Toward a Unified Mathematical and Computational Framework for Control and Mechanics[1)]

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Abstract This is a brief review on the recent book: *Duality System in Applied Mechanics and Optimal Control*, by Zhong Wan-Xie, published by *Kluwer Academic Publishers*, 2004. The book represents a significant effort to re-establish the historic and deep tie between control and mechanics by striving to connect and integrate concepts, methods, and algorithms in mechanics and control so that a unified framework can be established for both analytical and computational purposes. Clearly, it has demonstrated that the duality system method can be used as a mathematical and systematic foundation to deal with many important concepts and problems in both mechanics and control. This book is not only very useful for research and applications, but also extremely helpful for multidisciplinary curriculum development when students from one field are trying to learning and applying concepts and methods from the other field.

Key words Duality systems, symplectic methods, Hamiltor-Jacobi equation, computational mechanics, optimal control

1 Introduction

Control and mechanics have a deep and historical relationship ever since the birth of classical and modern control theory. Many great mathematicians had pioneered in both areas over the last two centuries. In early 1920s, Von Karman, a giant in both applied mathematics and mechanics, had made a significant contribution in bring in rigorous mathematics into engineering and opened the era of engineering science that still impacts our engineering education and research today. In my opinion, the very idea of engineering science is responsible at least partly for the birth of modern control theory, and it is not a coincident that exact 50 years ago it was Von Karman′s protege,Tsien H. S. (or Qian X. S.), the father of China′s space program, especially its rocketry, also a giant in mechanics and late in both control and systems engineering, wrote the first milestone book in modern control engineering and application, *Engineering Cybernetics* (McGraw-Hill Book Company, Inc., 1954), when Tsian was a professor at California Institute of Technology. One can see the obvious trace of mechanics and applied mathematics in Tsien′s control book. However, for many years, it seems that this historic and deep tie between mechanics and control, at least from the aspect of structural mechanics, has been overlooked and almost lost today. For many of us today, control is more of an EE-orientated than a ME disciplinary. For the benefit of both areas, especially with the new development of modern computational methods and tools, we must re-establish a close and strong tie between control and mechanics.

Duality System in Applied Mechanics and Optimal Control[1] represents one of successful and valuable attempts along this direction by a leading expert in both applied mechanics and control theory. The book strives to connect and integrate concepts, methods, and algorithms in mechanics and control so that a unified framework can be established for both analytical and computational purposes. The state space approach has been used as the mathematical basis for building the analogy and one to one correspondence between mechanics and control, especially structural mechanics and optimal control. From the aspect of mechanics, this enables us to reformulate basic equations for elasticity theory using Hamiltonian dual variables and transfer the corresponding mathematics from the traditional Euclidean geometric into symplectic geometric one. As a result, the symplectic eigen-function expansion approach can be used to solve analytically many problems in elasticity that are hard or impossible to solve using the traditional solution methodology by the try-and-error technique, called semi-inverse method. Likewise, from the aspect of control, the analogy leads to new understanding and insight of key control problems in terms of well-studied classical problems in mechanics, especially vibration problems in

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structural mechanics, so that many effective algorithms and computational methods in mechanics can be applied to solve the corresponding control problems. Clearly, the book has demonstrated that the duality system method can be used as a mathematical and systematic foundation to deal with many important concepts and problems in both mechanics and control. It must be pointed out that this is not only very useful for research and applications, but also extremely helpful for multidisciplinary curriculum development.

2 Review

The book is organized into 6 chapters, starting with a brief historical, mathematical, and somewhat philosophical introduction to developments in mechanics and control and the precise integration method, a key computational method in structural mechanics developed by the author and his research group. Roughly speaking, the first 4 chapters constitute the mathematical and mechanics foundation for the entire book, Chapter 5 focuses on the application of state space method in elasticity, specifically elastic systems with single continuous coordinates or one-dimensional systems, while Chapter 6 concentrates on the utilization of computational methods and algorithms for solving control problems, especially the use of the precise integration method for predication, filtering, smoothing, optimal control and robust control.

Chapter 1 presents the concepts and methods involving analytical dynamics, Lagrangian and Hamiltonian systems, Legendre transformation, dual variables, canonical transformation, symplectic systems, Poisson bracket, action, the Hamilton-Jacobi equation and separation of variables. The theme of this chapter is that the importance of Hamiltonian formulation goes well beyond analytical dynamics: it also provides a foundation for dealing with key problems in optimal control, robust control, elasticity, vibration, wave propagation, multi-body dynamics, and so on. One of the important objectives here is to establish a unified methodology under the sympletic frame for various disciplines based on the use of duality variables and reformulation of governing equations of dynamic systems into Hamiltonian forms.

Chapter 2 summaries key topics and problems in the structural vibration theory, especially the symplectic eigen-problem for gyrosocopic systems and the corresponding algorithms. Issues related to the eigen-value count are also addressed.

Chapters 3 and 4 provide the preliminaries for probability, stochastic processes, and random vibration of structures. Most of the materials in those two chapters are standard but the introduction and discussion of Lin′s Pseudo Excitation Method (PEM) for solving linear response problems are quite new and interesting. Compared with the traditional approach, the PEM has significantly improved the computational efficiency and enables us to solve more practical and complicated problems in non-stationary random vibrations of real-world structural systems.

Chapter 5 deals with elastic systems with single continuous coordinate or one-dimensional elastic systems. The key idea in this chapter is the establishment of the analogy between structural mechanics and optimal control by considering the single spatial coordinate as "time" coordinate and then transferring the original higher order spatial differential equations into a set of first-order "state space" equations. In this way, the traditional semi-analytical method and wave propagation problems in elasticity can now be solved analytically and systematically using the duality system theory. However, the original initial value problems for analytical dynamics become the two point boundary value problems for structural mechanics, which can be easily solved numerically with the precise integration algorithm Detailed discussions and procedures for solving matrix Riccati differential equations with precise integration as well as eigenvector based solutions of Riccati and Hamilton equations are illustrated.

Finally, Chapter 6 addresses issues of applying methods in mechanics for linear optimal control systems and corresponding computational problems. Actually, many methods used in this chapter have been discussed in the previous chapter since the same analogy between control and mechanics is still valid and applied here. For example, the precise integration algorithms are extensively used for solving various matrix differential equations in smoothing, filtering, and predicating operations, as well as in LQG optimal control and H_∞ robust control, with specific consideration for combing on-line light iterative calculation with off-line intensive one-time computation so that the real-time computational work can be reduced to minimum. Clearly, the use of internal mixed energy concept is essential for applying methods in mechanics to control problems. The physical interpretation of the solution of Riccati equation, *i.e.*, the solution matrix corresponds to the end flexibility matrix, offers useful information for its precise integration and analytical solution based on state eigen-vectors, as well as the observation that the eigen-solution based analytical method should be combined with the precise integration to solve Riccati differential equation and the filter equation. More interesting, it is pointed out that critical sub-optimal parameters in H_∞ robust control and filtering correspond to the extended Rayleigh quotients, thus many effective variational and numerical methods in structural mechanics, such as the W-W algorithm can be applied for robust control problems.

3 Concluding remarks

This book only represents part of Professor Zhong′s effort to bring control and mechanics together under a unified mathematical and computational framework, as one can see from his recent papers and two previous Chinese books: *Symplectic Elasticity*[2] (co-authored with Yao Wei-An, *Higher Education Press*, 2002) and *Computational Structural Mechanics and Optimal Control*[3] (*Dalian University of Technology Press*, 1993). Overall, this book is a very successful initial effort but more works are needed towards a completed and unified framework. In addition, there are still many open research topics involved, especially for problems in distributed parameter systems in both control and mechanics. As for the improvement of this book, I hope in its new version the English will be checked more carefully since sometimes one can see obvious traces of direct translation of Chinese into English.

More than 20 years ago, I was fortunate to have the opportunity to be Professor Zhong′s teaching assistant for his summer seminar course on Computational Structural Mechanics in Hangzhou, China. Today as a senior scientist in China, Prof. Zhong is still extremely active and engaged himself in exploring new topics and disciplinary. I must say that I deeply admire and respect his academic spirit, tireless effort, and endless energy in pursuing new knowledge and conducting solid research works.

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