPCA

By wavelet decomposition of each sensor′s output, MSPCA integrates PCA with multi-scale analysis of wavelet to discriminate the sensor′s faulty characteristics from others. The sensor′s fault characteristics are mainly included in the wavelet coefficients of coarse scale, while noise and other characteristics with normally operating correlation pattern are included among the wavelet coefficients of fine scales. To detect sensor failure more sensitively, MSPCA algorithm here changes the conventional PCA modeling with the output of sensors directly, meanwhile establishes the PCA model with the wavelet coefficients in the appropriate scale of each sensor′s data. For the wavelet coefficients in the coarse scale possess the faulty characteristics of sensor drifting, which is the fault mode studied here, we establish the PCA mode at the coarse scale. The accuracy and sensitivity for detection is improved, meanwhile the faulty sensor identification method with quantitative index of SVI is more convenient and effective than the identifying method with the contribution before.

The process of sensor fault diagnosis with MSPCA is below.

1) Ascertain the length of the moving window as $W = 2^N$, where N is a positive integer.

2) Acquire W samples of each sensor′s output.

3) Decompose the data in the moving window into L layers with wavelet transformation to get $W \times 2^{-L}$ coefficients in the coarse scale.

4) Establish the PCA model with the coefficients above, calculate the average and the standard variance of the coefficients and then save them.

5) Move the window to involve new data, meanwhile the oldest data are moved out of the window. So the length of the window W is constant.

6) Decompose the data in the moving window into L layers with wavelet transformation to get $W \times L^{-2}$ coefficients in the coarse scale. The last coefficient is kept.

7) Standardize the coefficients in 6) with the average and standard variance in 4).

8) Calculate the SPE_i and SVI_i with PCA model established above.

9) Goto 5), and decompose the new sensor data of the moving window with wavelet transformation, and so on.

The average and standard variance of the modeling data were not used to standardize the new data of the moving window in [6], but the average and standard variance of the new data. Now that PCA model is established with unitary data, for consistent contrast to the modeling data the data in real time should be standardized with the same scale. Hence, the average and standard variance of the modeling data are used to standardize the new data.

5 Diagnosis example

Modern industrial boiler is required to be equipped with monitor system. Hardware redundancy is often adopted to assure the reliability of the measurement values. For the purpose of cost efficiency, software redundancy is introduced in this paper. We sample the output of the sensors, and establish the MSPCA model with these correlative signals to implement sensor fault diagnosis to ensure the normal operation of the system. Three correlative signals of an industrial boiler are introduced — furnace pressure, stem flow, and feedwater flow.

With the method of MSPCA introduced above, we specify the length of moving window $W = 256$ first. Before we decompose the sensor data with wavelet transform, the wavelet base function and the scale of decomposition of wavelet transform should be specified. Harr wavelet is the orthogonal wavelet function which is the pioneer base with compactly supported characteristic. Meanwhile, the corresponding filter is causal. When the length of the moving window is a power of 2, wavelet transform with Harr will not cause any effectiveness of delay[11]. So the Harr wavelet is chosen to be the wavelet base function. By the orthogonal testing, the scale of decomposition is specified as $L = \log_2 W - 5 = 3$.

256 samples of each sensor in normal condition are acquired, then we decompose these samples with wavelet transformation to get the wavelet coefficients in coarse scale. Then, sensor fault diagnosis

can be implemented with the MSPCA model introduced above. To test the model for sensor fault diagnosis, the data from furnace pressure from the 100^{th} sample are involved with a 0.00006MPa per minute drift. The diagnosis results with PCA (Fig. 1) and MSPCA (Fig. 2) are below. By contrast, the approach with MSPCA can detect failure at about 120th point, while the approach with PCA can not detect any failure. So the result proves the sensitivity of MSPCA for failure detection.

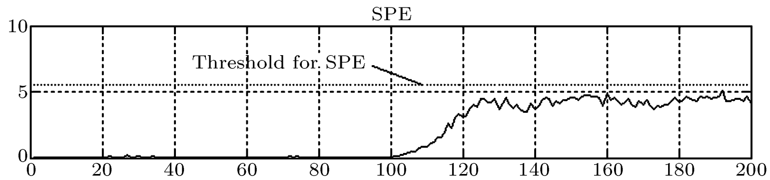

Fig. 1 The SPE curve with PCA

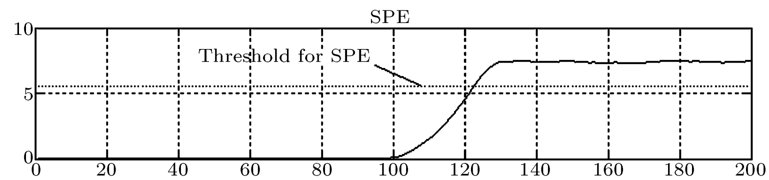

Fig. 2 The SPE curve with MSPCA

Each sensor′s contribution to SPE is presented in Fig. 3, and the corresponding SVI is presented in Fig. 4. The SVI values of furnace pressure approach 0 soon after 100^{th} sample, while the SVI values of the other two sensors still approach 1. Therefore, the furnace pressure sensor is discriminated. When contributions to SPE is applied to identify the faulty sensor, the contribution of furnace pressure is qualitatively larger than those of the other two sensors as in Fig. 3 without an explicit quantitative index from SVI.

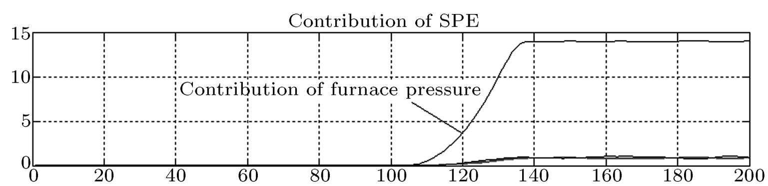

Fig. 3 The contribution to SPE

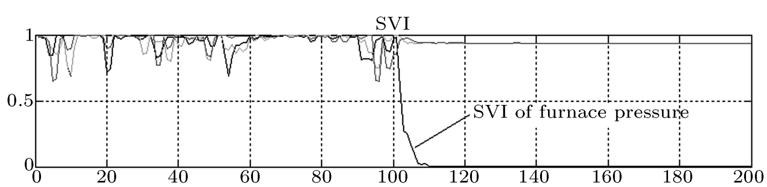

Fig. 4 The SVI curve

6 Conclusion

The method based on SVI to identify the faulty sensor possesses some advantages as below.

1) The value of SVI is between 0 and 1, so the threshold can be specified easily.

2) If the ith sensor is faulty, SPE_i will exceed its threshold. Yet, E_i will not change and η_i still approaches 0. So, this index is sensitive to sensor fault very much.

Moreover, the accuracy and effectiveness for sensor fault diagnosis are enhanced when SVI is integrated with multi-scale wavelet transformation to extract the characteristic information from the sensor signal.

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