An Improved Robust Adaptive Control Design for a Class of Neutral Delay Systems[1)]

SUN Xi-Ming SUN Wen-An ZHAO Jun

(*Key Laboratory of Process Industry Automation, Ministry of Education, School of Information Science and Engineering, Northeastern University, Shenyang* 110004)

(E-mail: wrsxm@126.com)

Abstract The problem of adaptive robust control is addressed for a class of neutral delay systems. All uncertainties are assumed to be bounded by unknown constants. An improved adaptation law is proposed to estimate the square of these unknown bounds. Then, by making use of the updated values of the squared unknown bounds, an adaptive controller is designed to make the solution of the resultant closed-loop system uniformly ultimately bounded. Furthermore, this method avoids chattering and improves the performance. An example is given to illustrate the effectiveness of this method.

Key words Neutral delay system, robust adaptive control, uniform ultimate boundedness

1 Introduction

Because neutral delay systems are of both theoretical and practical interest, in recent years, considerable attention has been focused on them, especially on stability analysis and controller synthesis[1~4]. On the other hand, in the control literature, the information of the upper bounds of uncertainties is usually assumed to be known. However, it is generally difficult to obtain such prior knowledge in practice because of the complexity of the structures of uncertainties. An effective way to deal with this problem is to introduce adaptive control law to estimate the bounds of uncertainties. Many forms of adaptive control schemes are available in the literature for uncertain systems without delay or with general non-neutral delay, see [5~7]. But for neutral delay systems, no results except our previous work[8] have been reported. In [8], by making use of the method of 1-norm of matrix, an adaptive controller was designed to make the solution of the resultant closed-loop system uniformly ultimately bounded. However, the use of the sign function in constructing the adaptive controller caused serious chattering and deteriorated the performance.

This paper presents an improved adaptive robust control law to fix this problem. The adaptive controller involving the estimate of the square of these unknown bounds makes the solution of the resultant closed-loop system uniformly ultimately bounded and can overcome some drawbacks of the method in [8]. Finally, simulation results are given to illustrate the effectiveness of the method.

2 System description and problem formulation

Consider the nonlinear uncertain neutral delay system which appeared in [8]:

$$\dot{\boldsymbol{x}}(t) = (A+\Delta A)\boldsymbol{x}(t) + (A_h+\Delta A_h)\boldsymbol{x}(t-h) + A_d\dot{\boldsymbol{x}}(t-d) + \boldsymbol{e}(t,\boldsymbol{x},\boldsymbol{x}(t-h)) + B\boldsymbol{u}(t) \tag{1a}$$

$$\boldsymbol{x}(\theta) = \boldsymbol{\varphi}(\theta), \theta\in[-\tau,0] \tag{1b}$$

where $\boldsymbol{x}(t)\in R^n$ is the state vector, $\boldsymbol{u}(t)\in R^m$ is the control input, $\boldsymbol{e}(t,\boldsymbol{x},\boldsymbol{x}(t-h))\in R^n$ is the nonlinear uncertainty, A, A_h, A_d are all known constant matrices with appropriate dimensions. ΔA and ΔA_h denote the unknown real-valued functions representing the time-varying parameter uncertainties of the matrices A and A_h, respectively. Scalars $h>0$ and $d>0$ denote the state delays. Let $\tau=\max\{h,d\}$. $\boldsymbol{\varphi}(\theta)\in R^n$ is a continuously differentiable vector-valued initial function on $[-\tau,0]$.

We also need the following standard assumptions.

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A1) The pair (A, B) is controllable.

A2) $\Delta A, \Delta A_h$ and $\boldsymbol{e}(t, \boldsymbol{x}, \boldsymbol{x}(t-h))$ are all continuously differentiable with respect to x, and piecewisely continuous in t[6].

A3) There exist unknown continuous functions matrices A_1, A_2, A_3 with appropriate dimensions, such that $\Delta A = BA_1, \Delta A_h = BA_2, \boldsymbol{e}(t, \boldsymbol{x}, \boldsymbol{x}(t-h)) = BA_3$[9,10].

A4) There exist unknown positive scalars g_1 and g_2 such that $\|A_1\boldsymbol{x} + A_2\boldsymbol{x}(t-h) + A_3\| \leqslant g_1\|\boldsymbol{x}\| + g_2\|\boldsymbol{x}(t-h)\|$.

Let $f = A_1\boldsymbol{x} + A_2\boldsymbol{x}(t-h) + A_3$. System (1) can be rewritten as

$$\dot{\boldsymbol{x}}(t) = A\boldsymbol{x}(t) + A_h\boldsymbol{x}(t-h) + A_d\dot{\boldsymbol{x}}(t-d) + B\boldsymbol{u}(t) + Bf \tag{2a}$$

$$\boldsymbol{x}(\theta) = \boldsymbol{\varphi}(\theta), \quad \theta \in [-\tau, 0] \tag{2b}$$

3 Main result

Now, under Assumptions A1)∼A4), we give the adaptive design scheme.

Let $\gamma = g_1^2$, $\theta = g_2^2$, and design the adaptive law by

$$\frac{\mathrm{d}\hat{\theta}}{\mathrm{d}t} = -\alpha\alpha_2\hat{\theta} + \frac{1}{2}\alpha\alpha_1^{-1}\|\wp^{\mathrm{T}}PB\|^2, \quad \frac{\mathrm{d}\hat{\gamma}}{\mathrm{d}t} = -\delta\delta_2\hat{\gamma} + \frac{1}{2}\delta\delta_1^{-1}\|\wp^{\mathrm{T}}PB\|^2 \tag{3a}$$

where α, α_2, δ and δ_2 are given positive constants, α_1 and δ_1 are positive constants to be chosen later.

Let

$$u_{adp} = -\frac{1}{2}(\delta_1^{-1}\hat{\gamma} + \alpha_1^{-1}\hat{\theta})B^{\mathrm{T}}P\wp \tag{3b}$$

$$\boldsymbol{u} = -\mu B^{\mathrm{T}}P\boldsymbol{x} + u_{adp} \tag{3c}$$

where $\wp$ is the difference operator defined as $\wp(\boldsymbol{x}(t)) = \boldsymbol{x}(t) - A_d\boldsymbol{x}(t-d)$, P is a positive matrix and will be chosen, μ is a given positive constant.

Define $\tilde{\gamma} = \hat{\gamma} - \gamma$, $\tilde{\theta} = \hat{\theta} - \theta$. Thus, $\dot{\tilde{\gamma}} = \dot{\hat{\gamma}}$, $\dot{\tilde{\theta}} = \dot{\hat{\theta}}$. For convenience, we adopt the notations

$$\boldsymbol{x}_h = \boldsymbol{x}(t-h), \ \boldsymbol{x}_d = \boldsymbol{x}(t-d), \ \wp = \wp(\boldsymbol{x}(t)), \ f = f(t, \boldsymbol{x}(t), \boldsymbol{x}(t-h))$$

and we refer to this method as 2-norm method in order to differentiate it from 1-norm method proposed in [8]. Now, we give the main result of this paper.

Theorem 1. Consider the neutral delay system (2) with Assumptions A1)∼A4). If there exist matrices $P > 0, Q > 0, S > 0$, positive constants α, δ_1 such that the matrix inequality

$$E = \begin{pmatrix} PA + A^{\mathrm{T}}P - 2\mu PBB^{\mathrm{T}}P + W & PA_h & (PA - \mu PBB^{\mathrm{T}}P + W)A_d \\ * & -S & 0 \\ * & * & -Q + A_d^{\mathrm{T}}WA_d \end{pmatrix} < 0 \tag{4}$$

holds, where $W = S + Q + \delta_1 I + \alpha_1 I$, then the solutions to closed-loop system (2)∼(3) are uniformly ultimately bounded for any delays h and d.

Proof. It is easy to show from (4) that $-Q + A_d^{\mathrm{T}}WA_d < 0$. Therefore, $A_d^{\mathrm{T}}QA_d - Q < 0$. Thus the operator $\wp$ is stable.

Let $\bar{A} = A - \mu BB^{\mathrm{T}}P$. Consider the following Lyapunov-Krasovskii candidate function:

$$V(\boldsymbol{x}_t, \tilde{\gamma}, \tilde{\theta}) = \wp^{\mathrm{T}}(\boldsymbol{x}(t))P\wp(\boldsymbol{x}(t)) + \int_{-h}^{0} \boldsymbol{x}^{\mathrm{T}}(t+\theta)(S + \alpha_1 I)\boldsymbol{x}(t+\theta)\mathrm{d}\theta +$$

$$\int_{-d}^{0} \boldsymbol{x}^{\mathrm{T}}(t+\theta)Q\boldsymbol{x}(t+\theta)\mathrm{d}s + \alpha^{-1}\tilde{\theta}^2 + \delta^{-1}\tilde{\gamma}^2$$

Taking the derivative of $V(\boldsymbol{x}_t, \tilde{\gamma}, \tilde{\theta})$ along the solutions of (2) and (3) results in

$$\dot{V}(\boldsymbol{x}_t, \tilde{\gamma}, \tilde{\theta}) = 2\wp^{\mathrm{T}}P\dot{\wp} + \boldsymbol{x}^{\mathrm{T}}(S + \alpha_1 I)\boldsymbol{x} - \boldsymbol{x}_h^{\mathrm{T}}(S + \alpha_1 I)\boldsymbol{x}_h + \boldsymbol{x}^{\mathrm{T}}Q\boldsymbol{x} - \boldsymbol{x}_d^{\mathrm{T}}Q\boldsymbol{x}_d + 2(\alpha^{-1}\tilde{\theta}\dot{\tilde{\theta}} + \delta^{-1}\tilde{\gamma}\dot{\tilde{\gamma}}) \leqslant$$

$$2\wp^{\mathrm{T}}P(\bar{A}\boldsymbol{x}+A_h\boldsymbol{x}_h)+2\wp^{\mathrm{T}}PB[-\frac{1}{2}(\delta_1^{-1}\hat{\gamma}+\alpha_1^{-1}\hat{\theta})B^{\mathrm{T}}P\wp]+2\wp^{\mathrm{T}}PBf+\boldsymbol{x}^{\mathrm{T}}(S+\alpha_1I+Q)\boldsymbol{x}-$$
$$\boldsymbol{x}_h^{\mathrm{T}}(S+\alpha_1I)\boldsymbol{x}_h-\boldsymbol{x}_d^{\mathrm{T}}Q\boldsymbol{x}_d+2\alpha^{-1}\tilde{\theta}[-\alpha\alpha_2\hat{\theta}+\frac{1}{2}\alpha\alpha_1^{-1}\|\wp^{\mathrm{T}}PB\|^2]+$$
$$2\delta^{-1}\tilde{\gamma}[-\delta\delta_2\hat{\gamma}+\frac{1}{2}\delta\delta_1^{-1}\|\wp^{\mathrm{T}}PB\|^2]\leqslant 2\wp^{\mathrm{T}}P(\bar{A}\boldsymbol{x}+A_h\boldsymbol{x}_h)+\boldsymbol{x}^{\mathrm{T}}(S+\alpha_1I+Q)\boldsymbol{x}-$$
$$\boldsymbol{x}_h^{\mathrm{T}}(S+\alpha_1I)\boldsymbol{x}_h-\boldsymbol{x}_d^{\mathrm{T}}Q\boldsymbol{x}_d-\alpha_1^{-1}\hat{\theta}\|\wp^{\mathrm{T}}PB\|^2-\delta_1^{-1}\hat{\gamma}\|\wp^{\mathrm{T}}PB\|^2-2\alpha_2\tilde{\theta}\hat{\theta}+$$
$$\alpha_1^{-1}\tilde{\theta}\|\wp^{\mathrm{T}}PB\|^2-2\delta_2\tilde{\gamma}\hat{\gamma}+\delta_1^{-1}\tilde{\gamma}\|\wp^{\mathrm{T}}PB\|^2+2\wp^{\mathrm{T}}PBf \quad (5)$$

Notice that

$$2\wp^{\mathrm{T}}PBf\leqslant 2\|\wp^{\mathrm{T}}PB\|\|f\|\leqslant 2\|\wp^{\mathrm{T}}PB\|(g_1\|\boldsymbol{x}\|+g_2\|\boldsymbol{x}_h\|)\leqslant$$
$$\delta_1^{-1}\gamma\|\wp^{\mathrm{T}}PB\|^2+\delta_1\|\boldsymbol{x}\|^2+\alpha_1^{-1}\theta\|\wp^{\mathrm{T}}PB\|^2+\alpha_1\|\boldsymbol{x}_h\|^2 \quad (6)$$

Applying (6) to (5) yields

$$\dot{V}(\boldsymbol{x}_t,\tilde{\gamma},\tilde{\theta})\leqslant 2\wp^{\mathrm{T}}P(\bar{A}\boldsymbol{x}+A_h\boldsymbol{x}_h)+\boldsymbol{x}^{\mathrm{T}}W\boldsymbol{x}-\boldsymbol{x}_h^{\mathrm{T}}S\boldsymbol{x}_h-\boldsymbol{x}_d^{\mathrm{T}}Q\boldsymbol{x}_d-2\alpha_2\tilde{\theta}\hat{\theta}-2\delta_2\tilde{\gamma}\hat{\gamma}=$$
$$2\wp^{\mathrm{T}}P[\bar{A}(\wp+A_d\boldsymbol{x}_d)+A_h\boldsymbol{x}_h]+(\wp+A_d\boldsymbol{x}_d)^{\mathrm{T}}W(\wp+A_d\boldsymbol{x}_d)-\boldsymbol{x}_h^{\mathrm{T}}S\boldsymbol{x}_h-\boldsymbol{x}_d^{\mathrm{T}}Q\boldsymbol{x}_d-$$
$$2\alpha_2\tilde{\theta}(\tilde{\theta}+\theta)-2\delta_2\tilde{\gamma}(\tilde{\gamma}+\gamma)\leqslant\wp^{\mathrm{T}}(\boldsymbol{x}(t))(P\bar{A}+\bar{A}^{\mathrm{T}}P+W)\wp(\boldsymbol{x}(t))+$$
$$2\wp^{\mathrm{T}}(\boldsymbol{x}(t))(W+P\bar{A})A_d\boldsymbol{x}_d+\boldsymbol{x}_d^{\mathrm{T}}(-Q+A_d^{\mathrm{T}}WA_d)\boldsymbol{x}_d-\boldsymbol{x}_h^{\mathrm{T}}S\boldsymbol{x}_h+2\wp^{\mathrm{T}}PA_h\boldsymbol{x}_h-\delta_2\tilde{\gamma}^2-$$
$$\alpha_2\tilde{\theta}^2+\delta_2\gamma^2+\alpha_2\theta^2\leqslant\boldsymbol{\zeta}\bar{E}\boldsymbol{\zeta}^{\mathrm{T}}+\delta_2\gamma^2+\alpha_2\theta^2 \quad (7)$$

where $\zeta=(\wp^{\mathrm{T}}\quad \boldsymbol{x}_h^{\mathrm{T}}\quad \boldsymbol{x}_d^{\mathrm{T}}\quad \tilde{\gamma}\quad \tilde{\theta})$, $\bar{E}=\mathrm{diag}(E\quad -\delta_2\quad -\alpha_2)$.

From (4), we know $\bar{E}<0$. Noting the stability of the operator $\wp$ and (7), we complete the proof by using Theorem 8.1 in [4]. □

Remark 1. In [8], the designed adaptive controller contains the sign function because the method of 1-norm is used. The application of such controller may give rise to undesirable chattering problem. A common method to overcome this drawback caused by the sign function in the literature is to exploit the saturation function or other similar function to replace the sign function[6,10]. But the use of such function reduces the tracking accuracy, and thus produces unsatisfactory performance. The application of the 2-norm method in this paper avoids these disadvantages, as shown in the proof of Theorem 1.

Remark 2. The information about the constant delays h and d must be known in order to design the adaptive controller by using 1-norm method, while for 2-norm method, from (3a), we can see that the constant delay h is not required to be known.

Remark 3. Noting the inequality $-PBB^{\mathrm{T}}P\leqslant NBB^{\mathrm{T}}N^{\mathrm{T}}-NBB^{\mathrm{T}}P-PBB^{\mathrm{T}}N^{\mathrm{T}}$, where N is an arbitrarily given matrix, we can easily turn (4) into an LMI.

Remark 4. If in Assumption A4), $g_1=g_2=g$, then by letting $\eta=g^2$, we can construct adaptive control law $\boldsymbol{u}=-\mu B^{\mathrm{T}}P\boldsymbol{x}+u_{adp}$, where $u_{adp}=-\beta_1^{-1}\hat{\eta}B^{\mathrm{T}}P\wp$, $\frac{\mathrm{d}\hat{\eta}}{\mathrm{d}t}=-\beta\beta_2\hat{\eta}+\frac{1}{2}\beta\beta_1^{-1}\|\wp^{\mathrm{T}}PB\|^2$, β and β_2 are given positive constants, β_1 is a positive constant to be chosen, and $\wp$, P and μ are defined as before.

4 Simulation

Consider the system (1) with parameters as follows:

$$A=\begin{pmatrix}-1 & 1\\ -2 & -3\end{pmatrix},\ \Delta A=\begin{pmatrix}0.1\sin(2t) & -0.3\sin(t)\\ -0.1\sin(t) & 0.075\sin(3t)\end{pmatrix},\ A_d=\begin{pmatrix}-0.5 & -0.27\\ 0.1 & 0\end{pmatrix},\ A_h=\begin{pmatrix}0 & -0.1\\ 0.5 & 1\end{pmatrix}$$
$$\Delta A_h=\begin{pmatrix}-0.2\sin(2t) & 0.1\sin(3t)\\ 0.1\sin(t) & -0.175\sin(t)\end{pmatrix},\ e(t,\boldsymbol{x},\boldsymbol{x}_h)=\begin{pmatrix}0.6\sin(3t)\\ 0.9\sin(3t)\end{pmatrix},\ B=\begin{pmatrix}1 & 0\\ 0 & 0.5\end{pmatrix}$$

When $A_d=0$, the system (1) reduces to the similar system discussed in [6], which is a water-quality dynamic model of the River Nile. It is easy to verify that Assumptions A1)∼A3) are all satisfied. According to Remark 3, taking $\mu=1, N=I$ and solving (4) leads to

$$P=\begin{pmatrix}7.9252 & 0.5572\\ 0.5572 & 4.1459\end{pmatrix},\ S=\begin{pmatrix}4.6107 & 0.2186\\ 0.2186 & 5.4786\end{pmatrix},\ Q=\begin{pmatrix}9.3346 & 2.2107\\ 2.2107 & 6.2790\end{pmatrix}$$

$$\alpha_1 = 4.0329,\ \delta_1 = 4.0339,\ \alpha = \delta = 0.2,\ \alpha_2 = \delta_2 = 5$$

Let $h = 1$ and $d = 2$; the adaptive law is then given by

$$\frac{\mathrm{d}\tilde{\theta}}{\mathrm{d}t} = -\hat{\theta} + \frac{1}{2} \times \frac{0.2}{4.0329}\|\wp^{\mathrm{T}}PB\|^2, \quad \frac{\mathrm{d}\tilde{\gamma}}{\mathrm{d}t} = -\hat{\gamma} + \frac{1}{2} \times \frac{0.2}{4.0339}\|\wp^{\mathrm{T}}PB\|^2$$

Therefore

$$\boldsymbol{u} = -\frac{\mu}{2}B^{\mathrm{T}}P\boldsymbol{x} - \frac{1}{2}(\delta^{-1}\hat{\gamma} + \alpha_1^{-1}\hat{\theta})B^{\mathrm{T}}P\wp \tag{8}$$

We take the initial value as $x(0) = (5 \quad -4)^{\mathrm{T}}$ and $\hat{\gamma}(0) = \hat{\theta}(0) = 2$, the simulation results are shown in Figs. 1~2. Fig. 1 shows that the original system states approach to a small bounded region in a finite time. Fig. 2 depicts the input control signals.

Based on the 1-norm method in [8], the simulation results are shown in Figs. 3~4. From Fig. 2 and Fig. 4, we can see that the chattering phenomenon is eliminated by using 2-norm method.

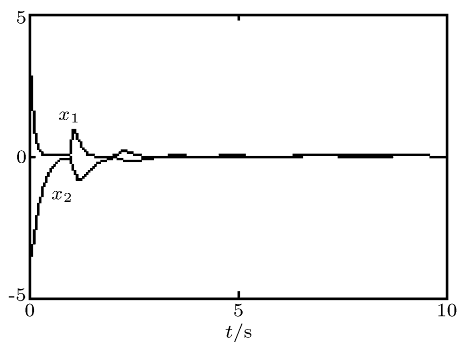

Fig. 1 The state responses (2-norm method)

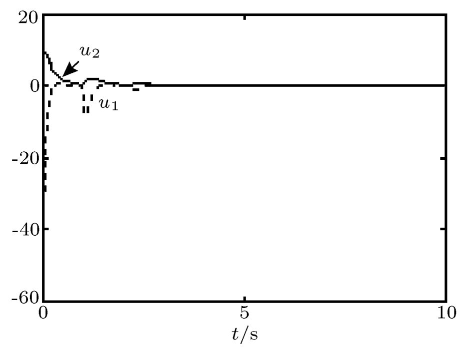

Fig. 2 Control law (2-norm method)

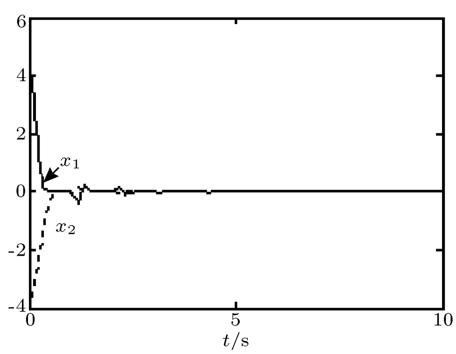

Fig. 3 The state responses (1-norm method)

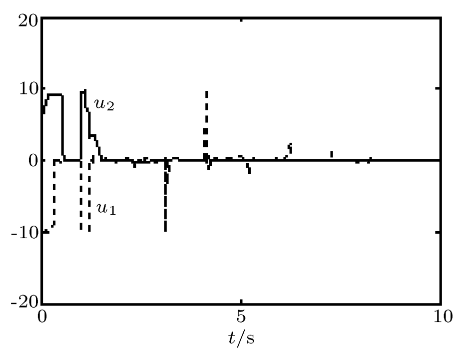

Fig. 4 Control law (1-norm method)

5 Conclusion

In this paper, an improved robust adaptive method is given for a class of neutral delay systems. The so-called 2-norm adaptive controller makes the resultant closed-loop system ultimately uniformly bounded, and provides better performance than the 1-norm method. Moreover, this adaptive controller avoids the use of sign function which may cause a serious chattering problem as in the 1-norm method.

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SUN Xi-Ming Ph. D. candidate at Northeastern University. His research interests include switched systems and delay systems.

SUN Wen-An Ph. D. candidate at Northeastern University. His research interests include hybrid systems and swiching control.

ZHAO Jun Professor at Northeastern University. His research interests include hybrid systems, nonlinear control theory, switching control, and robust control.