Hierarchical Switching Control of Multiple Models Based on Robust Control Theory

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Abstract A new hierarchical switching control system of multiple models based on robust control theory is designed for some plant with large uncertainties. The model set and controller set are designed by robust control theory and the characteristics of robust control system are taken into account. A new kind of switching index function by estimating uncertainty is designed. Furthermore, stability of the closed system is analyzed by the small gain theorem in the sense of exponentially weighted L^2 norm. And simulation is done on a plant with both parameter uncertainty and unmodeled dynamics. Both theoretical analysis and simulation results show that this new hierarchical switching control system can control the plant with large uncertainties effectively and has good performance of tracking and stability.

Key words Multiple model control, switching control, supervisory control, robust control

1 Introduction

Plant models are inherently inaccurate and if the model uncertainties are "sufficiently small", modern control theories can be used to design the controller that ensures satisfactory closed loop performance. But for " large" model uncertainties, common sense suggests and simple examples prove that no single, fixed-parameter controller can possibly control in a satisfactory way or stabilize the plant[1]. It is an efficient way to design hierarchical switching controller based on switching index function to control the plant with large uncertainties[1~5]. The switching index of present hierarchical switching control systems is mainly generated by some norm-squared output estimation errors. Motivation for this idea is obvious: the nominal model whose estimation error is the smallest can best approximate the plant and thus the candidate controller designed based on the model ought to be able to control the plant satisfactorily[1]. However, for different control theories, the indexes to evaluate model errors are different. Present hierarchical switching control systems do not take the characteristics of different control theories into account fully and their switching indexes are only generated by some norm-squared output estimation errors. This leads to the following disadvantages. First, they go against performance enhancement of the control system. Second, they do not fully utilize the robustness of the control system to reduce number of the models and a lot of models are always needed for a plant with some uncertainties. [6] uses 186 fixed models and 2 adaptive models for a plant with some parameter uncertainties. This prevents the hierarchical switching control of multiple models from practical application.

Considering the above problems, in this paper a new hierarchical switching control system of multiple models based on robust control theory is designed. The induced exponentially weighted L^2 norm is adopted to evaluate the uncertainties and the plant uncertainty is covered by a set of linear fractional uncertainty models. For each member of the model set, a robust controller is designed which can control the corresponding uncertainty model satisfactorily, and the control set is made up of these robust controllers. Considering the characteristics of robust control system, a switching index function by estimating the uncertainty is designed to determine which controller should be switched into the closed loop. The stability of the closed switching system is analyzed by the small gain theorem in the sense of exponentially weighted L^2 norm. Simulation is also done on a plant with both parameter uncertainty and unmodeled dynamics to validate the effectiveness of this new hierarchical switching control system. Both theoretical analysis and simulation results show that this new hierarchical switching control system can satisfactorily control the plant with large uncertainties and has good performance of tracking and stability.

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2 Control system design

The hierarchical switching control system of multiple models based on robust control theory is depicted in Fig. 1.

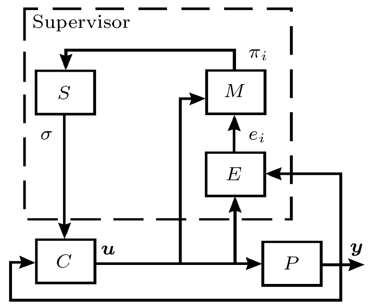


Fig. 1 The hierarchical switching control system

The control system has two layers: the upper layer is the supervisor, whose inputs are the input $\boldsymbol{u}$ and whose output $\boldsymbol{y}$ of the plant and whose output is the switching signal σ that determines which controller should be switched into the closed loop. The lower layer is the controller set C and the plant P to be controlled.

In this paper, the plant uncertainty is covered by the set P of linear fractional uncertainty models, viz. for any possible plant P, there exists $P_i \in P$ that can describe it. And the model set P is

$$P = \left\{ P_i(s) = M_{22i}(s) + M_{21i}(s)\Delta_i(I - M_{11i}(s)\Delta_i)^{-1}M_{12i}(s), \|\Delta_i\|_\infty^\delta < 1, i = 1 \cdots N \right\} \tag{1}$$

where $M_i(s) = \begin{bmatrix} M_{11i}(s) & M_{12i}(s) \\ M_{21i}(s) & M_{22i}(s) \end{bmatrix}$ is the time invariant part of the linear fractional model, Δ_i is the uncertainty part, $\| \bullet \|_\infty^\delta$ is induced exponentially weighted L^2 norm and defined as[7]

$$\|H\|_\infty^\delta = \sup_{\|\boldsymbol{u}\|_2^\delta = 1} \|H\boldsymbol{u}\|_2^\delta$$

where $\|\boldsymbol{u}\|_2^\delta = (\int_0^\infty e^{-\delta(t-\tau)}u^T(\tau)u(\tau)\mathrm{d}\tau)^{0.5}$ is exponentially weighted L^2 norm, δ is a positive constant. Each member of the model set P satisfies

A1) $M_{11i}(s) = 0$.

A2) $M_{21i}(s)$ is inversable and the inverse system is stable.

A3) $M_{21i}(s)\Delta_i M_{12i}(s)$ is analytical in $Re[s] > -0.5\delta$, *viz.* the real parts of all poles are small than -0.5δ.

For each member of the model set P, a robust controller can be designed and the controller set C is

$$C = \{K_i(s), i = 1 \cdots N\} \tag{2}$$

where $K_i(s)$ is the stable transfer function and satisfies

A4) $\|M_{12i}(s)(I - K_i(s)M_{22i}(s))^{-1}K_i(s)M_{21i}(s)\|_\infty^\delta < 1, i = 1 \cdots N$.

Assumption $A4$ means that $K_i(s)$ can robustly stabilize $P_i(s)$ in the sense of exponentially weighted L^2 norm. It′s very difficult to design the controller that satisfies $A4$ directly. Fortunately, induced exponentially weighted L^2 norm and induced L^2 norm have the relationship[7]:

$$\|H(s)\|_\infty^\delta = \|H(s - 0.5\delta)\|_\infty \tag{3}$$

where $H(s)$ is analytical in $Re[s] > -0.5\delta$. So the above controller design problem can be converted to a usual robust controller design problem. For practical use, the controller set $\boldsymbol{C}$ is described by a state shared dynamic system[1]

$$\dot{\boldsymbol{x}}_c = \begin{bmatrix} A_c & 0 \\ 0 & A_c \end{bmatrix} \boldsymbol{x}_c + \begin{bmatrix} B_{C1} \\ 0 \end{bmatrix} \boldsymbol{u} + \begin{bmatrix} 0 \\ B_{C2} \end{bmatrix} \boldsymbol{y}, \quad \boldsymbol{u} = C_{Ci}\boldsymbol{x}_C + D_{Ci}\boldsymbol{y} \tag{4}$$

where A_C is a stable constant matrix, B_{C1} and B_{C2} are constant matrixes, C_{Ci} and D_{Ci} satisfy

$$C_{Ci}\left[sI-\left(\begin{bmatrix}A_c & 0\\ 0 & A_c\end{bmatrix}+\begin{bmatrix}B_{C1}C_{Ci}\\ 0\end{bmatrix}\right)\right]^{-1}\begin{bmatrix}B_{C1}D_{Ci}\\ B_{C2}\end{bmatrix}+D_{Ci}=K_i(s) \tag{5}$$

The model set and the controller set should be designed interactively. The uncertainty of each member of the model set should be small enough so that the robust controller which satisfies assumption $A4$ exists. On the other hand, the model set should be modified according to the controller set and the uncertainty covered by each member of the model set should be as large as possible so as to reduce members of the model set.

The supervisor mainly consists of three subsystems:

Multi-estimator E: A dynamical system that estimates the output of the plant according to the input $\boldsymbol{u}$ and the output $\boldsymbol{y}$ of the plant and generates the estimation error $\boldsymbol{e_i}$. The multi-estimator E is described by a state shared dynamic system[1]

$$\dot{\boldsymbol{x}}_E=\begin{bmatrix}A_E & 0\\ 0 & A_E\end{bmatrix}\boldsymbol{x}_E+\begin{bmatrix}B_{E1}\\ 0\end{bmatrix}\boldsymbol{y}+\begin{bmatrix}0\\ B_{E2}\end{bmatrix}\boldsymbol{u},\quad \boldsymbol{y}_i=C_{Ei}\boldsymbol{x}_E+D_{Ei}\boldsymbol{u},\quad \boldsymbol{e}_i=\boldsymbol{y}_i-\boldsymbol{y} \tag{6}$$

where A_E is a stable constant matrix, B_{E1} and B_{E2} are constant matrixes, C_{Ei} and D_{Ei} satisfy

$$C_{Ei}\left[sI-\left(\begin{bmatrix}A_E & 0\\ 0 & A_E\end{bmatrix}+\begin{bmatrix}B_{E1}C_{Ei}\\ 0\end{bmatrix}\right)\right]^{-1}\begin{bmatrix}B_{E1}D_{Ei}\\ B_{E2}\end{bmatrix}+D_{Ei}=M_{22i}(s) \tag{7}$$

Monitoring signal generator M: It generates the monitoring signal π_i, *viz.* the switching index, according to the estimation error $\boldsymbol{e}_i$ and the input $\boldsymbol{u}$ of the plant. The switching index function of the monitoring signal generator M is the main difference between the hierarchical switching control system designed in this paper and the existing hierarchical switching control system. In this paper, considering the characteristics of robust control system, a new switching index function by estimating uncertainty is proposed.

$$\pi_i=\|\boldsymbol{M}_{21i}^{-1}(s)\boldsymbol{E}_i\|_2^\delta\Big/\left\|\frac{N_i(s)}{\Lambda(s)}\boldsymbol{M}_{12i}(s)\boldsymbol{U}(s)\right\|_2^\delta \tag{8}$$

where $\boldsymbol{E}_i(s)$ and $\boldsymbol{U}(s)$ are the Laplace transform of $\boldsymbol{e}_i$ and $\boldsymbol{u}$, respectively. $\Lambda(s)=|sI-A_E|$, $D_i(s)$ is a polynomial matrix, $N_i(s)$ is a polynomial and $D_i(s)/N_i(s)=M_{22i}(s)$. If the initial state is zero, π_i is the induced exponentially weighted L^2 norm of Δ_i under the current signal.

Switching logic S: A switched system whose inputs are performance signals π_i and output σ is a switching signal, which determines which controller should be connected to the plant. In the hierarchical switching control system presented in this paper, since the above switching index function by estimating uncertainty is adopted, the switching logic becomes very simple: The controller corresponding to the model yielding the minimum monitoring signal is used to compute the control input and σ is just the index of the controller, *viz.* if the minimum monitoring signal is π_j, the output of the switching logic S is $\sigma=j$.

By connecting the above three subsystems according to hierarchical switching control system depicted in Fig. 1, the hierarchical switching controller of multiple models based on robust control theory is obtained.

3 Stability analysis

In this section, stability of the hierarchical switching control system designed in Section 2 is analyzed in the sense of exponentially weighted L^2 norm. In order to simplify the analysis, it is assumed that the plant parameters vary slowly enough or the time between two changes is large enough if the parameters vary abruptly. So between switching times the switching control system can be considered as a linear time invariant system. Furthermore, since the uncertainty part of the plant is stable and the influence of initial state converges to zero exponentially, in the following analysis the initial states are assumed to be zero. For any $P_i\in P$, from (1) and (6) the estimated output is

$$\boldsymbol{Y}_i(s)=\frac{D_i(s)}{\Lambda(s)}\boldsymbol{U}(s)+\frac{\Lambda(s)-N_i(s)}{\Lambda(s)}\boldsymbol{Y}(s),\quad i=1\cdots N \tag{9}$$

where $\boldsymbol{Y}_i(s)$ and $\boldsymbol{Y}(s)$ are the Laplace transform of $\boldsymbol{y}_i$ and $\boldsymbol{y}$,respectively. Substituting (9) to the definition of estimation error $\boldsymbol{e}_i = \boldsymbol{y}_i - \boldsymbol{y}$, we have

$$\boldsymbol{E}_i(s) = \frac{D_i(s)}{\Lambda(s)}\boldsymbol{U}(s) - \frac{N_i(s)}{\Lambda(s)}\boldsymbol{Y}(s), \quad i = 1 \cdots N \tag{10}$$

It is assumed that the uncertainty model P_j can describe the current plant. Substituting (1) to (10) yields

$$M_{21j}^{-1}(s)\boldsymbol{E}_j(s) = -\frac{N_j(s)}{\Lambda(s)}\Delta_j M_{12j}(s)\boldsymbol{U}(s) \tag{11}$$

Since $\|\Delta_j\|_\infty^\delta < 1$, the following inequality can be obtained.

$$\boldsymbol{\pi}_j = \|M_{21j}^{-1}(s)E_j(s)\|_2^\delta \Big/ \left\| \frac{N_j(s)}{\Lambda(s)} M_{12j}(s)\boldsymbol{U}(s) \right\|_2^\delta \leqslant \|\Delta_j\|_\infty^\delta < 1 \tag{12}$$

It is assumed that the current controller is $K_\sigma(s)$. According to the switching logic, we have the inequality

$$\pi_\sigma = \|M_{21\sigma}^{-1}(s)\boldsymbol{E}_\sigma(s)\|_2^\delta \Big/ \left\| \frac{N_\sigma(s)}{\Lambda(s)} M_{12\sigma}(s)\boldsymbol{U}(s) \right\|_2^\delta < \pi_j < 1 \tag{13}$$

From (9) and $\boldsymbol{U}(s) = K_\sigma(s)\boldsymbol{Y}(s)$, the transfer function from $\boldsymbol{e}_\sigma$ to $\boldsymbol{u}$ can be obtained.

$$\boldsymbol{U}(s) = -\frac{\Lambda(s)}{N_\sigma(s)} \left(I - K_\sigma(s)M_{22\sigma}(s)\right)^{-1} K_\sigma(s)\boldsymbol{E}_\sigma(s) \tag{14}$$

Substituting (13) to (14), we have

$$\|M_{21\sigma}^{-1}(s)\boldsymbol{E}_\sigma(s)\|_2^\delta < \left\|M_{12\sigma}(s)(I - K_\sigma(s)M_{22\sigma}(s))^{-1}K_\sigma(s)M_{21\sigma}(s)\right\|_\infty^\delta \|M_{21\sigma}^{-1}(s)\boldsymbol{E}_\sigma(s)\|_2^\delta \tag{15}$$

All members of the controller set satisfy assumption $A4$, so

$$\|M_{21\sigma}(s)(I - K_\sigma(s)M_{22\sigma}(s))^{-1}K_\sigma(s)M_{21\sigma}(s)\|_\infty^\delta < 1 \tag{16}$$

viz. the exponentially weighted L^2 gain of the closed system of signal $M_{21\sigma}^{-1}(s)\boldsymbol{E}_\sigma(s)$ is smaller than 1. According to the small gain theorem, the exponentially weighted L^2 norm of signal $M_{21\sigma}^{-1}(s)\boldsymbol{E}_\sigma(s)$ is bounded. (14) can also be written as

$$\boldsymbol{U}(s) = -\frac{\Lambda(s)}{N_\sigma(s)}(I - K_\sigma(s)M_{22\sigma}(s))^{-1}K_\sigma(s)M_{21\sigma}(s)[M_{21\sigma}^{-1}(s)\boldsymbol{E}_\sigma(s)] \tag{17}$$

Substituting (14) to (10), so that

$$\boldsymbol{Y}(s) = -\frac{\Lambda(s)}{N_\sigma(s)}(I - M_{22\sigma}(s)K_\sigma(s))^{-1}M_{21\sigma}(s)[M_{21\sigma}^{-1}\boldsymbol{E}_\sigma(s)] \tag{18}$$

It is easy to find that the characteristic polynomials of the transfer functions in (17) and (18) are the same as that of $(I - K_\sigma(s)M_{22\sigma}(s))^{-1}M_{21\sigma}(s)$. Since the exponentially weighted L^2 norm of $M_{21\sigma}^{-1}(s)\boldsymbol{E}_\sigma(s)$ is bounded and $K_\sigma(s)$ can stabilize $P_\sigma(s)$ robustly in the sense of exponentially weighted L^2 norm, the exponentially weighted L^2 norm of the control input $\boldsymbol{u}$ and the output $\boldsymbol{y}$ of the plant are also bounded.

4 Simulation

The simulation is done on a first-order system with a pure time delay:

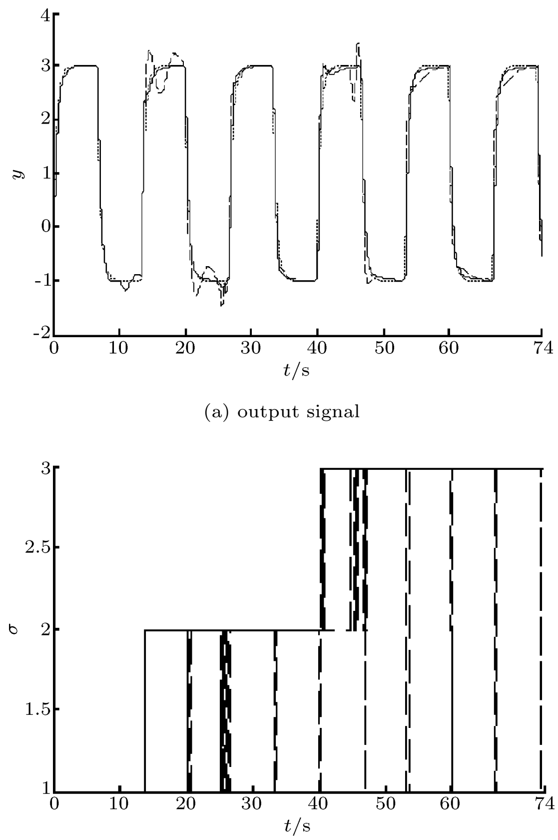
$$P(s) = \frac{b}{s+1}e^{-\tau s} \tag{19}$$

where $2 \leqslant b \leqslant 6$,and $0 \leqslant \tau \leqslant 0.1$. Three uncertainty models are used to cover the possible uncertainties of the above system. For each uncertainty model, a robust performance controller is designed by H_∞

control theory, which makes up the controller set. In the simulation, the exponentially weighted factor δ is set to 0.4, time delay τ is set to 0.1 and parameter b varies as

$$b = \begin{cases} 3, & 0 \leqslant t \leqslant 10 \\ 4, & 10 < t \leqslant 25 \\ 5, & 25 < t \leqslant 40 \\ 6, & 40 < t \leqslant 74 \end{cases} \tag{20}$$

Fig. 2 shows the simulation results.



(a) output signal

(b) switching signal

⋯⋯ desired output —— output of the switching control system presented in this paper
- - - - output of the switching control system based on hysteresis switching

Fig. 2 The results of simulation

From the simulation results, it is found that for the model set and controller set designed by robust control theory, the switching control system based on hysteresis switching is stable, but the system switches between controllers frequently, resulting in bad tracking performance. The reason is that the estimation error can′t evaluate the uncertainty between the nominal model and the plant properly. In this paper, the characteristics of robust control system are fully taken into account and a new kind of switching index function by estimating the uncertainty between the plant and the nominal model is used to determine when and which controller should be switched to the closed system, so the hierarchical switching system based on uncertainty estimation proposed in this paper has better performance of tracking and stability.

5 Conclusion

A new hierarchical switching control system of multiple models based on robust control theory is designed for some plant with large uncertainties in this paper. The model set and controller set are designed by robust control theory and the characteristics of robust control system are taken into account. A new kind of switching index function by estimating uncertainty is designed. The stability of the new switching control system is analyzed by the small gain theorem in the sense of exponentially weighted L^2 norm. And simulation is also done on a plant with both parameter uncertainty and unmodeled dynamics. Both theoretical analysis and simulation results show that the new hierarchical switching control system can control the plant with large uncertainties effectively. Comparing with the existing hierarchical switching control systems whose switching indexes are generated by some norm-squared output estimation errors, the new hierarchical switching control system based on uncertainty estimation has better performance of stability and tracking.

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