Observer-based Output-feedback Stabilization Control Design for a Class of Nonlinear Uncertain Systems[1)]

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Abstract The output-feedback stabilization control problem is investigated for a class of nonlinear uncertain systems. Based on the multivariable analog of circle criterion, an observer is designed to estimate the system states and hence the dynamical equations that the estimation error satisfies are derived first. Then, by using integral backstepping approach together with completing square technique, the output-feedback stabilization control is constructively designed such that the closed-loop system is asymptotically stable. Finally, an example is given to illustrate the main results of this paper.

Key words Nonlinear systems, uncertainties, output-feedback stabilization, integral backstepping approach, nonlinear observer

1 Introduction

The design of globally stabilizing controller for nonlinear systems is a research topic under intensive investigation[1∼5]. After the celebrated characterization of the feedback linearizable conditions obtained in [6], a breakthrough was achieved with the introduction of the integral backstepping design methodology[3]. This methodology provides a general recursive constructive tool to design globally stabilizing controllers for nonlinear systems that are given in a strict-feedback form, and for systems that are feedback equivalent to such systems. Ever since the early 1990s, plentiful research results have been obtained on the strict-feedback control systems[4,7,8].

Because of the incomplete information on the system states, output-feedback control problems are much more challenging, difficult and tough to realize[4]. The crucial of output-feedback control design is how to design state observers. The development of nonlinear observer has motivated the development of the nonlinear system output-feedback control in some degree. Before 2000, the results on nonlinear observer were obtained under much strong assumptions on the growth of nonlinearities of the unmeasured states. Earlier results required that nonlinearities satisfy global Lipschitz restriction[9], which excludes familiar nonlinearities such as x^3, e^x, $\sin(x)$, and so on. In recent decades, two classes of output-feedback nonlinear control systems have been investigated intensely: one is that nonlinearities are functions of measurable states, and for this class of systems, we can cancel these nonlinearities by output injection when design observer[10]; the other is that we can design a high gain observer, through which nonlinearities can be dominated by the linear high gains, but global conclusion seems to require global Lipschitz restrictions on nonlinearities of the unmeasured states. [11] showed that if the growth of nonlinearities is larger than 2, then there exists at least one counterexample for which no output-feedback control is existent. Recently, Arcak and Kokotovic eliminated this long-standing growth assumption on nonlinearities[12∼14]: instead of making restriction on the growth speed of nonlinearities, they transformed the convergence problem of observer error into solving an LMI and validating if the nonlinearities of the unmeasured states satisfy (multivariable) nonlinear monotony increasing condition or not.

In this paper, we consider the output-feedback stabilizing control design for a class of nonlinear

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systems as follows:

$$\begin{cases} \dot{x}_i = x_{i+1} + \sum_{j=1}^{i} a_{ij}x_j + f_i(\boldsymbol{x}_{[i]}) + \omega_i(\boldsymbol{x}), \ i = 1, \ldots, n-1 \\ \dot{x}_n = u + \sum_{j=1}^{n} a_{nj}x_j + f_n(\boldsymbol{x}) + \omega_n(\boldsymbol{x}) \\ y = x_1 \end{cases} \tag{1}$$

where $\boldsymbol{x} = [x_1, \ldots, x_n]^{\mathrm{T}}$ is the system state vector with initial value $\boldsymbol{x}_0$; $\boldsymbol{x}_{[i]}$ denotes $[x_1, \ldots, x_i]^{\mathrm{T}}$; $u \in \mathbb{R}$ is the system control input and $y \in \mathbb{R}$ is the system output; $a_{ij}, i, j = 1, \ldots, n, j \leqslant i$ are known constants; $f_i(\cdot)$, $i = 1, \ldots, n$, are known nonlinear functions depending on $\boldsymbol{x}_{[i]}$; $\omega_i(\cdot)$, $i = 1, \ldots, n$, are unknown nonlinear functions. Throughout this paper, we assume that only output $y = x_1$ is measurable.

System (1) can be written into the following compact form

$$\begin{cases} \dot{\boldsymbol{x}} = A\boldsymbol{x} + \boldsymbol{F}(\boldsymbol{x}) + \boldsymbol{\Omega}(\boldsymbol{x}) + \boldsymbol{B}u \\ y = \boldsymbol{C}\boldsymbol{x} \end{cases} \tag{2}$$

where $A = \{a_{ij}\} \in \mathbb{R}^{n\times n}$, $a_{i,i+1} = 1$, $a_{ij} = 0$, $i \geqslant j+2$; $\boldsymbol{F}(\boldsymbol{x}) = [f_1(\boldsymbol{x}_{[1]}), \ldots, f_n(\boldsymbol{x}_{[n]})]^{\mathrm{T}}$; $\boldsymbol{\Omega}(\boldsymbol{x}) = [\omega_1(\boldsymbol{x}), \ldots, \omega_n(\boldsymbol{x})]^{\mathrm{T}}$; $\boldsymbol{B} = [0, \ldots 0, 1]^{\mathrm{T}} \in \mathbb{R}^n$; $\boldsymbol{C} = [1, 0, \ldots, 0] \in \mathbb{R}^n$.

The main results of this paper are based on the following assumptions

A1. Nonlinear function $\boldsymbol{F}(\cdot)$ is smooth (*i.e.*, $\mathcal{C}^\infty$), and $\boldsymbol{F}(0) = 0$.

A2. There exists a nonsingular matrix G such that $\boldsymbol{F}(\boldsymbol{x}) = G\boldsymbol{\gamma}(\boldsymbol{x})$, where $\boldsymbol{\gamma}(\boldsymbol{x})$ is known and $\dfrac{\boldsymbol{\gamma}(\boldsymbol{x})}{\boldsymbol{x}} + \left(\dfrac{\boldsymbol{\gamma}(\boldsymbol{x})}{\boldsymbol{x}}\right)^{\mathrm{T}} \geqslant 0, \forall \boldsymbol{x} \in \mathbb{R}^n$.

A3. There exists smooth nonnegative function $\beta(y)$, $\beta(0) = 0$, such that the uncertain nonlinearity $\boldsymbol{\Omega}(\cdot)$ satisfies: $\boldsymbol{\Omega}(\boldsymbol{x}) \leqslant \beta(y)$.

Different from [13, 14], the nonlinear system to be considered is more general, which is with known and unknown nonlinearities. Especially, the known nonlinearities satisfy the multivariable nonlinear monotony increasing condition (see Assumption A2), and the unknown nonlinearities can be dominated by a known function only depending on the system output (see Assumption A3). This function can be canceled out in control design.

In this paper, an observer is introduced firstly[14], which gives estimates of all the true states, and then based on this observer, the output-feedback stabilizing control is constructively designed using integral backstepping approach. The controller designed preserves the equilibrium at the origin, and renders the closed-loop system to be globally asymptotically stable.

2 Output-feedback control design

Let us design the following observer[14] for system (2)

$$\dot{\hat{\boldsymbol{x}}} = A\hat{\boldsymbol{x}} + \boldsymbol{L}(\boldsymbol{C}\hat{\boldsymbol{x}} - y) + \boldsymbol{F}(\hat{\boldsymbol{x}} + \boldsymbol{K}(\boldsymbol{C}\hat{\boldsymbol{x}} - y)) + \boldsymbol{B}u \tag{3}$$

where $\boldsymbol{K} = [k_1, \ldots, k_n]^{\mathrm{T}}$ and $\boldsymbol{L} = [l_1, \ldots, l_n]^{\mathrm{T}}$ satisfy the following LMI's (Linear matrix inequalities)

$$P > 0, Q > 0, \begin{bmatrix} (A + \boldsymbol{LC})^{\mathrm{T}}P + P(A + \boldsymbol{LC}) + Q & PG + (I + \boldsymbol{KC})^{\mathrm{T}} \\ G^{\mathrm{T}}P + (I + \boldsymbol{KC}) & 0 \end{bmatrix} \leqslant 0 \tag{4}$$

Remark 1. LMI′s (4) give the set of observer design parameter vectors $\boldsymbol{K}$ and $\boldsymbol{L}$, *i.e.*, $\boldsymbol{K}$ and $\boldsymbol{L}$ should be chosen such that these LMI′s have symmetric positive definite solutions P and Q. For $\boldsymbol{K}$ and $\boldsymbol{L}$ satisfying LMI's (4), when no uncertainties, observer (3) will give the open-loop asymptotical estimates of the states of system (2), otherwise, from the following analysis, we know that there exists a suitable controller under which observer (3) will give the closed-loop asymptotical estimates of the states of system (2).

For the state observer systems (3), we have the following lemma.

Lemma 1. Consider the nonlinear system (1)(or (2)) and the state observer system (3). Let $\tilde{\boldsymbol{x}} = \boldsymbol{x} - \hat{\boldsymbol{x}}$ be the observer error. Then, the function $V_0 = \tilde{\boldsymbol{x}}^{\mathrm{T}}P\tilde{\boldsymbol{x}}$ satisfies

$$\dot{V}_0 \leqslant -\delta\|\tilde{\boldsymbol{x}}\|^2 + y^2\phi(y) \tag{5}$$

where $0 < \delta < \lambda_{\min}(Q)$, $\phi(y) = \dfrac{\|P\|^2\beta^2(y)}{\lambda_{\min}(Q)-\delta}$, P and Q are symmetric positive definite matrices satisfying (4), $\bar{\beta}(y)$ is a smooth function such that $\beta(y) = y\bar{\beta}(y)$.

Proof. Because of the space limit, the proof is omitted here.

Remark 2. Since the existence of $\phi(y)y^2 \geqslant 0$ in (5), we can not determine whether $\dot{V}_0$ is negative definite or not. Therefore, if the control input is not suitable, the observer error may be diverge, *i.e.*, the observer cannot give the estimates of the true states. Inequality (5) also shows that the observer error system is output-to-state stable. Thus, if a controller has been designed such that y, as well as $y^2\phi(y)$, is asymptotically stable, then the convergence of the observer error system will be guaranteed and the observer (3) will give the estimates of the true states.

Starting from the following "overall system", we design the output-feedback controller $u(y, \hat{\boldsymbol{x}})$ for system (1) (or (2)) such that the resulting closed-loop system is globally asymptotically stable:

$$\begin{cases} \dot{\tilde{\boldsymbol{x}}} = (A + \boldsymbol{LC})\tilde{\boldsymbol{x}} + G\boldsymbol{\varphi}(\boldsymbol{x}, \hat{\boldsymbol{x}}) + \boldsymbol{\Omega}(\boldsymbol{x}) \\ \dot{y} = \hat{x}_2 + \tilde{x}_2 + a_{11}x_1 + f_1(y) + \omega_1(\boldsymbol{x}) \\ \dot{\hat{x}}_i = \hat{x}_{i+1} - l_i\tilde{x}_1 + \displaystyle\sum_{j=1}^{i} a_{ij}\hat{x}_j + f_i(\hat{\boldsymbol{x}}_{[i]} - \boldsymbol{k}_{[i]}\tilde{x}_1), \quad i = 2, \ldots, n-1 \\ \dot{\hat{x}}_n = u - l_n\tilde{x}_1 + \displaystyle\sum_{i=1}^{n} a_{n,i}\hat{x}_i + f_n(\hat{\boldsymbol{x}} - \boldsymbol{K}\tilde{x}_1) \end{cases} \tag{6}$$

where $\boldsymbol{\varphi}(\boldsymbol{x}, \hat{\boldsymbol{x}}) = \boldsymbol{\gamma}(\boldsymbol{x}) - \boldsymbol{\gamma}(\hat{\boldsymbol{x}} + \boldsymbol{K}(\boldsymbol{C}\hat{\boldsymbol{x}} - y))$.

The reason that system (6) is called the "overall system" is that merely from it, the control objective of this paper described earlier can be completely realized: Obtaining the reconstruction of the original system states and designing the output-feedback stabilizing controller such that the closed-loop system is globally asymptotically stable.

Define new state variables as

$$\begin{cases} z_1 = y, \quad z_2 = \hat{x}_2 - \alpha_1(y) \\ z_i = \hat{x}_i - \alpha_{i-1}(\hat{\boldsymbol{x}}_{[i-1]}, y), i = 3, \ldots, n \end{cases} \tag{7}$$

For notional convenience, denote $\alpha_0 = 0$, $z_{n+1} = 0$. Here $\alpha_i, i = 1, \ldots, n-1$ are smooth functions called virtual controllers to be determined later; $\alpha_n = u(\hat{\boldsymbol{x}}, y)$ is the actual controller to be determined later as well; $\alpha_i, i = 1, \ldots, n$, preserve the equilibrium at the origin ($\hat{\boldsymbol{x}} = 0, y = 0$), that is, $\alpha_1(0) = \alpha_2(0,0) = \cdots = \alpha_n(0,0) = 0$.

In z coordinates, system (6) is transformed into

$$\begin{cases} \dot{\tilde{\boldsymbol{x}}} = (A + \boldsymbol{LC})\tilde{\boldsymbol{x}} + G\boldsymbol{\varphi}(\boldsymbol{x}, \hat{\boldsymbol{x}}) + \boldsymbol{\Omega}(\boldsymbol{x}) \\ \dot{z}_1 = z_2 + \alpha_1 + f_1 + a_{11}x_1 + \tilde{x}_2 \\ \dot{z}_i = z_{i+1} + \alpha_i + F_i - \dfrac{\partial\alpha_{i-1}}{\partial y}\tilde{x}_2, \quad i = 2, \ldots, n-1 \\ \dot{z}_n = u + F_n - \dfrac{\partial\alpha_{n-1}}{\partial y}\tilde{x}_2 \end{cases} \tag{8}$$

where $F_1 = f_1(y) + a_{11}x_1$, $F_i = f_i(\hat{\boldsymbol{x}}_{[i]} - \boldsymbol{k}_{[i]}\tilde{x}_1) - l_i\tilde{x}_1 + \sum_{j=1}^{i} a_{ij}\hat{x}_j - \sum_{j=1}^{i-1}\dfrac{\partial\alpha_{i-1}}{\partial\hat{x}_j}\dot{\hat{x}}_j - \dfrac{\partial\alpha_{i-1}}{\partial y}(\hat{x}_2 + f_1(y) + a_{11}x_1)$, $i = 2, \ldots, n$.

Obviously, the subsystem $z_1, \ldots, z_n$ of system (8) is in the low triangular form regardless of $\tilde{x}$. Thus by using integral backstepping method[3,8] together with completing square technique, we can recursively construct $\alpha_1, \ldots, \alpha_{n-1}$ in the following forms

$$\alpha_1 = \left\{-z_1 - F_1 - \frac{1}{4\varepsilon}z_1 - \phi(y)z_1 - \frac{1}{2}z_1 - \frac{n}{2}z_1\bar{\beta}^2(y)\right\}_{z_1=y} \tag{9}$$

$$\alpha_i = \left\{ -z_i - F_i - z_{i-1} - \frac{1}{4\varepsilon}\left(\frac{\partial\alpha_{i-1}}{\partial y}\right)^2 z_i - \left(\frac{1}{2\sqrt{\varepsilon}}\sum_{j=1}^{i-2}\frac{\partial\alpha_j}{\partial y}z_{j+1} - \frac{1}{2\sqrt{\varepsilon}}z_1\right)\right.$$

$$\cdot \frac{1}{\sqrt{\varepsilon}}\frac{\partial \alpha_{i-1}}{\partial y} - \frac{1}{2}z_i\left(\frac{\partial \alpha_{i-1}}{\partial y}\right)^2\Bigg\}_{z_1=y,\dots,z_i=\hat{x}_i-\alpha_{i-1}}, \quad i=2,\dots,n-1 \tag{10}$$

where parameter ε is chosen such that $\varepsilon < \delta$.

Remark 3. When designing $\alpha_i, i=1,\dots,n-1$, we have used the Lyapunov function candidate $V_i = V_0 + \sum_{j=1}^{i}\frac{1}{2}z_j^2$ and also employed the following technique of completing square to stabilize the term "$-\frac{\partial \alpha_{i-1}}{\partial y}z_i\tilde{x}_2$" appearing in $\dot{V}_i$

$$-\left(\sqrt{\varepsilon}\tilde{x}_2 - \frac{1}{2\sqrt{\varepsilon}}z_1 + \frac{1}{2\sqrt{\varepsilon}}\sum_{j=1}^{i-2}\frac{\partial \alpha_j}{\partial y}z_{j+1}\right)^2 - \frac{\partial \alpha_{i-1}}{\partial y}z_i\tilde{x}_2 = -\Bigg(\sqrt{\varepsilon}\tilde{x}_2 - \frac{1}{2\sqrt{\varepsilon}}z_1 + \frac{1}{2\sqrt{\varepsilon}}\sum_{j=1}^{i-1}\frac{\partial \alpha_j}{\partial y}z_{j+1}\Bigg)^2 + \frac{1}{4\varepsilon}\left(\frac{\partial \alpha_{i-1}}{\partial y}\right)^2 z_i^2 - \frac{1}{2\varepsilon}\frac{\partial \alpha_{i-1}}{\partial y}z_i\left(z_1 - \sum_{j=1}^{i-2}\frac{\partial \alpha_j}{\partial y}z_{j+1}\right)$$

If no completing square technique is used, it is necessary to use the inequality: $-\frac{\partial \alpha_{i-1}}{\partial y}z_i\tilde{x}_2 \leqslant \frac{\varepsilon_{i-1}}{2}\tilde{x}_2^2 + \frac{z_i^2}{2\varepsilon_{i-1}}\left(\frac{\partial \alpha_{i-1}}{\partial y}\right)^2$. This will cause a new design parameter $\varepsilon_{i-1} > 0$ to appear and hence the performance of the control system will decay.

Because $u = \alpha_n$, and according to integral backstepping method, one can know that the expression of α_i is also available to formulating α_n. Then from (10), we can easily obtain the output-feedback stabilizing controller u, *i.e.*,

$$u = \Bigg\{ - z_n - F_n - z_{n-1} - \frac{1}{4\varepsilon}\left(\frac{\partial \alpha_{n-1}}{\partial y}\right)^2 z_n - \left(\frac{1}{2\sqrt{\varepsilon}}\sum_{j=1}^{n-2}\frac{\partial \alpha_j}{\partial y}z_{j+1} - \frac{1}{2\sqrt{\varepsilon}}z_1\right) \cdot \frac{1}{\sqrt{\varepsilon}}\frac{\partial \alpha_{n-1}}{\partial y} - \frac{1}{2}z_n\left(\frac{\partial \alpha_{n-1}}{\partial y}\right)^2\Bigg\}_{z_1=y,\dots,z_n=\hat{x}_n-\alpha_{n-1}} \tag{11}$$

The designed virtual controllers $\alpha_1,\dots,\alpha_{n-1}$ and the actual controller u such that $V = V_0 + \sum_{i=1}^{n}\frac{1}{2}z_i^2$ satisfies:

$$\dot{V} \leqslant -\sum_{j=1}^{n}z_j^2 - \left(\sqrt{\varepsilon}\tilde{x}_2 - \frac{1}{2\sqrt{\varepsilon}}z_1 + \frac{1}{2\sqrt{\varepsilon}}\sum_{j=1}^{n-1}\frac{\partial \alpha_j}{\partial y}z_{j+1}\right)^2 - \delta\|\tilde{\boldsymbol{x}}\|^2 + \varepsilon\tilde{x}_2^2 \tag{12}$$

The main results of the paper are summarized in the following theorem.

Theorem 1. Consider the nonlinear system (1), as well as its compact form (2). Suppose that the system satisfies Assumptions A1, A2 and A3. Then based on observer (3), the output-feedback controller (11), which preserves the equilibrium at the origin, guarantees that closed-loop system is globally asymptotically stable.

Proof. From the expression of (11) it is easy to validate the controller preserves the equilibrium at the origin. In the following, we will prove the globally asymptotical stability of the closed-loop system.

Let $c = \delta - \varepsilon$. Because $\varepsilon < \delta$, it is clear that $c > 0$. Furthermore, from (12) it follows that $\dot{V} \leqslant -c\|\tilde{\boldsymbol{x}}\|^2 - \sum_{j=1}^{n}z_j^2$. This implies the asymptotical stability of the state vector z and the observer error $\tilde{x}$ of (8), *i.e.*,

$$\lim_{t\to\infty}\boldsymbol{z}(t) = 0 \quad \text{and} \quad \lim_{t\to\infty}\tilde{\boldsymbol{x}}(t) = 0 \tag{13}$$

If $\boldsymbol{x}$ is asymptotically stable, then from $\hat{\boldsymbol{x}} = \boldsymbol{x} - \tilde{\boldsymbol{x}}$ together with (13), one can easily achieve the asymptotical stability of $\hat{\boldsymbol{x}}$, and hence the closed-loop system is asymptotically stable as well.

Now we prove the asymptotical stability of x by induction. From $x_1 = y = z_1$ and (13) we know that x_1 is asymptotically stable. Suppose that $[x_1,\dots,x_{i-1}]^{\mathrm{T}}$, where $i = 2,3,\dots,n$, is asymptotically stable. Then by $x_i = z_i + \alpha_{i-1}(\hat{\boldsymbol{x}}_{[i-1]}, y)$ and the smoothness of α_{i-1}, we know that x_i, as well as

$[x_1, \dots, x_i]^{\mathrm{T}}$, is asymptotically stable. Therefore by induction, $\boldsymbol{x} = [x_1, \dots, x_n]^{\mathrm{T}}$ is asymptotically stable. □

3 Simulation example

According to the design procedure given in Section 2, this section considers the control design for the following third-order nonlinear system and illustrates the dynamical behaviors of the closed-loop states and control input by Fig. 1, Fig. 2, Fig. 3 and Fig. 4.

$$\begin{cases} \dot{x}_1 = x_2 + x_1^2 \sin(x_3) \\ \dot{x}_2 = -x_2 + x_3 - \frac{1}{3}x_2^3 \\ \dot{x}_3 = -\frac{1}{2}x_3 - \frac{1}{3}x_3^3 - x_2^2 x_3 + u \\ y = x_1 \end{cases} \tag{14}$$

The initial conditions of system (14) and the corresponding observer are $x_1(0) = x_3(0) = 0$, $x_2(0) = 0.2$, and $\hat{x}_1(0) = \hat{x}_2(0) = \hat{x}_3(0) = 0$, respectively. The design parameters are chosen as $\varepsilon = 0.18$ and $\varepsilon_0 = 0.14$. The simulation results validate the rightness of the results obtained in this paper and show the effectiveness of the designed controller.

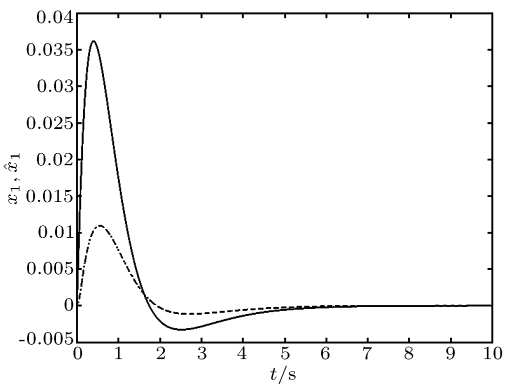

Fig. 1 x_1 (solid) and $\hat{x}_1$ (dashdotted)

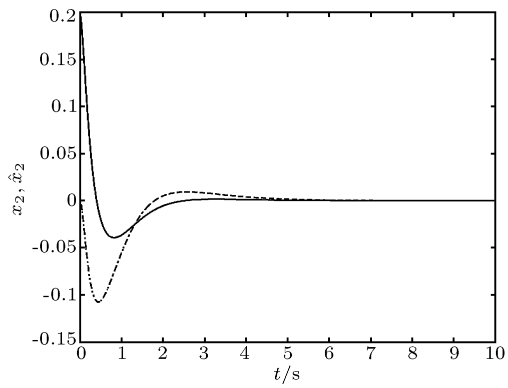

Fig. 2 x_2 (solid) and $\hat{x}_2$ (dashdotted)

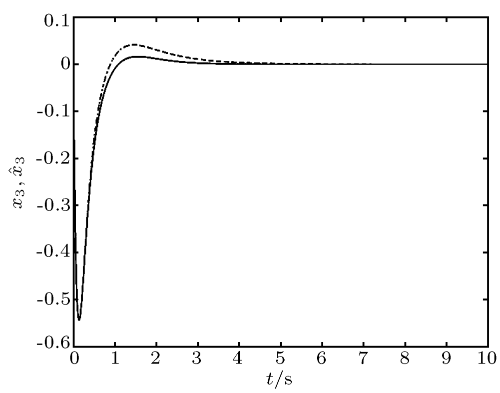

Fig. 3 x_3 (solid) and $\hat{x}_3$ (dashdotted)

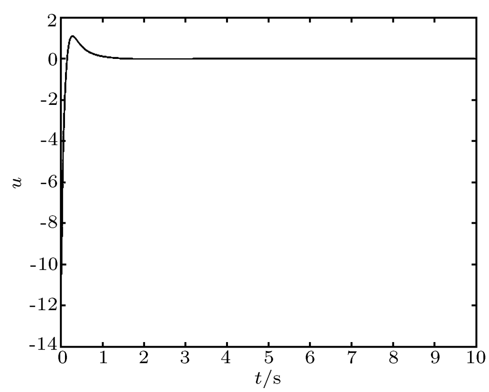

Fig. 4 Control input u

4 Concluding remarks

The output-feedback stabilization control design is investigated for a class of nonlinear uncertain systems. Based on the multivariable analog of circle criterion, an observer is designed to estimate the system states and moreover, the dynamical equations of the estimation error are derived. Then, by using integral backstepping approach together with the completing square technique, the output-feedback stabilization control is constructively designed such that the closed-loop system is asymptotically stable. The further research along this direction will be: (1) to design an output-feedback controller such that the system output asymptotically tracks the given signal, and (2) to design the adaptive output-feedback controller when the uncertainties can be linearly parameterized.

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