Robust Stabilization of Nonlinear Time Delay Discrete-time Systems Based on T-S Model[1)]

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Abstract A robust stabilization problem is considered for time delay nonlinear discrete-time systems based on T-S fuzzy model. A necessary and sufficient condition for the existence of such controllers is given through Lyapunov stability theorem. And it is further shown that this condition is equivalent to the solvability of a certain linear matrix inequality, which can be solved easily by using the LMI toolbox of Matlab. At last, an illustrative example of truck-trailer is presented to show the feasibility and effectiveness of the proposed method.

Key words Discrete-time systems, fuzzy control, linear matrix inequality, T-S fuzzy model

1 Introduction

Recently, fuzzy control has been one of the useful control techniques for uncertain and nonlinear complicated systems. The conventional fuzzy control is composed of some if-then linguistic rules. The property of it makes the control algorithm easily understood. Its main drawback, however, comes from the lack of a systematic control design methodology. Particularly, the stability analysis and robustness are not easy. To solve these problems, the idea that a linear system is adopted as the consequent part of a fuzzy rule has evolved into the T-S model[1], which becomes quite popular today.

Time delays are common in engineering field and are a source of instability and poor performance even in a nonlinear mode, so there are many results to deal with time delay problem[2,3]. With development of computer, the discrete system has attracted great attention[4~7], and the fuzzy control has been extended to nonlinear time delay discrete system, but research results are too limited for reference. The robust stabilization of linear system in [4] is discussed by using LMI techniques. The stability of nonlinear system is considered by using fuzzy control[2,8,9], in which the consequent part of T-S model is linear normal system without uncertainties. In [2] the analysis and synthesis problem is investigated including continuous and discrete time delay systems, but there is only time delay part in the T-S model. The fuzzy robust tracking control is discussed in [10] for uncertain nonlinear system, the parametric uncertainty is employed to the consequent part of the T-S model, and so the T-S model can represent the original system exactly. However, it does not apply the method to the discrete time delay system. In [11], the stabilization problem is discussed for a class of nonlinear discrete systems with parameter uncertainty, without considering the time delay term.

So far the class of nonlinear time delay discrete systems have not yet been discussed by using the T-S fuzzy control method, but time delay and uncertainty occur in practical engineering field. Based on these intentions, in this paper, the robust stabilization problem will be considered for nonlinear time delay discrete systems. And it will be shown that this stabilization problem is equivalent to the solvability of a certain linear matrix inequality. Finally, an illustrative example of truck-trailer will show the feasibility of the proposed method.

2 Problem formulation

The consequent part of T-S model has exact mathematics description, so the fuzzy T-S model as in [12] is used in this paper. The ith rule of the fuzzy model for the nonlinear discrete system is of the following form:

Plant Rule i: If $\boldsymbol{z}_1(k)$ is F_1^i, $\cdots$, and $\boldsymbol{z}_n(k)$ is F_n^i then

$$\begin{aligned}&\boldsymbol{x}(k+1)=(A_i+\Delta A_i)\boldsymbol{x}(k)+A_{1i}\boldsymbol{x}(k-\tau)+(B_i+\Delta B_i)\boldsymbol{u}(k)\\&\boldsymbol{y}(k)=C_i\boldsymbol{x}(k),\quad i=1,\cdots,q\end{aligned}\tag{1}$$

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where $\boldsymbol{x}(k)=\psi(k)$, for $k=-\tau,\cdots,-1,0$, is the initial value, $\boldsymbol{z}(k)=[z_1(k)\cdots z_n(k)]^{\mathrm{T}}$ is the premise variable vector, $\boldsymbol{x}(k)=[x_1(k),\cdots,x_n(k)]^{\mathrm{T}}$ is the state vector, $(j=1,\cdots,n)$, F_j^i is a fuzzy set, $\tau>0$ is a constant, $A_i\in R^{n\times n}$ is the system matrix, $B_i\in R^{n\times m}$ and $C_i\in R^{l\times n}$ are input and output matrices, respectively, ΔA_i and ΔB_i are matrices with appropriate dimensions, representing parametric uncertainties in the plant model, and q is the number of rules of this T-S fuzzy model.

Here, by using a standard fuzzy inference method that is using a singleton fuzzifier product fuzzy inference and weighted average defuzzifier for system (1), the final state of the fuzzy system is inferred as follows.

$$\begin{aligned}\boldsymbol{x}(k+1)&=\sum_{i=1}^{q}h_i(z(k))[A_ix(k)+\Delta A_ix(k)+A_{1i}\boldsymbol{x}(k-\tau)+B_i\boldsymbol{u}(k)+\Delta B_i\boldsymbol{u}(k)]\\ \boldsymbol{y}(k)&=\sum_{i=1}^{q}h_i(z(k))C_i\boldsymbol{x}(k)\end{aligned}\tag{2}$$

where

$$w_i(z(k))=\prod_{j=1}^{n}F_j^i(z_j(k)),\ h_i(z(k))=\frac{w_i(z(k))}{\sum_{i=1}^{q}w_i(z(k))},\ w_i(k)\geqslant 0,\ \sum_{i=1}^{q}w_i(k)>0,\ i=1,2,\cdots,q$$

and F_j^i is the fuzzy set, h_i is the grade of membership of F_j^i.

The main intention of this paper is to design the fuzzy T-S model controller, which can make system (2) stabilized.

For the convenience of proof, the assumption is given as follows

Assumption 1. The parametric uncertainties in system (1) are norm bounded, satisfying the followings

$$[\Delta A_i,\Delta B_i]=D_iF_i(k)[E_{i1},E_{i2}],\quad F_i^{\mathrm{T}}(k)F_i(k)\leqslant I$$

where D_i, E_{i1}, E_{i2} are known real constant matrices of appropriate dimensions, and $F_i(k)$ is an unknown matrix function with Lebesgue-measerable element, I is the identity matrix of appropriate dimension.

3 Fuzzy state feedback control design

Based on the parallel distributed compensation (PDC), we consider the following fuzzy control law for the fuzzy model (2)

Regulator Rule i:

$$\begin{aligned}&\text{If } \boldsymbol{z}_1(\text{k}) \text{ is } F_1^i \text{ and } \cdots \text{ and } \boldsymbol{z}_n(k) \text{ is } F_n^i, \text{ then}\\ &\boldsymbol{u}(k)=K_i\boldsymbol{x}(k),\quad i=1,\cdots,q\end{aligned}\tag{3}$$

where $K_i\in R^{m\times n}$ is a constant gain feedback to be determined.

The overall state feedback fuzzy control law is represented by

$$\boldsymbol{u}(k)=\sum_{i=1}^{q}h_i(z(k))K_i\boldsymbol{x}(k)\tag{4}$$

Substituting (4) into (2) yields

$$\boldsymbol{x}(k+1)=\sum_{i=1}^{q}\sum_{j=1}^{q}h_i(z(k))h_j(z(k))((A_i+\Delta A_i)\boldsymbol{x}(k)+A_{1i}\boldsymbol{x}(k-\tau)+(B_i+\Delta B_i)K_j\boldsymbol{x}(k))\tag{5}$$

For the necessary of proof, one lemma is given as follows.

Lemma 1. Given matrices H, F, and E with appropriate dimensions, $F^{\mathrm{T}}F\leqslant I$, and $P>0$, for any $\varepsilon>0$, if $P^{-1}-\varepsilon HH^{\mathrm{T}}>0$, then $(A+HFE)^{\mathrm{T}}P(A+HFE)\leqslant A^{\mathrm{T}}(P^{-1}-\varepsilon HH^{\mathrm{T}})^{-1}A+\varepsilon^{-1}E^{\mathrm{T}}E$.

The main results on the fuzzy robust stabilization of T-S model with parametric uncertainties are summarized in the following theorem.

Theorem 1. If there exist a symmetric and positive definite matrix $P > 0$ and some matrices $K_i (i = 1, 2, \cdots, q)$, such that the following matrix inequalities are satisfied, then the stability of system (4) is guaranteed via the T-S fuzzy model based state-feedback controller (3).

$$\begin{bmatrix} -X & * & * & * & * \\ 0 & -H & * & * & * \\ A_iX + B_iM_i & A_{1i}H & -X + \varepsilon_i D_i D_i^{\mathrm{T}} & * & * \\ E_{i1}X + E_{i2}M_i & 0 & 0 & -\varepsilon_i I & * \\ X & 0 & 0 & 0 & -H \end{bmatrix} < 0 \tag{6}$$

and

$$\begin{bmatrix} -X & * & * & * & * & * \\ 0 & -H & * & * & * & * \\ \dfrac{A_iX + B_iM_j + A_jX + B_jM_i}{2} & \dfrac{A_{1i} + A_{1j}}{2}H & -X + \varepsilon_{ij}(D_iD_i^{\mathrm{T}} + D_jD_j^{\mathrm{T}}) & * & * & * \\ \dfrac{E_{i1}X + E_{i2}M_j}{2} & 0 & 0 & -\varepsilon_{ij}I & * & * \\ \dfrac{E_{j1}X + E_{j2}M_i}{2} & 0 & 0 & 0 & -\varepsilon_{ji}I & * \\ X & 0 & 0 & 0 & 0 & -H \end{bmatrix} < 0 \tag{7}$$

Proof. Select Lyapunov function as

$$V(k) = \boldsymbol{x}^{\mathrm{T}}P\boldsymbol{x} + \sum_{\alpha=k-\tau}^{k-1} \boldsymbol{x}^{\mathrm{T}}(\alpha)S\boldsymbol{x}(\alpha) \tag{8}$$

where $X = P^{-1}$, $H = S^{-1}$, $\bar{\boldsymbol{x}} = \begin{bmatrix} \boldsymbol{x}(k) \\ \boldsymbol{x}(k-\tau) \end{bmatrix}$, $G = \begin{bmatrix} -P + S & 0 \\ 0 & -S \end{bmatrix}$, $\bar{A}_{ij} = A_i + \Delta A_i + (B_i + \Delta B_i)K_j$, $M_j = K_jX$. Giving the difference of (8), we get

$$\Delta V(k) = V(k+1) - V(k) = \boldsymbol{x}^{\mathrm{T}}(k+1)P\boldsymbol{x}(k+1) - \boldsymbol{x}^{\mathrm{T}}(k)P\boldsymbol{x}(k) + \boldsymbol{x}^{\mathrm{T}}(k)S\boldsymbol{x}(k) - \boldsymbol{x}^{\mathrm{T}}(k-\tau)S\boldsymbol{x}(k-\tau) \tag{9}$$

Then substituting (5) into (9) yields

$$\begin{aligned} \Delta V(k) = & \sum_{i=1}^{q}\sum_{j=1}^{q}\sum_{k=1}^{q}\sum_{l=1}^{q} h_ih_jh_kh_l\{\boldsymbol{x}^{\mathrm{T}}(k)[\bar{A}_{ij}^{\mathrm{T}}P\bar{A}_{kl} - P + S]\boldsymbol{x}(k) + \boldsymbol{x}^{\mathrm{T}}(k)\bar{A}_{ij}^{\mathrm{T}}P\bar{A}_{1k}\boldsymbol{x}(k-\tau) + \\ & \boldsymbol{x}^{\mathrm{T}}(k-\tau)\bar{A}_{1i}^{\mathrm{T}}P\bar{A}_{kl}\boldsymbol{x}(k) + \boldsymbol{x}^{\mathrm{T}}(k-\tau)\bar{A}_{1i}^{\mathrm{T}}P\bar{A}_{1k}\boldsymbol{x}(k-\tau) - \boldsymbol{x}^{\mathrm{T}}(k-\tau)S\boldsymbol{x}(k-\tau)\} = \\ & \sum_{i=1}^{q}\sum_{j=1}^{q}\sum_{k=1}^{q}\sum_{l=1}^{q} h_ih_jh_kh_l\bar{\boldsymbol{x}}^{\mathrm{T}}(k)([\bar{A}_{ij} \quad A_{1i}]^{\mathrm{T}}P[\bar{A}_{kl} \quad A_{1k}] + G)\bar{x}(k) = \\ & \frac{1}{4}\sum_{i=1}^{q}\sum_{j=1}^{q}\sum_{k=1}^{q}\sum_{l=1}^{q} h_ih_jh_kh_l\bar{\boldsymbol{x}}^{\mathrm{T}}\{[\bar{A}_{ij} + \bar{A}_{ji} \quad A_{1i} + A_{1j}]^{\mathrm{T}}P[\bar{A}_{kl} + \bar{A}_{lk} \quad A_{1k} + A_{1l}] + 4G\}\bar{\boldsymbol{x}} \leqslant \\ & \frac{1}{4}\sum_{i=1}^{q}\sum_{j=1}^{q}\sum_{k=1}^{q}\sum_{l=1}^{q} h_ih_jh_kh_l\bar{\boldsymbol{x}}^{\mathrm{T}}\{[\bar{A}_{ij} + \bar{A}_{ji} \quad A_{1i} + A_{1j}]^{\mathrm{T}}P[\bar{A}_{ij} + \bar{A}_{ji} \quad A_{1i} + A_{1j}] + 4G\}\bar{\boldsymbol{x}} = \\ & \sum_{i=1}^{q} h_i^2\bar{\boldsymbol{x}}^{\mathrm{T}}(k)([\bar{A}_{ii} \quad A_{1i}]^{\mathrm{T}}P[\bar{A}_{ii} \quad A_{1i}] + G)\bar{\boldsymbol{x}}(k) + \\ & 2\sum_{i=1}^{q}\sum_{j=1}^{q} h_ih_j\bar{\boldsymbol{x}}^{\mathrm{T}}(k)\left(\left[\frac{\bar{A}_{ij} + \bar{A}_{ji}}{2} \quad \frac{A_{1i} + A_{1j}}{2}\right]^{\mathrm{T}}P\left[\frac{\bar{A}_{ij} + \bar{A}_{ji}}{2} \quad \frac{\bar{A}_{ij} + \bar{A}_{ji}}{2}\right] + G\right)\bar{\boldsymbol{x}}(k) \end{aligned} \tag{10}$$

So system (2) can be stabilized if and only if there exist a symmetric and positive definite matrix $P > 0$ and some matrices K_i such that the following inequalities are satisfied

$$[\bar{A}_{ii} \quad A_{1i}]^{\mathrm{T}}P[\bar{A}_{ii} \quad A_{1i}] + G < 0 \tag{11}$$

$$\begin{bmatrix} \dfrac{\bar{A}_{ij}+\bar{A}_{ji}}{2} & \dfrac{A_{1i}+A_{1j}}{2} \end{bmatrix}^{\mathrm{T}} P \begin{bmatrix} \dfrac{\bar{A}_{ij}+\bar{A}_{ji}}{2} & \dfrac{\bar{A}_{ij}+\bar{A}_{ji}}{2} \end{bmatrix} + G \leqslant 0,\ i<j \tag{12}$$

It is noted that Theorem 1 gives the sufficient condition of ensuring the stability of the fuzzy system (2). However, it does not give the methods of obtaining the solution of a common matrix P and gain feedback K_i. Fortunately, (11) and (12) can be transferred into LMIs, so parametric P and K_i can be determined by the solubility of LMI.

From Assumption 1, we can see $[\Delta A_i, \Delta B_i] = D_i F_i(k)[E_{i1}, E_{i2}]$, so (11) is equivalent to

$$([A_i+B_iK_i \quad A_{1i}]+D_iF_i[E_{i1}+E_{i2}K_i \quad 0])^{\mathrm{T}} P([A_i+B_iK_i \quad A_{1i}]+D_iF_i[E_{i1}+E_{i2}K_i \quad 0])+G<0 \tag{13}$$

From Lemma 1, (13) holds if there exists some real constant such that, $P^{-1}-\varepsilon_i D_i D_i^{\mathrm{T}}>0$, and

$$G+[A_i+B_iK_i \quad A_{1i}]^{\mathrm{T}}(P^{-1}-\varepsilon_i D_i D_i^{\mathrm{T}})[A_i+B_iK_i \quad A_{1i}]+\varepsilon_i^{-1}[E_{1i}+E_{2i}K_i \quad 0]^{\mathrm{T}}[E_{1i}+E_{2i}K_i \quad 0]<0 \tag{14}$$

By using Schur complement, the inequality (14) can be rewritten as

$$\begin{bmatrix} -P+S & 0 & (A_i+B_iK_i)^{\mathrm{T}} & (E_{1i}+E_{2i}K_i)^{\mathrm{T}} \\ 0 & -S & A_{1i}^{\mathrm{T}} & 0 \\ (A_i+B_iK_i) & A_{1i} & -P^{-1}-\varepsilon_i D_i D_i^{\mathrm{T}} & 0 \\ E_{1i}+E_{2i}K_i & 0 & 0 & -\varepsilon_i I \end{bmatrix} < 0 \tag{15}$$

Left multiplying and right multiplying (15) by diag$[X \quad H \quad I \quad I]$, and then applying Schur complement, (15) is then equivalent to

$$\begin{bmatrix} -X & * & * & * & * \\ 0 & -H & * & * & * \\ A_iX+B_iM_i & A_{1i}H & -X+\varepsilon_i D_i D_i^{\mathrm{T}} & * & * \\ E_{i1}X+E_{i2}M_i & 0 & 0 & -\varepsilon_i I & * \\ X & 0 & 0 & 0 & -H \end{bmatrix} < 0$$

It is easy to transfer matrix (12) into the following

$$\begin{bmatrix} -X & * & * & * & * & * \\ 0 & -H & * & * & * & * \\ \dfrac{A_iX+B_iM_j+A_jX+B_jM_i}{2} & \dfrac{A_{1i}+A_{1j}}{2}H & -X+\varepsilon_{ij}(D_iD_i^{\mathrm{T}}+D_jD_j^{\mathrm{T}}) & * & * & * \\ \dfrac{E_{i1}X+E_{i2}M_j}{2} & 0 & 0 & -\varepsilon_{ij}I & * & * \\ \dfrac{E_{j1}X+E_{j2}M_i}{2} & 0 & 0 & 0 & -\varepsilon_{ji}I & * \\ X & 0 & 0 & 0 & 0 & -H \end{bmatrix} < 0$$

By using LMI software of Matlab, the parametric uncertainty X and K_i can be solved, so the controller can be determined.

Then we conclude the computational procedure as follows:

Step 1. Use T-S fuzzy implications and the fuzzy reasoning method to express the real plant model.

Step 2. Based on the parallel distributed compensation, the fuzzy control law for the fuzzy T-S model is constructed.

Step 3. The global fuzzy model can be obtained from Step 1 and Step 2.

Step 4. The stability analysis of the global fuzzy closed-loop system can be transferred into the solution of linear matrix inequality (LMI).

Step 5. The LMI can be solved by the Matlab simulink toolbox.

It should be noted that the above procedure belongs to fuzzy modeling and fuzzy control process. The solution of the LMI plays a crucial role in arriving at the desired solution. This procedure is illustrated by the following example.

4 Simulation example

To illustrate the proposed fuzzy robust control approach, a control problem of truck-trailer[12] with the time-delay state $x_1(k)$ is considered, *i.e.*,

$$x_1(k+1) = (1-v\frac{\bar{k}}{L})x_1(k) + (1-v\frac{\bar{k}}{L})x_1(k-\tau) + v\frac{\bar{k}}{l}u(k) + a(k)x_1(k)$$

$$x_2(k+1) = x_2(k) + v\frac{\bar{k}}{L}x_1(k) + v\frac{\bar{k}}{L}x_1(k-\tau) + a(k)x_2(k)$$

$$x_3(k+1) = x_3(k) + v\bar{k}\sin(x_2(k) + v\frac{\bar{k}}{2L}x_1(k) + v\frac{\bar{k}}{2L}x_1(k-\tau)) + a(k)x_3(k)$$

where l is the length of the truck, L is the length of the trailer, $\bar{k}$ is the sampling time, and v is the constant speed of the backward movement, $a(k) = 0.2\sin(k)$ is the parameter uncertainty. Take fuzzy model rule as follows.

$$R^1 \text{ If } \boldsymbol{z}(k) = x_2(k) + v\frac{\bar{k}}{2L}x_1(k) + v\frac{\bar{k}}{2L}x_1(k-\tau) \text{ is about } 0$$

$$\text{Then } \boldsymbol{x}(k+1) = (A_1 + \Delta A_1)\boldsymbol{x}(k) + A_{11}\boldsymbol{x}(k-\tau) + (B_1 + \Delta B_1)\boldsymbol{u}(k)$$

$$R^2 \text{ If } \boldsymbol{z}(k) = x_2(k) + \nu\frac{\bar{k}}{2L}x_1(k) + \nu\frac{\bar{k}}{2L}x_1(k-\tau) \text{ is about } \pi \text{ or } -\pi$$

$$\text{Then } \boldsymbol{x}(k+1) = (A_2 + \Delta A_2)\boldsymbol{x}(k) + A_{12}\boldsymbol{x}(k-\tau) + (B_2 + \Delta B_2)\boldsymbol{u}(k)$$

where

$$A_1 = \begin{bmatrix} 1-\frac{v\bar{k}}{L} & 0 & 0 \\ \frac{v\bar{k}}{L} & 1 & 0 \\ \frac{v^2\bar{k}^2}{2L} & v\bar{k} & 1 \end{bmatrix},\ B_1 = \begin{bmatrix} \frac{v\bar{k}}{l} \\ 0 \\ 0 \end{bmatrix},\ \Delta A_1 = \begin{bmatrix} 0.2\sin(k) & 0 & 0 \\ 0 & 0.2\sin(k) & 0 \\ 0 & 0 & 0.2\sin(k) \end{bmatrix}$$

$$A_{11} = \begin{bmatrix} 1-v\frac{\bar{k}}{L} & 0 & 0 \\ v\frac{\bar{k}}{L} & 0 & 0 \\ \frac{v^2\bar{k}^2}{2L} & 0 & 0 \end{bmatrix},\ A_2 = \begin{bmatrix} 1-\frac{v\bar{k}}{L} & 0 & 0 \\ \frac{v\bar{k}}{L} & 1 & 0 \\ \frac{dv^2\bar{k}^2}{2L} & dv\bar{k} & 1 \end{bmatrix},\ B_2 = \begin{bmatrix} \frac{v\bar{k}}{l} \\ 0 \\ 0 \end{bmatrix}$$

$$\Delta A_2 = \begin{bmatrix} 0.2\sin(k) & 0 & 0 \\ 0 & 0.2\sin(k) & 0 \\ 0 & 0 & 0.2\sin(k) \end{bmatrix},\ A_{12} = \begin{bmatrix} 1-v\frac{\bar{k}}{L} & 0 & 0 \\ v\frac{\bar{k}}{L} & 0 & 0 \\ \frac{dv^2\bar{k}^2}{2L} & 0 & 0 \end{bmatrix}$$

and $l = 2.8$, $L = 5.5$, $v = -1.0$, $\bar{k} = 2.0$, $a = 0.7$, $\tau = 0.5$, $d = 0.01/\pi$, $\Delta B_1 = 0$, $\Delta B_2 = 0$.

From the parallel distributed compensation principle, the fuzzy control law can be constructed as

$$R^1 \text{ If } \boldsymbol{z}(k) = x_2(k) + v\frac{\bar{k}}{2L}x_1(k) + v\frac{\bar{k}}{2L}x_1(k-\tau) \text{ is about } 0$$

$$\text{Then } u(k) = K_1\boldsymbol{x}(k)$$

$$R^2 \text{ If } \boldsymbol{z}(k) = x_2(k) + \nu\frac{\bar{k}}{2L}x_1(k) + \nu\frac{\bar{k}}{2L}x_1(k-\tau) \text{ is about } \pi \text{ or } -\pi$$

$$\text{Then } u(k) = K_2\boldsymbol{x}(k)$$

The membership function is taken as follows.

$$h_1(z(k)) = (1 - \frac{1}{1+\exp\{-3[z(k)-\pi/2]\}}) \times \frac{1}{1+\exp\{-3[z(k)-\pi/2]\}}$$

$$h_2(z(k)) = 1 - h_1(z(k))$$

$$z(k) = x_2(k) + v\frac{\bar{k}}{2L}x_1(k) + v\frac{\bar{k}}{2L}x_1(k-\tau)$$

where ΔA_1 and ΔA_2 are parameter uncertainties.

From (6) and (7), the state feedback matrices K_i and $X = P^{-1}$ can be solved by the LMI software of Matlab as follows.

$$X = \begin{bmatrix} 0.0104 & 0.0026 & 0.0000 \\ 0.0026 & 0.0013 & 0.0007 \\ 0.0000 & 0.0007 & 0.0029 \end{bmatrix},\ K_1 = [3.3673 \quad -4.5910 \quad 1.0729],\ K_2 = [3.3673 \quad -4.5910 \quad 1.0729]$$

From the solutions of the LMI, the controller of the system can be determined.

The simulation figures are as follows.

From the above figures, it is seen that Fig. 1 is about the closed-loop system without time-delay, the trajectories of states x_1, x_2, x_3 are asymptotical tend to zero by using the fuzzy controller designed in this paper. At the same time, the state trajectories of closed-loop system with time-delay can also be stabilized by using the designed controller here in Fig. 2. Based on these results, it is obvious that the controller proposed in this paper can stabilize the closed-loop system, so the methods presented here are simple and feasible.

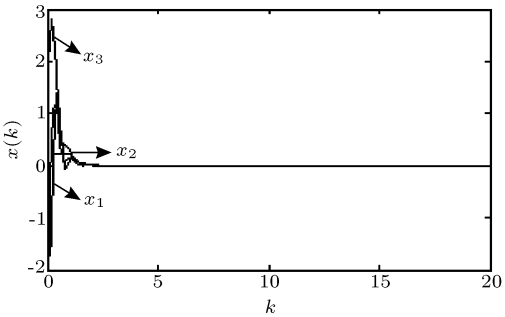

Fig. 1 The trajectories of the state variables x_1, x_2, and x_3

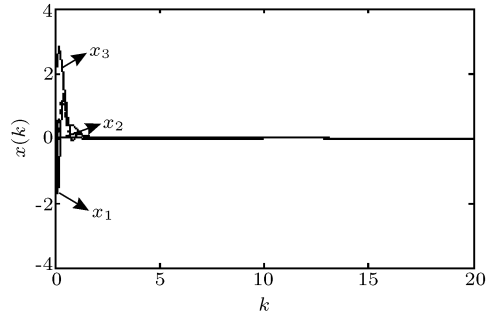

Fig. 2 The trajectories with time delay of the state variables x_1, x_2, and x_3

5 Conclusions

In this paper, the robust stabilization for certain of nonlinear time delay discrete systems is considered. A stability condition is deduced based on Lyapunov theorem and this condition can be transferred into the solubility of the LMI problem. In the last, it is shown that the method proposed in this paper is feasible by the simulation of truck-trailer[12] based on the LMI toolbox of Matlab.

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