Observer-based H_∞ Filtering of 2-D Singular System Described by Roesser Models[1)]

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Abstract This paper discusses the problem of the H_∞ filtering for discrete time 2-D singular Roesser models (2-D SRM). The purpose is to design an observer-based 2-D singular filter such that the error system is acceptable, jump modes free and stable, and satisfies a pre-specified H_∞ performance level. By general Riccati inequality and bilinear matrix inequalities (BMI), a sufficient condition for the solvability of the observer-based H_∞ filtering problem for 2-D SRM is given. A numerical example is provided to demonstrate the applicability of the proposed approach.

Key words 2-D singular systems, jump modes, general Riccati inequality, bilinear matrix inequalities

1 Introduction

In the past decades, 2-D singular systems have received much interest due to their extensive applications in many practical areas[1~4]. An asymptotic stability theory based on the concept of jump modes was proposed in [1]. In [3] a singular observer design approach was developed while the problem of robust H_∞ control for uncertain 2-D singular Roesser models (2-D SRM) was considered in [2]. However, there is no remarkable progress to be reported on the problem of H_∞ filtering for 2-D singular systems. This motivates the present study.

In this paper, we consider the problem of observer-based H_∞ filtering for 2-D singular Roesser models (2-D SRM). Attention is focused on the design of 2-D singular filters such that the resulting closed-loop system is acceptable, jump modes free and stable, and satisfies a prespecified H_∞ performance level. General Riccati inequality approach and bilinear matrix inequalities(BMI) approach are presented for the design of observer-based H_∞ filters.

2 Problem formulation and preliminaries for 2-D singular systems

Consider the following 2-D SRM (Σ):

$$E\begin{bmatrix}\boldsymbol{x}^h(i+1,j)\\ \boldsymbol{x}^v(i,j+1)\end{bmatrix} = A\begin{bmatrix}\boldsymbol{x}^h(i,j)\\ \boldsymbol{x}^v(i,j)\end{bmatrix} + L\boldsymbol{d}(i,j) \tag{1}$$

$$\boldsymbol{y}(i,j) = C\begin{bmatrix}\boldsymbol{x}^h(i,j)\\ \boldsymbol{x}^v(i,j)\end{bmatrix} + D\boldsymbol{d}(i,j) \tag{2}$$

$$\boldsymbol{z}(i,j) = H\begin{bmatrix}\boldsymbol{x}^h(i,j)\\ \boldsymbol{x}^v(i,j)\end{bmatrix} \tag{3}$$

with the zero boundary conditions:

$$\boldsymbol{x}^h(0,j) = 0, \quad \boldsymbol{x}^v(i,0) = 0 \tag{4}$$

where $\boldsymbol{x}^h(i,j) \in R^{n_1}$, $\boldsymbol{x}^v(i,j) \in R^{n_2}$ are the horizontal and vertical states, respectively, $\boldsymbol{y}(i,j) \in R^l$ is the measured output, $\boldsymbol{z}(i,j) \in R^p$ is the signal to be estimated, $\boldsymbol{d}(i,j) \in R^q$ is the noise signal which

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belongs to $l_2\{[0,\infty),[0,\infty)\}$. A, L, C, D, and H are real matrices of appropriate dimensions. $E \in R^{n\times n}$ is possibly singular, satisfying the 2-D regular pencil condition, *i.e.*, for some finite pair (z,w):

$$\det[EI(z,w)-A]=\sum_{k=0}^{\bar{n}_1}\sum_{l=0}^{\bar{n}_2} a_{kl}z^k w^l \neq 0$$

where $I(z,w)=\mathrm{diag}\{zI_{n_1}, wI_{n_2}\}$. When $a_{\bar{n}_1,\bar{n}_2}\neq 0$, system (Σ) is called acceptable.

In terms of the singular observer theory[4], the following observer-based 2-D singular filter is adopted for studying the H_∞ filtering problem of 2-D SRM $(\hat{\Sigma})$:

$$E\begin{bmatrix}\hat{\boldsymbol{x}}^h(i+1,j)\\ \hat{\boldsymbol{x}}^v(i,j+1)\end{bmatrix}=A\begin{bmatrix}\hat{\boldsymbol{x}}^h(i,j)\\ \hat{\boldsymbol{x}}^v(i,j)\end{bmatrix}+K\left\{\boldsymbol{y}(i,j)-C\begin{bmatrix}\hat{\boldsymbol{x}}^h(i,j)\\ \hat{\boldsymbol{x}}^v(i,j)\end{bmatrix}\right\} \tag{5}$$

$$\hat{\boldsymbol{z}}(i,j)=H\begin{bmatrix}\hat{\boldsymbol{x}}^h(i,j)\\ \hat{\boldsymbol{x}}^v(i,j)\end{bmatrix} \tag{6}$$

$$\hat{\boldsymbol{x}}^h(0,j)=0,\quad \hat{\boldsymbol{x}}^v(i,0)=0 \tag{7}$$

Assumption 1. Systems (Σ) and $(\hat{\Sigma})$ are acceptable.

Remark 1. [3] presented that the assumption of acceptability is needed for the 2-D singular systems, because unacceptable systems are ill-posed in some ways.

Denote the state estimation error as

$$\boldsymbol{e}(i,j)=\begin{bmatrix}\boldsymbol{e}^h(i,j)\\ \boldsymbol{e}^v(i,j)\end{bmatrix}=\begin{bmatrix}\boldsymbol{x}^h(i,j)\\ \boldsymbol{x}^v(i,j)\end{bmatrix}-\begin{bmatrix}\hat{\boldsymbol{x}}^h(i,j)\\ \hat{\boldsymbol{x}}^v(i,j)\end{bmatrix}$$

and the estimation error for the signal z as

$$\boldsymbol{z}_e(i,j)=\boldsymbol{z}(i,j)-\hat{\boldsymbol{z}}(i,j)$$

Then, from (Σ) and $(\hat{\Sigma})$, we have the following dynamic error systems (Σ_e)

$$E\begin{bmatrix}\boldsymbol{e}^h(i,j)\\ \boldsymbol{e}^v(i,j)\end{bmatrix}=(A-KC)\begin{bmatrix}\boldsymbol{e}^h(i,j)\\ \boldsymbol{e}^v(i,j)\end{bmatrix}+(L-KD)\ \boldsymbol{d}(i,j) \tag{8}$$

$$\boldsymbol{z}_e(i,j)=H\begin{bmatrix}\boldsymbol{e}^h(i,j)\\ \boldsymbol{e}^v(i,j)\end{bmatrix} \tag{9}$$

and $G(z,w)=H[EI(zw)-A_k]^{-1}L_k$ is the transfer function matrix from the disturbances $\boldsymbol{d}(i,j)$ to the controlled output $z_e(i,j)$, where $A_k=A-KC$, $L_k=L-KD$, and $\|G(z,w)\|_\infty=\sup\limits_{\omega_1,\omega_2\in[0,2\pi)}\bar{\sigma}[G(e^{j\omega_1},e^{j\omega_2})]$, in which $\bar{\sigma}(\cdot)$ represents the maximum singular value of matrix $(\cdot)$.

The observer-based H_∞ filtering problem to be addressed in this paper can be formulated as follows: given a scalar $\gamma>0$ and the 2-D SRM in (Σ), find a 2-D singular filter of the form $(\hat{\Sigma})$, such that dynamic error systems (Σ_e) is acceptable, internally stable and jump modes free, and $\|G(z,w)\|_\infty<\gamma$.

Consider the following system of 2-D SRM (Σ_0)

$$E\begin{bmatrix}\boldsymbol{x}^h(i+1,j)\\ \boldsymbol{x}^v(i,j+1)\end{bmatrix}=A\begin{bmatrix}\boldsymbol{x}^h(i,j)\\ \boldsymbol{x}^v(i,j)\end{bmatrix}+L\boldsymbol{d}(i,j) \tag{10}$$

$$\boldsymbol{z}(i,j)=H\begin{bmatrix}\boldsymbol{x}^h(i,j)\\ \boldsymbol{x}^v(i,j)\end{bmatrix} \tag{11}$$

with zero boundary condition . We then have the following lemmas.

Lemma 1[1]. 2-D SRM (Σ_0) is acceptable and internally stable if and only if

$$p(z,w)\neq 0,\quad 0<|z|\leqslant 1,\quad 0<|w|\leqslant 1 \tag{12}$$

where $p(z,w)=\det[E-AI(z,w)]$.

3 Observer-based 2-D singular H_∞ filter design

In this section, we shall present two approaches for the observer-based 2-D H_∞ filtering. They are the general Riccati inequality approach and bilinear matrix inequalities (BMI) approach.

Lemma 2. Given a positive scalar $\gamma > 0$, the 2-D SRM (Σ_0) with zero boundary condition is acceptable, internally stable and jump modes free, and satisfies $\|G(z,w)\|_\infty < \gamma$ if there exists a symmetric block-diagonal matrix $P = \mathrm{diag}\{P_h, P_v\} \in R^{n\times n}$ such that the following LMIs hold

$$EPE^{\mathrm{T}} \geqslant 0 \tag{13}$$

$$\begin{bmatrix} APA^{\mathrm{T}} - EPE^{\mathrm{T}} + LL^{\mathrm{T}} & APH^{\mathrm{T}} \\ HPA^{\mathrm{T}} & HPH^{\mathrm{T}} - \gamma^2 I \end{bmatrix} < 0 \tag{14}$$

$$\gamma^2 I - HPH^{\mathrm{T}} > 0 \tag{15}$$

where $P_h \in R^{n_1\times n_1}$ and $P_v \in R^{n_2\times n_2}$.

Proof. From (14), it is easy to see that

$$APA^{\mathrm{T}} - EPE^{\mathrm{T}} < 0 \tag{16}$$

By this and (13), we assert that system (Σ_0) is acceptable, internally stable, and jump modes free. To show this, we suppose that there exist complex numbers z_1 and w_1 with $0 < |z_1| \leqslant 1, 0 < |w_1| \leqslant 1$ such that

$$p(z_1, w_1) = \det[E - AI(z_1, w_1)] = 0 \tag{17}$$

that is,

$$\det[E^{\mathrm{T}} - I^H(z_1, w_1)A^{\mathrm{T}}] = 0 \tag{18}$$

This implies that there exist some complex numbers z_0 and w_0 with $1 \leqslant |z_0| < \infty, 1 \leqslant |w_0| < \infty$ and a vector $x_0 \neq 0$ such that

$$(I(z_0, w_0)E^{\mathrm{T}} - A^{\mathrm{T}})x_0 = 0 \tag{19}$$

By equation (19) one gets

$$x_0^H(APA^{\mathrm{T}} - EPE^{\mathrm{T}})x_0 = (E^{\mathrm{T}}x_0)^H \mathrm{diag}\{(|z_0| - 1)P_h, (|w_0| - 1)P_v\}(E^{\mathrm{T}}x_0) \geqslant 0 \tag{20}$$

This contradicts with (16). Hence, we have that 2-D SRM (Σ_0) is acceptable, stable internally, and jump modes free. Noting this and following a similar line as in the proof of Theorem 1 in [2], the desired result follows immediately. □

3.1 General Riccati inequality approach

Assumption 2. $DD^{\mathrm{T}} > 0$.

Note that Assumption 2 is standard in the Kalman and H_∞ filtering for 1-D systems. It implies that all the measurements are corrupted by noise.

Theorem 1. Consider the system (Σ) satisfying Assumption 2 and zero boundary condition. Given a prescribed level of H_∞ noise attenuation $\gamma > 0$, the H_∞ filtering problem is solvable if there exists a symmetric block-diagonal matrix $Q = \mathrm{diag}\{Q_h, Q_v\} \in R^{n\times n}$, $(Q_h \in R^{n_1\times n_1}, Q_v \in R^{n_2\times n_2})$ such that

$$EQE^{\mathrm{T}} \geqslant 0 \tag{21}$$

$$AQA^{\mathrm{T}} - EQE^{\mathrm{T}} - (AQ\bar{C}^{\mathrm{T}} + L\bar{D}^{\mathrm{T}})(\bar{C}Q\bar{C}^{\mathrm{T}} + \bar{R})^{-1}(\bar{C}QA^{\mathrm{T}} + \bar{D}L^{\mathrm{T}}) + LL^{\mathrm{T}} < 0 \tag{22}$$

where $\bar{C} = \begin{bmatrix} C \\ H \end{bmatrix}$, $\bar{D} = \begin{bmatrix} D \\ 0 \end{bmatrix}$, $\bar{R} = \begin{bmatrix} DD^{\mathrm{T}} & 0 \\ 0 & -\gamma^2 I \end{bmatrix}$.

In this situation, a suitable filter gain of (5) is given by

$$K = (AVC^{\mathrm{T}} + LD^{\mathrm{T}})(CVC^{\mathrm{T}} + DD^{\mathrm{T}}) \tag{23}$$

where $V = Q + QH^{\mathrm{T}}\Theta^{-1}HQ$, $\Theta = \gamma^2 I - HQH^{\mathrm{T}} > 0$.

Proof. From Lemma 2, the error system (Σ_e) is acceptable, internally stable and jump modes free, and satisfies $\|G(z,w)\|_\infty < \gamma$ if there exists a symmetric block-diagonal matrix $Q = \mathrm{diag}\{Q_h, Q_v\} \in R^{n\times n}$, $(Q_h \in R^{n_1\times n_1}, Q_v \in R^{n_2\times n_2})$ such that

$$EQE^{\mathrm{T}} \geqslant 0 \tag{24}$$

$$(A-KC)Q(A-KC)^{\mathrm{T}} - EQE^{\mathrm{T}} + (L-KD)(L-KD)^{\mathrm{T}} + \\ (A-KC)QH^{\mathrm{T}}(\gamma^2 I - HQH^{\mathrm{T}})^{-1}HQ(A-KC)^{\mathrm{T}} < 0 \tag{25}$$

$$\Theta = \gamma^2 I - HQH^{\mathrm{T}} > 0 \tag{26}$$

let $\Gamma_A = HQA^{\mathrm{T}}$, $\Gamma_L = CQA^{\mathrm{T}} + DL^{\mathrm{T}}$, $\Gamma_C = CQA^{\mathrm{T}}$

$$X_1 = DD^{\mathrm{T}} + CQC^{\mathrm{T}} + \Gamma_C^{\mathrm{T}}\Theta^{-1}\Gamma_C, \quad X_2 = \Gamma_L + \Gamma_C^{\mathrm{T}}\Theta^{-1}\Gamma_A \tag{27}$$

Then (25) can be rewritten as

$$AQA^{\mathrm{T}} - EQE^{\mathrm{T}} + LL^{\mathrm{T}} + KX_1K^{\mathrm{T}} - KX_2 - X_2^{\mathrm{T}}K^{\mathrm{T}} + \Gamma_A^{\mathrm{T}}\Theta^{-1}\Gamma_A < 0$$

In view of Assumption 2, $X_1 > 0$. Thus, it is easy to obtain

$$AQA^{\mathrm{T}} - EQE^{\mathrm{T}} + LL^{\mathrm{T}} - X_2^{\mathrm{T}}X_1^{-1}X_2 + \Gamma_A^{\mathrm{T}}\Theta^{-1}\Gamma_A \\ (-K^{\mathrm{T}} + X_1^{-1}X_2)^{\mathrm{T}}X_1(-K^{\mathrm{T}} + X_1^{-1}X_2) < 0 \tag{28}$$

From (22) and (27), it can be shown that $K = X_2^{\mathrm{T}}X_1^{-1}$ and (22) is equivalent to

$$AQA^{\mathrm{T}} - EQE^{\mathrm{T}} + LL^{\mathrm{T}} - X_2^{\mathrm{T}}X_1^{-1}X_2 + \Gamma_A^{\mathrm{T}}\Theta^{-1}\Gamma_A < 0 \tag{29}$$

Hence, Q and K in (22) and (23) satisfy (29) or equivalently (29), *i.e.*, the H_∞ filtering problem is solvable. This completes the proof. □

Remark 3. Solving a general Riccati inequality to obtain a block-diagonal solution may not be easy. In the following subsection, we shall propose a BMI approach to computing the filter gain.

3.2 BMI approach

Theorem 2. Consider the system (Σ) with zero boundary condition. Given a prescribed level of H_∞ noise attenuation $\gamma > 0$, the H_∞ filtering problem is solvable if there exist matrices $S \in R^{n\times n}$, $R_1 \in R^{n\times n}$, $R_2 \in R^{l\times n}$ and a symmetric block-diagonal matrix $P = \mathrm{diag}\{P_h, P_v\} \in R^{n\times n}$, $(P_h \in R^{n_1\times n_1}, P_v \in R^{n_2\times n_2})$ such that

$$EPE^{\mathrm{T}} \geqslant 0 \tag{30}$$

$$\gamma^2 I - HPH^{\mathrm{T}} > 0 \tag{31}$$

$$\begin{bmatrix} -EPE^{\mathrm{T}} + A_kR_1^{\mathrm{T}} + R_1A_k^{\mathrm{T}} & A_kR_2^{\mathrm{T}} + R_1H^{\mathrm{T}} & -R_1 + A_kS & L_k \\ R_2A_k^{\mathrm{T}} + HR_1^{\mathrm{T}} & -\gamma^2 I + HR_2^{\mathrm{T}} + R_2H^{\mathrm{T}} & -R_2 + HS & 0 \\ -R_1^{\mathrm{T}} + S^{\mathrm{T}}A_k^{\mathrm{T}} & -R_2^{\mathrm{T}} + S^{\mathrm{T}}H^{\mathrm{T}} & P - S - S^{\mathrm{T}} & 0 \\ L_k^{\mathrm{T}} & 0 & 0 & -I \end{bmatrix} < 0 \tag{32}$$

where $A_k = A - KC$, $L_k = L - KD$.

Proof. By the Schur complement formula, (32) equivalent to

$$\begin{bmatrix} -EPE^{\mathrm{T}} + LL^{\mathrm{T}} + A_kR_1^{\mathrm{T}} + R_1A_k^{\mathrm{T}} & A_kR_2^{\mathrm{T}} + R_1H^{\mathrm{T}} & -R_1 + A_kS \\ R_2A_k^{\mathrm{T}} + HR_1^{\mathrm{T}} & -\gamma^2 I + HR_2^{\mathrm{T}} + R_2H^{\mathrm{T}} & -R_2 + HS \\ -R_1^{\mathrm{T}} + S^{\mathrm{T}}A_k^{\mathrm{T}} & -R_2^{\mathrm{T}} + S^{\mathrm{T}}H^{\mathrm{T}} & P - S - S^{\mathrm{T}} \end{bmatrix} < 0 \tag{33}$$

Let $W = \begin{bmatrix} -EPE^{\mathrm{T}} + LL^{\mathrm{T}} & 0 \\ 0 & -\gamma^2 I \end{bmatrix}$ and $\Psi = \lfloor A_k^{\mathrm{T}} \quad H^{\mathrm{T}} \rfloor$, $R = \begin{bmatrix} R_1 \\ R_2 \end{bmatrix}$

Then (32) can be written as

$$\begin{bmatrix} W + \Psi^{\mathrm{T}}R^{\mathrm{T}} + R\Psi & -R + \Psi^{\mathrm{T}}S \\ -R^{\mathrm{T}} + S^{\mathrm{T}}\Psi & P - S - S^{\mathrm{T}} \end{bmatrix} < 0 \tag{34}$$

Note that this is in turn equivalent to[4] $W+\Psi^{\mathrm{T}}P\Psi<0$

Then

$$W+\Psi^{\mathrm{T}}P\Psi=\begin{bmatrix}-EPE^{\mathrm{T}}+LL^{\mathrm{T}}+A_kPA_k^{\mathrm{T}} & A_kPH^{\mathrm{T}}\\ HPA_k^{\mathrm{T}} & -\gamma^2I+HPH^{\mathrm{T}}\end{bmatrix}<0 \tag{35}$$

Therefore, the desired result follows immediately from (30), (31) and Lemma 1. This completes the proof. □

From Theorem 2, the following iterative algorithm can be used to solve the uncertain 2-D SRM H_∞ filtering problem.

4 Algorithm

Step 1. Choose an initial K, and solve the following convex optimization problem:

$$\min_{(P,S,R)}\ \{\mu\}$$

$$\text{Such that}\ \begin{bmatrix}W+\Psi^{\mathrm{T}}R^{\mathrm{T}}+R\Psi & -R+\Psi^{\mathrm{T}}S\\ -R^{\mathrm{T}}+S^{\mathrm{T}}\Psi & P-S-S^{\mathrm{T}}\end{bmatrix}<\mu I$$

and (29)∼(30) are hold. If $\mu\leqslant 0$, then the problem is solved; otherwise, go to Step 2.

Step 2. With the obtained matrices P, S, and R, solve the above optimization with respect to K. Again, if $\mu\leqslant 0$, the problem is solved; otherwise, go to Step 1.

5 Numerical example

Consider a 2-D SRM (Σ) with parameters: $(n_1=1, n_2=2)$

$$E=\begin{bmatrix}1&0&1\\0&1&0\\0&0&0\end{bmatrix},\ A=\begin{bmatrix}2&1&0\\0&0&1\\0&0&1\end{bmatrix},\ C=[1\quad 0.2\quad 1],\ L=\begin{bmatrix}0.1\\0.2\\0.3\end{bmatrix},\ H=[0.1\quad 0.1\quad 0.2],\ D=0.2$$

It is easy to see that this system is an acceptable, unstable system with jump-mode.

Let $\gamma=0.5$. By the above algorithm, the solution to BMIs (29)∼(31) is as follows.

$$P=\begin{bmatrix}6.1293&0&0\\0&2.6159&-2.2767\\0&-2.2767&-2.1312\end{bmatrix},\ R_1=\begin{bmatrix}-1050.7&878.3&749.0\\878.3&734.1&-625.5\\749.0&-625.5&-533.9\end{bmatrix}$$

$$R_2=[159.8490\quad -133.4886\quad -113.5674]$$

The corresponding filter gain can be obtained as: $K=[2.7718\quad -0.1603\quad -1.0000]^{\mathrm{T}}$

Fig. 1 shows the frequency response of the error system (Σ_e) over all frequencies. It can be observed that the amplitude response of the filtering error transfer function is below the prescribed H_∞ noise attenuation level $\gamma=0.5$.

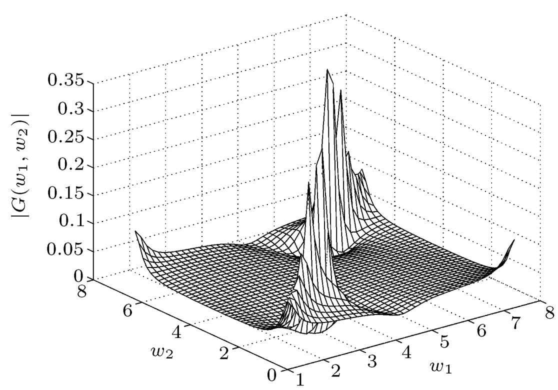

Fig. 1 Frequency response of the filtering error system

6 Conclusions

This paper has solved the H_∞ filtering problem for 2-D singular Roesser models. Both the general Riccati inequality and bilinear matrix inequalities (BMI) have been developed for the design of an observer-based 2-D singular H_∞ filter. Numerical example is provided to demonstrate the applicability of the proposed approach.

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