Distributed Reduced-order Optimal Fusion Kalman Filters for Stochastic Singular Systems[1)]

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Abstract Based on the optimal fusion algorithm weighted by matrices in the linear minimum variance (LMV) sense, a distributed full-order optimal fusion Kalman filter (DFFKF) is given for discrete-time stochastic singular systems with multiple sensors, which involves the inverse of a high-dimension matrix to compute matrix weights. To reduce the computational burden, a distributed reduced-order fusion Kalman filter (DRFKF) is presented, which involves in parallel the inverses of two relatively low-dimension matrices to compute matrix weights. A simulation example shows the effectiveness.

Key words Multisensor, information fusion, distributed reduced-order fusion filter, cross-covariance, stochastic singular system

1 Introduction

Recently, state estimation for stochastic singular systems has attracted considerable attention due to extensive application backgrounds including robotics, economics, chemical systems, *etc*[1~5]. When multiple sensors measure the state of a stochastic singular system, we can combine all measurement vectors from different sensors into one measurement vector, and then we can obtain the centralized filter. But the centralized filter can bring large computational burden in the fusion center. In recent years, the distributed filters have been widely investigated due to the parallel structures to increase the input data rates and reliability[6~14], including the fusion filter of two sensors[6], the distributed parallel filter with feedback[7], the federal Kalman filter[8], the optimal fusion filter in the maximum likelihood (ML) sense under the assumption of the normal distribution[9,10], the suboptimal fusion steady-state Kalman filter under the assumption of local estimation errors to be uncorrelated[11], the unified fusion rules based on a unified linear model for centralized, distributed, and hybrid fusion architectures in weighted least square (WLS) and best linear unbiased estimation (BLUE) sense[12], the measurement fusion filter with the same dimension measurement matrices[13] and the optimal fusion weighted by matrices in the LMV sense[14], which is equivalent to the ML fusion[9] and the standard distributed BLUE fusion[12]. The above results about fusion estimation are mainly focused on nonsingular systems. But the distributed fusion estimation problem for stochastic singular systems is seldom reported, however, it has a widely application background.

In this paper, we present a distributed full-order optimal fusion Kalman filter (DFFKF) for stochastic singular system based on the optimal fusion algorithm weighted by matrices in the LMV sense, which involves the inverse of a high-dimension matrix to compute matrix weights. To reduce the computational burden, we also present a distributed reduced-order optimal fusion Kalman filter (DRFKF), which involves the inverses of two low-dimension matrices.

2 Problem formulation

Consider the discrete-time stochastic singular linear system with multiple sensors

$$M\boldsymbol{x}(t+1) = \Phi\boldsymbol{x}(t) + \Gamma\boldsymbol{w}(t) \tag{1}$$

$$\boldsymbol{y}^{(i)}(t) = H^{(i)}\boldsymbol{x}(t) + \boldsymbol{v}^{(i)}(t), \quad i = 1, 2, \cdots, l \tag{2}$$

where the state $\boldsymbol{x}(t) \in R^n$, the measurements $\boldsymbol{y}^{(i)}(t) \in R^{m_{(i)}}$, $i = 1, 2, \cdots, l$. $\boldsymbol{w}(t) \in R^r$ and $\boldsymbol{v}^{(i)}(t) \in R^{m_{(i)}}$, $i = 1, 2, \cdots, l$ are independent white noises with zero mean and variances Q_w and $Q_{v^{(i)}}$. $M, \Phi, \Gamma, H^{(i)}$ are the constant matrices with compatible dimensions, l is the number of sensors, and the superscript (i) denotes the ith sensor.

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Assumption 1. M is a singular square matrix, $\text{rank}M = n_1 < n$, $\text{rank}\Phi \geqslant n_2$ and $n_1 + n_2 = n$.

Assumption 2. System (1) is regular, *i.e.*, $\det(zM - \Phi) \not\equiv 0$ where z is an arbitrary complex.

Assumption 3. The initial state $\boldsymbol{x}(0)$ with mean $\boldsymbol{\mu}_0$ and variance P_0 is independent of $\boldsymbol{w}(t)$ and $\boldsymbol{v}^{(i)}(t)$, $i = 1, 2, \cdots, l$.

Our aim is to find the distributed reduced-order fusion Kalman filter $\hat{\boldsymbol{x}}^{(o)}(t|t)$ of the state $\boldsymbol{x}(t)$ based on measurements $(\boldsymbol{y}^{(i)}(t), \cdots, \boldsymbol{y}^{(i)}(1))$, $i = 1, 2, \cdots, l$.

For system (1) and (2), there are nonsingular matrices L and R[15], such that

$$LMR = \begin{bmatrix} M_1 & 0 \\ M_2 & 0 \end{bmatrix},\ L\Phi R = \begin{bmatrix} \Phi_1 & 0 \\ \Phi_2 & \Phi_3 \end{bmatrix},\ L\Gamma = \begin{bmatrix} \Gamma_1 \\ \Gamma_2 \end{bmatrix},\ H^{(i)}R = [H_1^{(i)} \quad H_2^{(i)}] \tag{3}$$

where $M_1 \in R^{n_1 \times n_1}$ is nonsingular lower-triangular, $\Phi_1 \in R^{n_1 \times n_1}$ is quasi-lower-triangular, $\Phi_3 \in R^{n_2 \times n_2}$ is nonsingular lower-triangular. By introducing the transformation $\boldsymbol{x}(t) = R[\boldsymbol{x}_1^{\rm T}(t) \quad \boldsymbol{x}_2^{\rm T}(t)]^{\rm T}$ with $\boldsymbol{x}_1(t) \in R^{n_1}$ and $\boldsymbol{x}_2(t) \in R^{n_2}$, where T denotes the transpose, the singular system (1) and (2) is transferred into the following two reduced-order subsystems:

$$\begin{cases} \boldsymbol{x}_1(t+1) = \Phi_0 \boldsymbol{x}_1(t) + \Gamma_0 \boldsymbol{w}(t) \\ \boldsymbol{y}^{(i)}(t) = \bar{H}^{(i)} \boldsymbol{x}_1(t) + \boldsymbol{\eta}^{(i)}(t) \end{cases}, \quad i = 1, 2, \cdots, l \tag{4}$$

$$\boldsymbol{x}_2(t) = B\boldsymbol{x}_1(t) + C\boldsymbol{w}(t) \tag{5}$$

where $\Phi_0 = M_1^{-1}\Phi_1$, $\Gamma_0 = M_1^{-1}\Gamma_1$, $\bar{H}^{(i)} = H_1^{(i)} + H_2^{(i)}B$, $\eta^{(i)}(t) = \Gamma_3^{(i)}w(t) + v^{(i)}(t)$, $\Gamma_3^{(i)} = H_2^{(i)}C$, $B = \Phi_3^{-1}M_2M_1^{-1}\Phi_1 - \Phi_3^{-1}\Phi_2$, $C = \Phi_3^{-1}M_2M_1^{-1}\Gamma_1 - \Phi_3^{-1}\Gamma_2$. Also we have

$$\mathrm{E}\left\{\begin{bmatrix} \boldsymbol{w}(t) \\ \boldsymbol{\eta}^{(i)}(t) \end{bmatrix} [\boldsymbol{w}^{\rm T}(k) \quad \boldsymbol{\eta}^{(j)^{\rm T}}(k)]\right\} = Q^{(ij)}\delta_{tk}, \quad Q^{(ij)} = \begin{bmatrix} Q_w & S^{(j)} \\ S^{(i)^{\rm T}} & Q_{\eta^{(ij)}} \end{bmatrix} \tag{6}$$

where $S^{(i)} = Q_w\Gamma_3^{(i)^{\rm T}}$, $Q_{\eta^{(ii)}} = Q_{\eta^{(i)}} = \Gamma_3^{(i)}Q_w\Gamma_3^{(i)^{\rm T}} + Q_{v^{(i)}}$ and $Q_{\eta^{(ij)}} = \Gamma_3^{(i)}Q_w\Gamma_3^{(j)^{\rm T}}$, $i \neq j$. E is the expectation, and δ_{tk} is the Kronecker delta function.

3 Distributed fusion filters

For every sensor subsystem of system (4) with multiple sensors, from [16] we can obtain the local Kalman filter $\hat{\boldsymbol{x}}_1^{(i)}(t|t)$ for the reduced-order state $\boldsymbol{x}_1(t)$, the filtering gain $K^{(i)}(t)$, the filtering error covariance $P_1^{(i)}(t|t)$, innovation $\boldsymbol{\varepsilon}^{(i)}(t)$ with covariance $Q_{\varepsilon^{(i)}}(t)$ and the white noise filter $\hat{\boldsymbol{w}}(t|t)$. So, from (5) we have the filter of the reduced-order state $\boldsymbol{x}_2(t)$ as

$$\hat{\boldsymbol{x}}_2^{(i)}(t|t) = B\hat{\boldsymbol{x}}_1^{(i)}(t|t) + C\hat{\boldsymbol{w}}^{(i)}(t|t) \tag{7}$$

3.1 Computation of cross covariance

From (4)$\sim$(7) and [16], we can obtain the prediction and filtering error equations as follows:

$$\tilde{\boldsymbol{x}}_1^{(i)}(t+1|t) = \bar{\Phi}_0^{(i)}[I_{n_1} - K^{(i)}(t)\bar{H}^{(i)}]\tilde{\boldsymbol{x}}_1^{(i)}(t|t-1) + \Gamma_0\boldsymbol{w}(t) - (\bar{\Phi}_0^{(i)}K^{(i)}(t) + J^{(i)})\boldsymbol{\eta}^{(i)}(t) \tag{8}$$

$$\tilde{\boldsymbol{x}}_1^{(i)}(t|t) = [I_{n_1} - K^{(i)}(t)\bar{H}^{(i)}]\tilde{\boldsymbol{x}}_1^{(i)}(t|t-1) - K^{(i)}(t)\boldsymbol{\eta}^{(i)}(t) \tag{9}$$

$$\tilde{\boldsymbol{x}}_2^{(i)}(t|t) = F^{(i)}(t)\tilde{\boldsymbol{x}}_1^{(i)}(t|t-1) + D^{(i)}(t)[\boldsymbol{w}^{\rm T}(t), \boldsymbol{\eta}^{(i)^{\rm T}}(t)]^{\rm T} \tag{10}$$

where $\tilde{\boldsymbol{x}}_1^{(i)}(t|t-1) = \boldsymbol{x}_1(t) - \hat{\boldsymbol{x}}_1^{(i)}(t|t-1)$, $\tilde{\boldsymbol{x}}_1^{(i)}(t|t) = \boldsymbol{x}_1(t) - \hat{\boldsymbol{x}}_1^{(i)}(t|t)$, $\tilde{\boldsymbol{x}}_2^{(i)}(t|t) = \boldsymbol{x}_2(t) - \hat{\boldsymbol{x}}_2^{(i)}(t|t)$, $\bar{\Phi}_0^{(i)} = \Phi_0 - J^{(i)}\bar{H}^{(i)}$, $J^{(i)} = \Gamma_0 S^{(i)} Q_{\eta^{(i)}}^{-1}$, $F^{(i)}(t) = B(I_{n_1} - K^{(i)}(t)\bar{H}^{(i)}) - CS^{(i)}Q_{\varepsilon^{(i)}}^{-1}(t)\bar{H}^{(i)}$ and $D^{(i)}(t) = [C, -BK^{(i)}(t) - CS^{(i)}Q_{\varepsilon^{(i)}}^{-1}(t)]$. I_{n_1} is an $n_1 \times n_1$ identity matrix. Using (8)$\sim$(10) and projection theory[16], we easily obtain the following Lemmas 1 and 2.

Lemma 1. For system (4) with multiple sensors, the cross-covariance matrices of prediction and filtering errors for state $\boldsymbol{x}_1(t)$ between the ith and the jth sensor subsystems are given by

$$\begin{aligned} P_1^{(ij)}(t+1|t) = {} & \bar{\Phi}_0^{(i)}[I_{n_1} - K^{(i)}(t)\bar{H}^{(i)}]P_1^{(ij)}(t|t-1)[I_{n_1} - K^{(j)}(t)\bar{H}^{(j)}]^{\rm T}\bar{\Phi}_0^{(j)^{\rm T}} + \\ & [\Gamma_0, -\bar{\Phi}_0^{(i)}K^{(i)}(t) - J^{(i)}]Q^{(ij)}[\Gamma_0, -\bar{\Phi}_0^{(j)}K^{(j)}(t) - J^{(j)}]^{\rm T} \end{aligned} \tag{11}$$

$$P_1^{(ij)}(t|t) = [I_{n_1} - K^{(i)}(t)\bar{H}^{(i)}]P_1^{(ij)}(t|t-1)[I_{n_1} - K^{(j)}(t)\bar{H}^{(j)}]^{\rm T} + K^{(i)}(t)Q_{\eta^{(ij)}}K^{(j)^{\rm T}}(t) \tag{12}$$

with the initial value $P_1^{(ij)}(0|-1)=P_{01}$ where P_{01} is the first $n_1\times n_1$ block of $R^{-1}P_0R^{-\mathrm{T}}$.

Lemma 2. For system (5) with multiple sensors, the covariance matrix of the filtering errors for state $\boldsymbol{x}_2(t)$ between the ith and the jth sensor subsystems is given by

$$P_2^{(ij)}(t|t)=F^{(i)}(t)P_1^{(ij)}(t|t-1){F^{(j)}}^{\mathrm{T}}(t)+D^{(i)}(t)Q^{(ij)}{D^{(j)}}^{\mathrm{T}}(t) \tag{13}$$

where $P_2^{(ii)}(t|t)$ is the filtering error variance of $\boldsymbol{x}_2(t)$ based on the ith sensor, *i.e.*, $P_2^{(i)}(t|t)$.

3.2 Full-order fusion filter

Theorem 1. For singular system (1) and (2) with multiple sensors, we have the distributed full-order optimal fusion filter

$$\hat{\boldsymbol{x}}^o(t|t)=\sum_{i=1}^{l}\bar{A}^{(i)}(t)\hat{\boldsymbol{x}}^{(i)}(t|t) \tag{14}$$

The optimal matrix weights $\bar{A}^{(i)}(t)$, $i=1,2,\cdots,l$ are computed by

$$\bar{A}(t)=\Sigma^{-1}(t)e(e^{\mathrm{T}}\Sigma^{-1}(t)e)^{-1} \tag{15}$$

where $\bar{A}(t)=[\bar{A}^{(1)}(t),\cdots,\bar{A}^{(l)}(t)]^{\mathrm{T}}$ and $e=[I_n\cdots I_n]^{\mathrm{T}}$ are both $nl\times n$ matrices. $\Sigma(t)=(P^{(ij)}(t|t))_{nl\times nl}$ is an $nl\times nl$ matrix. Covariance matrix $P^{(ij)}(t|t)$ between $\tilde{\boldsymbol{x}}^{(i)}(t|t)$ and $\tilde{\boldsymbol{x}}^{(j)}(t|t)$ is computed by

$$P^{(ij)}(t|t)=R\begin{bmatrix}P_1^{(ij)}(t|t) & P_{12}^{(ij)}(t|t)\\ P_{21}^{(ij)}(t|t) & P_2^{(ij)}(t|t)\end{bmatrix}R^{\mathrm{T}} \tag{16}$$

where the correlated matrix $P_{12}^{(ij)}(t|t)$ between $\tilde{\boldsymbol{x}}_1^{(i)}(t|t)$ and $\tilde{\boldsymbol{x}}_2^{(j)}(t|t)$ is computed by

$$P_{12}^{(ij)}(t|t)=(I_{n_1}-K^{(i)}(t)\bar{H}^{(i)})P_1^{(ij)}(t|t-1){F^{(j)}}^{\mathrm{T}}(t)+[0,-K^{(i)}(t)]Q^{(ij)}{D^{(j)}}^{\mathrm{T}}(t) \tag{17}$$

with $P_{12}^{(ij)}(t|t)={P_{21}^{(ji)}}^{\mathrm{T}}(t|t)$. $\hat{\boldsymbol{x}}^{(i)}(t|t)$ is computed by

$$\hat{\boldsymbol{x}}^{(i)}(t|t)=R[{\hat{\boldsymbol{x}}_1^{(i)}}^{\mathrm{T}}(t|t),{\hat{\boldsymbol{x}}_2^{(i)}}^{\mathrm{T}}(t|t)]^{\mathrm{T}} \tag{18}$$

and the variance matrix of the optimal fusion filter $\hat{\boldsymbol{x}}^o(t|t)$ is computed by

$$P^o(t|t)=(e^{\mathrm{T}}\Sigma^{-1}(t)e)^{-1} \tag{19}$$

and we have $P^o(t|t)\leqslant P^{(i)}(t|t)$, $i=1,2,\cdots,l$.

Proof. Taking projection on $\boldsymbol{x}(t)=R[\boldsymbol{x}_1^{\mathrm{T}}(t)\quad \boldsymbol{x}_2^{\mathrm{T}}(t)]^{\mathrm{T}}$ gives (18). We have the filtering error

$$\tilde{\boldsymbol{x}}^{(i)}(t|t)=R[{\tilde{\boldsymbol{x}}_1^{(i)}}^{\mathrm{T}}(t|t),{\tilde{\boldsymbol{x}}_2^{(i)}}^{\mathrm{T}}(t|t)]^{\mathrm{T}} \tag{20}$$

From (20) we have the covariance matrix of the filtering errors as (16). Using (9) and (10) gives (17). Using the optimal fusion algorithm[14], we have (14), (15), and (19). □

3.3 Reduced-order fusion filters

Theorem 1 gives a distributed full-order optimal fusion Kalman filter (DFFKF). It requires the inverse of an $nl\times nl$ high-dimension matrix $\Sigma(t)$. To reduce the computational burden, we will give a distributed reduced-order fusion Kalman filter (DRFKF).

Theorem 2. For two reduced-order subsystems (4) and (5) with multiple sensors, we have the reduced-order optimal fusion filters

$$\hat{\boldsymbol{x}}_k^{(o)}(t|t)=(e_k^{\mathrm{T}}\Sigma_k^{-1}(t)e_k)^{-1}e_k^{\mathrm{T}}\Sigma_k^{-1}(t)[{\hat{\boldsymbol{x}}_k^{(1)}}^{\mathrm{T}}(t|t),{\hat{\boldsymbol{x}}_k^{(2)}}^{\mathrm{T}}(t|t),\cdots,{\hat{\boldsymbol{x}}_k^{(l)}}^{\mathrm{T}}(t|t)]^{\mathrm{T}},\quad k=1,2 \tag{21}$$

where $e_k=[I_{n_k},\cdots,I_{n_k}]^{\mathrm{T}}$ is an $n_kl\times n_k$ matrix. $\Sigma_k(t)=(P_k^{(ij)}(t|t))_{n_kl\times n_kl}$, $i,j=1,2,\cdots,l$ is an $n_kl\times n_kl$ positive definite matrix. Variances of $\hat{\boldsymbol{x}}_k^{(o)}(t|t)$, $k=1,2$ are given by

$$P_k^{(o)}(t|t)=(e_k^{\mathrm{T}}\Sigma_k^{-1}(t)e_k)^{-1},\quad k=1,2 \tag{22}$$

and we have $P_k^{(o)}(t|t)\leqslant P_k^{(i)}(t|t)$, $i=1,2,\cdots,l$; $k=1,2$.

Proof. These follow from the optimal fusion algorithm in [14]. □

Theorem 3. For singular system (1) and (2) with multiple sensors, we have the fusion filter

$$\hat{\boldsymbol{x}}^{(o)}(t|t) = R[\hat{\boldsymbol{x}}_1^{(o)^{\mathrm{T}}}(t|t) \quad \hat{\boldsymbol{x}}_2^{(o)^{\mathrm{T}}}(t|t)]^{\mathrm{T}} \tag{23}$$

The variance of the filtering error of $\hat{\boldsymbol{x}}^{(o)}(t|t)$ is computed by

$$P^{(o)}(t|t) = R\begin{bmatrix} P_1^{(o)}(t|t) & P_{12}^{(o)}(t|t) \\ P_{21}^{(o)}(t|t) & P_2^{(o)}(t|t) \end{bmatrix} R^{\mathrm{T}} \tag{24}$$

where $P_1^{(ij)}(t|t)$ and $P_2^{(ij)}(t|t)$ are computed by Lemmas 1 and 2. Correlated matrix $P_{12}^{(o)}(t|t)$ between filtering errors $\tilde{\boldsymbol{x}}_1^{(o)}(t|t)$ and $\tilde{\boldsymbol{x}}_2^{(o)}(t|t)$ is computed by

$$P_{12}^{(o)}(t|t) = P_1^{(o)}(t|t)e_1^{\mathrm{T}}\Sigma_1^{-1}(t)\Sigma_{12}(t)\Sigma_2^{-1}(t)e_2P_2^{(o)}(t|t) \tag{25}$$

where $P_{12}^{(o)}(t|t) = P_{21}^{(o)^{\mathrm{T}}}(t|t)$ and $\Sigma_{12}(t) = (P_{12}^{(ij)}(t|t))_{n_1l\times n_2l}$. $P_{12}^{(ij)}(t|t)$ is computed by (17).

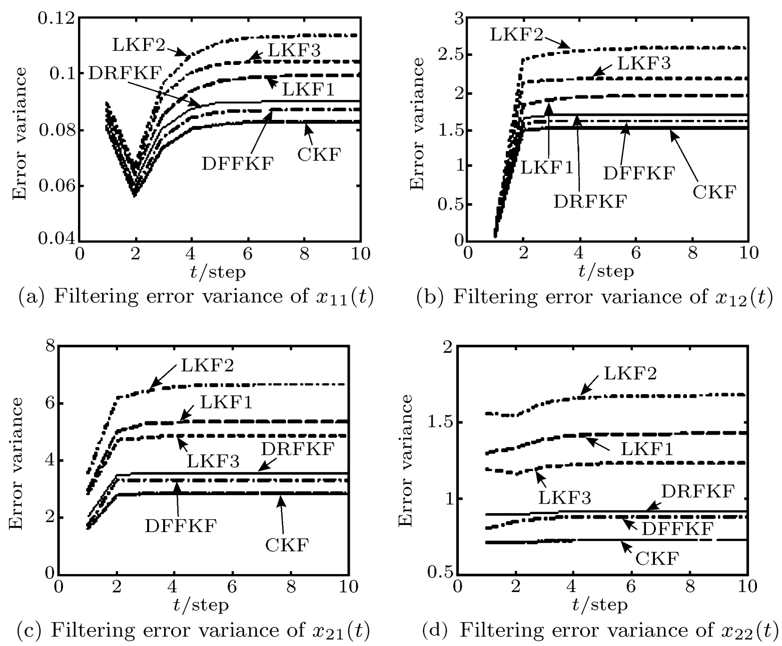
Proof. (23) and (24) can be obtained easily. Using (21) gives (25). □

So far, we have given the DFFKF by Theorem 1 and DRFKF by Theorems 2 and 3. We have the results that the precision of DFFKF is higher than that of DRFKF, however, DRFKF can reduce the computation burden since it only requires in parallel the inverses of two $n_kl \times n_kl$, $k = 1, 2$ relatively low-dimension matrices with $n = n_1 + n_2$ to compute the weights.

4 Simulation example

For simplification, we consider a stochastic singular system with three sensors as in (3) where $M_1 = \begin{bmatrix} -2.13 & 0 \\ 1 & 0.5 \end{bmatrix}$, $M_2 = \begin{bmatrix} 1 & 0.5 \\ 0 & -1 \end{bmatrix}$, $\varPhi_1 = \begin{bmatrix} -1 & 0.2 \\ -0.5 & 0 \end{bmatrix}$, $\varPhi_2 = \begin{bmatrix} 1 & -0.5 \\ 0 & -1 \end{bmatrix}$, $\varPhi_3 = \begin{bmatrix} -0.5 & 0 \\ -1 & 2 \end{bmatrix}$, $\varGamma_1 = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.8 \end{bmatrix}^{\mathrm{T}}$, $\varGamma_2 = \begin{bmatrix} 0.8 & 0 \\ 0 & -0.6 \end{bmatrix}^{\mathrm{T}}$, $H_1^{(1)} = \begin{bmatrix} 1 & 0.5 \\ 0 & 1 \end{bmatrix}$, $H_2^{(1)} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, $H_1^{(2)} = \begin{bmatrix} 1 & 0 \\ 0 & 0.8 \end{bmatrix}$, $H_2^{(2)} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$, $H_1^{(3)} = \begin{bmatrix} 0.1 & 0.6 \\ 1 & 0.1 \end{bmatrix}$, $H_2^{(3)} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$, $Q_w = I_2$, $Q_{v^{(1)}} = 8I_2$, $Q_{v^{(2)}} = 12I_2$ and $Q_{v^{(3)}} = 15I_2$. States $\boldsymbol{x}_1(t) = [x_{11}(t) \quad x_{12}(t)]^{\mathrm{T}}$ and $\boldsymbol{x}_2(t) = [x_{21}(t) \quad x_{22}(t)]^{\mathrm{T}}$ with initial values $\boldsymbol{x}_1(0) = \boldsymbol{x}_2(0) = [0 \quad 0]^{\mathrm{T}}$ and $P_{10} = 0.1I_2$. Our aim is to find DRFKF $\hat{\boldsymbol{x}}_k^{(o)}(t|t)$, $k = 1, 2$.

Variances of DFFKF, DRFKF, local Kalman filters (LKF) and the centralized Kalman filter (CKF) are shown in Fig. 1. Fig.1 shows that the precision of DRFKF and DFFKF is higher than that



(a) Filtering error variance of $x_{11}(t)$ (b) Filtering error variance of $x_{12}(t)$

(c) Filtering error variance of $x_{21}(t)$ (d) Filtering error variance of $x_{22}(t)$

Fig. 1 Comparison of the precision of LKF, DFFKF, DRFKF and CKF of each LKF, but lower than that of CKF. However, DRFKF and DFFKF can increase the input data rate and have reliability since the distributed parallel structure is used. Further, the precision of DFFKF is higher than that of DRFKF, but DFFKF requires the inverse of a 12×12 matrix to compute the weights, however, DRFKF requires in parallel the inverses of two 6×6 matrices to compute the weights. So DRFKF can reduce the computational burden.

5 Conclusion

For stochastic singular systems with multiple sensors, a distributed full-order optimal fusion Kalman filter is given based on the optimal fusion algorithm weighted by matrices in the LMV sense, which brings the computational burden since the inverse of a high-dimension matrix is required to compute weights at each step. To reduce the computational burden, a distributed reduced-order fusion Kalman filter is presented, which can improve the real-time property since it involves in parallel the inverses of two relatively low-dimension matrices to compute weights at each step.

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