H_∞ Control of Bio-dissimilation Process of Glycerol to 1,3-Propanediol[1)]

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Abstract A robust controller is designed by using the bilinear transformation and H_∞ mixed sensitivity method for bio-dissimilation process of glycerol to 1,3-propanediol. Under the controller the system works near an optimal steady-state for the volumetric productivity of 1,3-propanediol attaining its maximization. The design procedure is carried out by tuning the transformation parameter and DC gain of the performance weighted function, which is an iterative and optimal search process. Simulation results are presented which show that the designed robust controller not only ensures the robust stability of the system in face of the parametric variations in the model, but also makes the system have a favourable robust tracking performance. The validity of the proposed H_∞ controller has been tested.

Key words H_∞ control, mixed sensitivity problem, bilinear transformation, bioprocess control

1 Introduction

Bio-dissimilation process of glycerol to 1,3-propanediol is a bioprocess with strong nonlinearity. The experimental investigations showed that as the substrate for cell growth, glycerol is an activator at its low concentration whereas it is a potential inhibitor at its high concentration. High glycerol concentration means that multiple inhibitions of substrate and products occur in the system[1]. Under given operational conditions of a relatively high dilution rate and initial glycerol concentration in feed, decrease from high values to low values in feed substrate concentration will cause multiplicity[2]. This is also the case for the converse. When an abrupt change in external circumstances such as initial glycerol concentration in feed, dilution rate or pH value, oscillatory phenomena of biomass, substrate and products can be observed[3]. Using the excess kinetic models, [4,5] described the substrate, energy consumption and partial products formation in the bioconversion of glycerol. Considering there is a transport process in cells ingesting substrate and secreting products through cell wall or membrane, [6,7] introduced a continuous time delay into the model in [2] and qualitatively characterized the oscillatory and transient phenomena occurring in experiment. [8] investigated the model-guided optimization of glycerol to 1,3-propanediol. Significant efforts from biochemical mechanism analysis and mathematical modeling have been made to improve the dissimilation process of glycerol, but the control of this process was not reported.

Due to the intrinsic complexity of biological system, it is difficult to determine its exact process model. Even if the mathematical model is built up, model parameters will vary with the working conditions. In addition, external disturbance signals also have an important effect on the system. These uncertain factors can deteriorate the performance of a system and lead to the instability of the process. One efficient approach to solving such problems is to design a robust controller via the H_∞ control theory[9~17]. The H_∞ control approach integrates the uncertainty involved in model parameters and external disturbance with the half-baked information of uncertainty to synthesize a control law which maintains real plants to work within desired performance specifications despite the effects of uncertainty on the system.

In this paper we propose a robust control strategy for bio-dissimilation process of glycerol to 1,3-propanediol. Utilizing the bilinear transformation and H_∞ mixed sensitivity method, a robust controller is designed to keep the system work at an optimal steady-state for the volumetric productivity of 1,3-propanediol attaining its maximization with a favourable tracking performance and ensure the robust stability of the process in face of the variations in model parameters. Firstly, we shift the imaginary

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axis poles of nominal model to the right-half plane via a bilinear transformation. Then we compute the H_∞ mixed sensitivity problem for the transformed plant to find a feedback controller. Finally, we map the above obtained controller back to the original plane via the inverse bilinear transformation and attain the final H_∞ controller. The whole design procedure is implemented by regulating the transformation parameter and DC gain of the performance weighted function. The former is the key parameter in placing the dominant closed-loop poles at the desired locations. And the latter determines the steady-state tracking error of the system. Simulation results indicate that the presented controller is valid.

2 Mathematical model

2.1 Nonlinear model

The material balance equations of continuous microbial cultures are written as follows[8]:

$$\frac{\mathrm{d}X}{\mathrm{d}t} = (\mu - D)X \tag{1}$$

$$\frac{\mathrm{d}C_S}{\mathrm{d}t} = D(C_{SF} - C_S) - q_S X \tag{2}$$

$$\frac{\mathrm{d}C_{PD}}{\mathrm{d}t} = q_{PD}X - DC_{PD} \tag{3}$$

$$\frac{\mathrm{d}C_{HAc}}{\mathrm{d}t} = q_{HAc}X - DC_{HAc} \tag{4}$$

$$\frac{\mathrm{d}C_{EtOH}}{\mathrm{d}t} = q_{EtOH}X - DC_{EtOH} \tag{5}$$

where X is the biomass, gl^{-1}; D is the dilution rate, h^{-1}; C_{SF} and C_S are the substrate concentrations (glycerol) in feed and reactor, respectively, mmoll^{-1}; C_{PD}, C_{HAc} and C_{EtOH} are the concentrations of products 1,3-propanediol, acetic acid and ethanol, respectively, mmoll^{-1}; t is the fermentation time, h; $\mu, q_S, q_{PD}, q_{HAc}$ and q_{EtOH} are the specific growth rate of cells, specific consumption rate of substrate, specific formation rate of products 1,3-propanediol, acetic acid and ethanol, respectively, $\mathrm{mmolg}^{-1}\mathrm{h}^{-1}$, which can be expressed as

$$\mu = \mu_m \frac{C_S}{K_S + C_S}\left(1 - \frac{C_S}{C_S^*}\right)\left(1 - \frac{C_{PD}}{C_{PD}^*}\right)\left(1 - \frac{C_{HAc}}{C_{HAc}^*}\right)\left(1 - \frac{C_{EtOH}}{C_{EtOH}^*}\right) \tag{6}$$

$$q_S = m_S + \frac{\mu}{Y_S^m} + \Delta q_S^m \frac{C_S}{C_S + K_S^*} \tag{7}$$

$$q_{PD} = m_{PD} + \mu Y_{PD}^m + \Delta q_{PD}^m \frac{C_S}{C_S + K_{PD}^*} \tag{8}$$

$$q_{HAc} = m_{HAc} + \mu Y_{HAc}^m + \Delta q_{HAc}^m \frac{C_S}{C_S + K_{HAc}^*} \tag{9}$$

$$q_{EtOH} = q_S\left(\frac{b_1}{c_1 + DC_S} + \frac{b_2}{c_2 + DC_S}\right) \tag{10}$$

For Klebsiella pneunoniae cultivated under anaerobic conditions at 37°C and pH 7.0, the maximum specific growth rate μ_m and the saturation constant for glycerol are $0.67\mathrm{h}^{-1}$ and $0.28\mathrm{mmoll}^{-1}$, respectively. The critical concentrations denoted as C^* in glycerol, 1,3-propanediol, acetic acid and ethanol are 2039, 939.5, 1026 and $360.9\mathrm{mmoll}^{-1}$, respectively. In addition, the parameters b_1, b_2, c_1 and c_2 in (10) are 0.025, 5.18, 0.06 and 50.45 $\mathrm{mmoll}^{-1}\mathrm{h}^{-1}$, respectively, while the ones for (7), (8) and (9) are listed in Table 1.

Table 1 Parameters in models (7)~(9)

Substrate/products	m	Y^m	Δq^m	K^*
Glycerol	2.20	0.0082	28.58	11.43
1,3-propanediol	−2.69	67.69	26.59	15.50
Acetic acid	−0.97	33.07	5.74	85.71

2.2 Calculation of the optimal operating point

To impose the fermentation process to run at some steady-state and maximize the volumetric productivity of 1,3-propanediol, we present the following steady-state optimization problem

$$\begin{aligned} \max\ & DC_{PD} \\ \text{s.t.}\ & (\mu - D)X = 0 \\ & D(C_{SF} - C_S) - q_S X = 0 \\ & q_{PD}X - DC_{PD} = 0 \\ & q_{HAc}X - DC_{HAc} = 0 \\ & q_{EtOH}X - DC_{EtOH} = 0 \\ & 0 < D \leqslant 0.5 \\ & 0 < C_{SF} \leqslant 2000 \end{aligned} \tag{11}$$

From [8] we know that wash-out is easy to occur when the dilution rate is larger than 0.5h^{-1}. So the constraint on D is given by (11). It can be attained that the maximum volumetric productivity of 1,3-propanediol DC_{PD} is 114.3 $\text{mmoll}^{-1}\text{h}^{-1}$ at a dilution rate of 0.29h^{-1} and an initial glycerol concentration of 730.8 mmoll^{-1}. Now the optimal steady-state operating point is $(X_0, C_{S0}, C_{PD0}, C_{HAc0}, C_{EtOH0}) = (2.89, 98.1, 400.1, 116.6, 42.33)$.

2.3 Obtaining a linear model

The process dynamics (1)~(5) is represented as a linear model with uncertain parameters

$$\dot{\boldsymbol{x}} = A\boldsymbol{x} + Bu \tag{12}$$

$$y = C\boldsymbol{x} \tag{13}$$

where $\boldsymbol{x} = (X, C_S, C_{PD}, C_{HAc}, C_{EtOH})^{\mathrm{T}}$ is used for the vector of states, $u = D$ is the control input, $y = C_S$ is the measured output, and

$$A = \begin{bmatrix} \mu & 0 & 0 & 0 & 0 \\ -q_S & 0 & 0 & 0 & 0 \\ q_{PD} & 0 & 0 & 0 & 0 \\ q_{HAc} & 0 & 0 & 0 & 0 \\ q_{EtOH} & 0 & 0 & 0 & 0 \end{bmatrix},\ \boldsymbol{B} = \begin{bmatrix} -X \\ C_{SF} - C_S \\ -C_{PD} \\ -C_{HAc} \\ -C_{EtOH} \end{bmatrix},\ \boldsymbol{C} = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}^{\mathrm{T}}$$

All parameters in A and $\boldsymbol{B}$ may vary within known bounds. Considering the uncertain parameters $D, X, C_S, C_{PD}, C_{HAc}$ and C_{EtOH}, we allow their changes of up to 20% around the nominal values. If the nominal values of uncertain parameters are selected as the ones at their optimum steady-state, then all parameters can be uniformly denoted as

$$p = p_0(1 + 0.2\Delta_p)$$

where $p = D, X, C_S, C_{PD}, C_{HAc}, C_{EtOH}, |\Delta_p| \leqslant 1$.

By (12) and (13), the process transfer function can be derived as

$$G_p(s) = \boldsymbol{C}(sI - A)^{-1}\boldsymbol{B} = \frac{(C_{SF} - C_S)s + q_S X - \mu(C_{SF} - C_S)}{s(s - \mu)}$$

Let $C_{SF} = C_{SF0}$, then the plant′s nominal model in a transfer function form is expressed as

$$G_0(s) = \frac{632.7s + 0.2713}{s(s - 0.2857)}$$

Hence, the multiplicative uncertainty can be written as

$$\Delta_m = \frac{G_p(s) - G_0(s)}{G_0(s)}$$

3 $\boldsymbol{H}_\infty$ mixed sensitivity problem

For a general SISO feedback control system, the sensitivity function must be small in order to reject the effect of disturbance on the output and to reduce the tracking error. The smaller the function values, the better the tracking, whereas from a view of guaranteeing the stability of controlled plant despite model uncertainty, a smaller complementary function implies a good robust stability. Unfortunately, it conflicts with the previous requirement of disturbance rejection because these two quantities must add to unity. For this reason, we need to have a tradeoff between them. Therefore, Kwakernaak proposed the so-called mixed sensitivity problem[18]. It is formulated as the problem of finding a feedback controller that stabilizes the closed-loop system shown in Fig. 1 and minimizes the H_∞-norm of closed-loop transfer function T_{zw} from the exogenous input $w(w=r)$ to the regulated outputs $z(z=[z_1, z_2]^{\mathrm{T}})$, namely

$$\gamma_{opt} = \min_{K} \|T_{zw}(s)\|_\infty \tag{14}$$

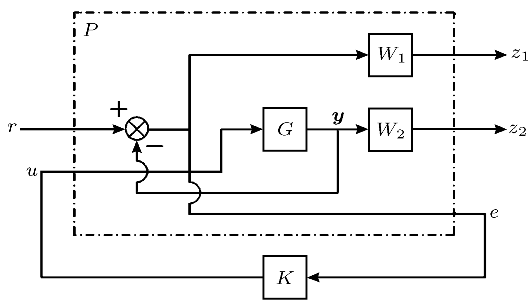


Fig. 1 Mixed sensitivity configuration

(14) is called the H_∞ optimal control problem, where

$$T_{zw}(s) = \begin{bmatrix} W_1(s)S(s) \\ W_2(s)T(s) \end{bmatrix} = P_{11} + P_{12}K(I - P_{22}K)^{-1}P_{21}$$

where $S(s) = (1 + G(s)K(s))^{-1}$ and $T(s) = G(s)K(s)S(s)$ are the sensitivity function and the complementary function, respectively; $G(s)$ is the nominal model that have no imaginary axis zeros and/or poles; $W_1(s)$, $W_2(s)$ and P are performance weighting function, robustness weighting function and augmented plant, respectively.

$$P = \begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix} = \begin{bmatrix} W_1 & -W_1G \\ 0 & W_2G \\ 1 & -G \end{bmatrix}$$

where

$$P_{11} = \begin{bmatrix} W_1 \\ 0 \end{bmatrix}, \; P_{12} = \begin{bmatrix} -W_1G \\ W_2G \end{bmatrix}, \; P_{21} = 1, \; P_{22} = -G$$

If G, W_1 and W_2G have the following state-space realizations:

$$G = \begin{bmatrix} A_g & B_g \\ C_g & D_g \end{bmatrix}, \; W_1 = \begin{bmatrix} A_{w_1} & B_{w_1} \\ C_{w_1} & D_{w_1} \end{bmatrix}, \; W_2G = \begin{bmatrix} A_g & B_g \\ C_{w_2} & D_{w_2} \end{bmatrix}$$

then the augmented plant has a state-space realization as follows:

$$P(s) = \begin{bmatrix} A_p & B_1 & B_2 \\ C_1 & D_{11} & D_{12} \\ C_2 & D_{21} & D_{22} \end{bmatrix}$$

where $A_p = \begin{bmatrix} A_g & 0 \\ B_{w_1}C_g & A_{w_1} \end{bmatrix}$, $B_1 = \begin{bmatrix} 0 \\ B_{w_1} \end{bmatrix}$, $B_2 = \begin{bmatrix} B_g \\ B_{w_1}D_g \end{bmatrix}$, $C_1 = \begin{bmatrix} D_{w_1}C_g & C_{w_1} \\ C_{w_2} & 0 \end{bmatrix}$, $D_{11} = \begin{bmatrix} D_{w_1} \\ 0 \end{bmatrix}$, $D_{12} = \begin{bmatrix} D_{w_1}D_g \\ D_{w_2} \end{bmatrix}$, $C_2 = \lfloor C_g \quad 0 \rfloor$, $D_{21} = 1$, $D_{22} = D_g$.

By [9,10], for the H_∞ optimal control problem (14) to have a solution $K(s)$ the following assumptions must be satisfied.

1) the pair (A_p, B_2) is stabilizable and (A_p, C_2) is detectable.

2) rank(D_{12})=rank(u)=1 and rank(D_{21})=rank$(\boldsymbol{y})$=1.

3) rank$\begin{bmatrix} A_p - j\omega I & B_2 \\ C_1 & D_{12} \end{bmatrix} = n + \text{dim}u$, $\forall \omega \in R$, rank$\begin{bmatrix} A_p - j\omega I & B_1 \\ C_2 & D_{21} \end{bmatrix} = n + \text{dim}y$, $\forall \omega \in R$, where n is the order of matrix A_p.

Assumption a ensures the stability of a synthesized H_∞ controller. The second assumption guarantees the designed H_∞ controller is a proper and real rational function. The final assumption is a technical condition in mathematics that enables both $P_{12}(s)$ and $P_{21}(s)$ have no invariant zeros on the imaginary axis[11].

4 Bilinear transformation

Before solving the H_∞ optimal control problem (14), we must deal with the nominal model G_0 technically to keep G_0 to have no zeros and poles because G_0 has a pole on the imaginary axis. An available approach was presented in [12] to work out this nicely. Firstly, using the bilinear transformation

$$s = \frac{\tilde{s} + r_1}{\tilde{s}/r_2 + 1} \tag{15}$$

the pole of G_0 on the imaginary axis can be mapped onto a circle in $\tilde{s}$-plane centered at

$$-(r_1 + r_2)/2$$

where $r_1, r_2 < 0$. Secondly, for the transformed plant $\tilde{T}_{zw}(\tilde{s})$, solve the H_∞ optimal control problem

$$\tilde{\gamma}_{opt} = \min_{\tilde{K}} \|\tilde{T}_{zw}(\tilde{s})\|_\infty \tag{16}$$

Thirdly, map $\tilde{K}(\tilde{s})$ back to s-plane from $\tilde{s}$-plane via the inverse bilinear transformation

$$\tilde{s} = \frac{-s - r_1}{s/r_2 - 1} \tag{17}$$

Note that r_1 and r_2 are chosen such that G_0 has no zeros and poles on the imaginary axis in $\tilde{s}$-plane after a bilinear transformation. The parameter r_1 is the determinant parameter for the quality of dynamic behavior in the closed-loop system. In addition, compared with problem (14), the H_∞ controller $K(s)$ is just a suboptimal solution.

5 Weighting function selection

The selection of the weighting functions $W_1(s)$ and $W_2(s)$ observes the following basic rules

1) Choose a low-order weighting function, otherwise a high-order H_∞ controller can be achieved.

2) As the perturbation bound of the uncertainty Δ_m, the choice of robustness weighting function $W_2(s)$ depends also on whether the nominal model is a strictly proper and real rational function. Usually, $W_2(s)$ is chosen to be an improper and real rational function because most systems in the world are strictly proper. Though $W_2(s)$ cannot be realized in state-space form, $W_2(s)G(s)$ has a state-space realization since it is a proper structure. This ensures D_{12} has a full rank.

3) The performance weighting function $W_1(s)$ is usually a stable, proper and real rational function.

4) The 0 dB crossover frequency for the Bode plot of $W_1(s)$ should be below the 0 dB crossover frequency for the Bode plot of $W_2(s)$. More precisely, for $\forall \omega \in R$, we require $\bar{\sigma}(W_1^{-1}(j\omega)) + \bar{\sigma}(W_2^{-1}(j\omega)) > 1$, where $\bar{\sigma}$ denotes the maximum singular values of a transfer function, otherwise the performance requirements will not be achievable. It is necessary to note that a strategy was proposed to determine $W_1(s)$ by tuning its crossover frequency in [14]. If the crossover frequency of the established $W_2(s)$ is ω_{c2}, the crossover frequency of $W_1(s)$ is given by $\omega_{c1} = 10^{k-1}\omega_{c2}$, where k should not be larger than 1. This fact was not shown clearly by [14].

Based on the above-mentioned rules concerning the choice of the weighting function, robustness weighting function $W_2(s)$ can be chosen as $W_2(s) = s$ whose crossover frequency is $\omega_{c2} = 1$rad/s; performance weighting function $W_1(s)$ is a second-order filter with

$$W_1(s) = \frac{\beta(\alpha s^2 + 2\zeta_1\omega_{c1}\sqrt{\alpha}s + \omega_{c1}^2)}{\beta s^2 + 2\zeta_2\omega_{c1}\sqrt{\beta}s + \omega_{c1}^2} \tag{18}$$

where β: DC gain of the filter (controls the disturbance rejection).

$\alpha = 0.5$: high frequency gain (controls the response peak overshoot).

$\omega_{c1} = 0.3\text{rad/s}$: filter crossover frequency.

$\zeta_1 = 0.6, \zeta_2 = 0.7$: damping ratios of the corner frequencies.

Obviously, $W_1^{-1}(0) = 1/\beta$ is the steady-state tracking error, and $\lim\limits_{s\to\infty} W_1^{-1}(s) = 1/\alpha = 2$ is the corresponding amplification factor of the high frequency disturbances.

6 $\boldsymbol{H_\infty}$ controller design

Summarizing the previous description, the design procedure of the H_∞ controller can be formulated as follows:

1) Transform the nominal model $G_0(s)$ *via* the bilinear transformation (15), where $r_1 = -2$, $r_2 = -\infty$.

2) Design performance weighting function $\tilde{W}_1(\tilde{s})$ by adjusting β, where $\beta \in [10, 400]$ and the other parameters are exposed in (18).

3) Build up the augmented plant $\tilde{P}(\tilde{s})$, and find a controller $\tilde{K}(\tilde{s})$ if the H_∞ optimal control problem (16) has a solution. Else go to Step 2).

4) Shift back the controller $\tilde{K}(\tilde{s})$ to $K(s)$ *via* the inverse bilinear transformation (17).

5) Return Step 1) and reset r_1 until obtaining the desired performance specifications, where $r_1 \in [-2, 0)$.

By using MATLAB the augmented plant $\tilde{P}(\tilde{s})$ has the following state-space realization under $r_1 = 0.0001$ and $\beta = 204.9$:

$$\tilde{A}_p = \begin{bmatrix} 0.28580 & 0 & 0 & 0 \\ 1.00000 & 0.00010 & 0 & 0 \\ -632.70 & -0.27130 & -0.02934 & -0.00044 \\ 0 & 0 & 1.00000 & 0 \end{bmatrix}, \ \tilde{\boldsymbol{B}}_1 = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \ \tilde{\boldsymbol{B}}_2 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\tilde{C}_1 = \begin{bmatrix} -316.350 & -0.135650 & 0.239888 & 0.089780 \\ 181.097 & 0.000027 & 0 & 0 \end{bmatrix}, \ \tilde{\boldsymbol{C}}_2 = [-632.700 \quad -0.271300 \quad 0 \quad 0]$$

$$\tilde{\boldsymbol{D}}_{11} = \begin{bmatrix} 0.5 \\ 0 \end{bmatrix}, \ \tilde{\boldsymbol{D}}_{12} = \begin{bmatrix} 0 \\ 632.7 \end{bmatrix}, \ \tilde{D}_{21} = 1, \ \tilde{D}_{22} = 0$$

After 8 iterations $\tilde{\gamma}_{opt}$ is found to be 0.9922. The corresponding H_∞ controller is

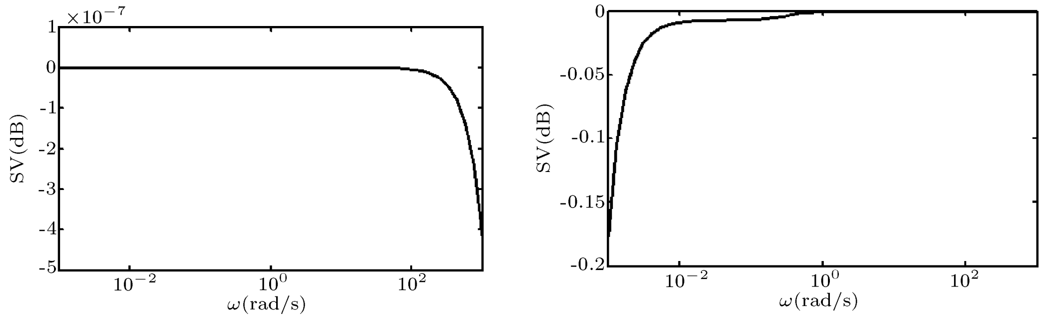
$$K(s) = \frac{3835s^3 + 892.2s^2 + 112.6s + 0.02238}{s^4 + 2773000s^3 + 83090s^2 + 1261s + 0.5257}$$

Fig. 2 shows the singular values Bode plots of cost functions $\tilde{T}_{zw}(\tilde{s})$ and $T_{zw}(s)$. As shown, both two cost functions are all-pass, *i.e.*, $\bar{\sigma}(\tilde{T}_{zw}(j\omega)) = 1$ and $\bar{\sigma}(T_{zw}(j\omega)) = 1$ hold for all $\omega \in R$. The results of the singular values analysis for the sensitivity function $S(s)$, the complementary sensitivity function $T(s)$ and their associated weighting functions $W_1^{-1}(s)$ and $W_2^{-1}(s)$ are illustrated in Fig. 3. It can be observed that S is below its upper bound W_1^{-1} at a low frequency whereas T locates below its upper bound W_2^{-1} at a high frequency, *i.e.*, $\bar{\sigma}(S(j\omega)) \leqslant W_1^{-1}(j\omega)$ and $\bar{\sigma}(T(j\omega)) \leqslant W_2^{-1}(j\omega)$ hold. These results not only indicate that the closed-loop system has a favourable performance of disturbance reduction but also guarantee the robust stability of controlled system in face of the parametric uncertainty in model.

To detect the dynamic tracking performance of the system, we consider a reference input as follows:

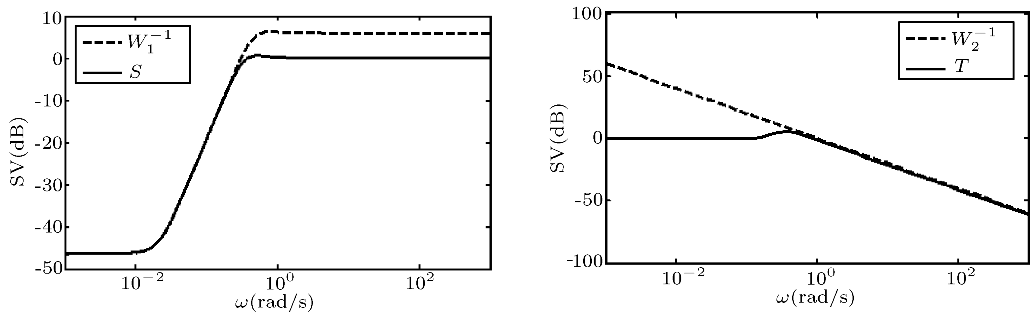
$$r(t) = \begin{cases} 98.10, & 0 \leqslant t < 10 \\ 98.10 + 0.2(t-10)\dfrac{98.10}{40}, & 10 \leqslant t < 50 \\ 98.10(1+0.2), & t \geqslant 50 \end{cases}$$

Then the dynamic response curves of the output y and control input u are plotted in Fig. 4. From Fig. 4 (a), it can be seen that the output y tracks favourably the reference input r, while Fig. 4 (b) demonstrates the control input u behaves within the interval $[0.8D_0, 1.2D_0]$. These results imply that the H_∞ controller $K(s)$ has a good control action on the presented bioprocess.



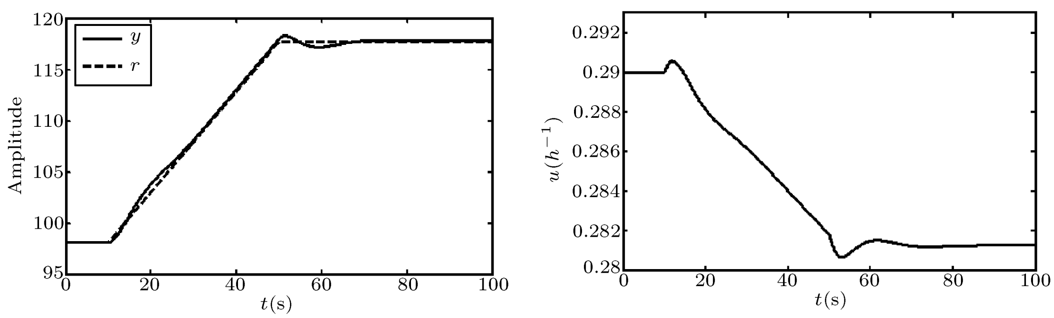
(a) (b)

Fig. 2 Singular value Bode plots of cost functions $\tilde{T}_{zw}$ and T_{zw}



(a) (b)

Fig. 3 Singular value Bode plots of S, T and their associated weighting functions $W_1^{-1}(s)$ and $W_2^{-1}(s)$



(a) Output response (b) Input response

Fig. 4 Output (glycerol concentration) and input (dilution rate) response

7 Conclusions

In this work an H_∞ control strategy for bio-dissimilation process of glycerol has been presented. Since the nominal model has a imaginary axis pole, a robust controller that enables the bioprocess to work around the optimal steady-state is proposed by employing an approach of integrated the bilinear transformation and H_∞ controller design. The design procedure is performed *via* optimizing the parameters r_1 and β, thus it possesses a strong maneuverability and can be applied to other similar processes with uncertain parameters. The simulation shown that the designed H_∞ controller is effective.

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