Robust Sliding-mode Filtering for a Class of Uncertain Nonlinear Discrete-time State-delayed Systems[1)]

WU Li-Gang WANG Chang-Hong ZENG Qing-Shuang GAO Hui-Jun

(*Space Control and Inertial Technology Research Center, Harbin Institute of Technology, Harbin* 150001)
(E-mail: ligangwu@hit.edu.cn)

Abstract This paper is concerned with the problem of robust sliding-mode filtering for a class of uncertain nonlinear discrete-time systems with time-delays. The nonlinearities are assumed to satisfy global Lipschitz conditions and parameter uncertainties are supposed to reside in a polytope. The resulting filter is of the Luenberger type with the discontinuous form. A sufficient condition with delay-dependency is proposed for existence of such a filter. And the desired filter can be found by solving a set of matrix inequalities. The resulting filter adapts for the systems whose noise input is real functional bounded and not be required to be energy bounded. A numerical example is given to illustrate the effectiveness of the proposed design method.

Key words Sliding-mode filter, discrete-time systems, state-delayed, nonlinear, linear matrix inequalities (LMIs)

1 Introduction

The robust filtering problem has received considerable attention for the past decades. The current efforts on this issue mainly focus on Kalman filtering and H_∞ filtering. In using Kalman filtering, the system′s disturbances are assumed to be Gaussian noise with known statistics. When the noise sources are arbitrary signals with bounded energy, the H_∞ filtering approach provides a guaranteed noise attention level. On the other hand, time-delay and nonlinearity are often encountered in various industrial systems, such as electrical networks, rolling mills, chemical processes, nuclear reactors, *etc.* The existence of time-delay is often a source of poor performance, even instability[1], while nonlinearity always brings much difficulty for the system stabilization, filtering, fault detection, *etc.* So, the study of the robust filtering for the nonlinear time-delay system has great theoretical and practical importance. For the past few years, a rich literature has been dedicated to the time-delay system filtering[2~4]. However, they all deal with the linear system, and when the system is nonlinear the design of robust filter turns out to be much more difficult. To the best of the author′s knowledge, so far, the related results arising from nonlinear systems with time-delay are very limited.

This paper is interested in robust filtering for uncertain nonlinear discrete-time systems with time-delays. We design a new filter, namely the sliding-mode filter, which has a special function of dealing with the system nonlinearities and uncertainties because there is a nonlinear discontinuous term injected into the filter depending on the state estimation error and their differential. This kind of filter is more robust than the aforementioned Kalman filter and H_∞ filter, as this discontinuous term enables the filter to reject the effect of system nonlinearity and to drive the trajectories of the filter so that the state estimation error vector is forced onto and subsequently remains on a sliding surface defined in the filtering error space[5~7]. The motion on this surface is referred to as the sliding mode. Once the sliding mode is achieved the system will experience a reduced-order motion, which is insensitive to system parameter uncertainties and external disturbance. This is an inherent property of the sliding mode control[8]. Moreover, different from Kalman and H_∞ filtering, the sliding mode filtering does not require the system′s disturbances $\omega(t)$ to be Gaussian noise with known statistics or bounded energy, and only positive real function bounded is required, *i.e.*, satisfies $\|\omega(t)\| \leqslant \rho(t)$, where $\rho(t)$ is a known positive real function.

2 Problem formulation

Consider the following uncertain nonlinear discrete-time system with multiple delays in the state

$$\boldsymbol{x}(k+1) = A_0\boldsymbol{x}(k) + \sum_{j=1}^{q} A_j\boldsymbol{x}(k-d_j) + Ff(\boldsymbol{x}_k) + B\boldsymbol{\omega}(k)$$

1) Supported by National Natural Science Foundation of P. R. China (69874008)
Received December 17, 2004; in revised form October 9, 2005

$$\boldsymbol{y}(k) = C_0\boldsymbol{x}(k) + \sum_{j=1}^{q} C_j\boldsymbol{x}(k-d_j) + Gg(\boldsymbol{x}_k) + D\boldsymbol{\omega}(k)$$
$$\boldsymbol{x}(k) = \boldsymbol{\phi}(k), \quad k = -2\bar{d}, -2\bar{d}+1, \cdots, 0, \quad \bar{d} = \max\{d_j, j = 1, 2, \cdots, q\} \tag{1}$$

where $\boldsymbol{x}(k) \in \mathbb{R}^n$ is the state vector, $\boldsymbol{y}(k) \in \mathbb{R}^q$ is the measured output, $\boldsymbol{\omega}(k) \in \mathbb{R}^l$ is the noise input, $\boldsymbol{x}(k) = \boldsymbol{\phi}(k) : k = -2\bar{d}, -2\bar{d}+1, \cdots, 0$, is the given initial condition sequence, $d_j \geqslant 0, j = 1, 2, \cdots, q$, are the known constant time delays; $f(\boldsymbol{x}_k) := f(x, x_{d1}, \cdots, x_{dq})$ and $g(\boldsymbol{x}_k) := g(x, x_{d1}, \cdots, x_{dq})$ represent system nonlinearities. For convenience, we let $\boldsymbol{x}_k := \{\boldsymbol{x}(k), x(k-d_1), \cdots, x(k-d_q)\}$.

The system matrices reside anywhere in the uncertainty polytope Ω defined by

$$\Omega := [A_0, A_1, \cdots, A_q, F, B, C_0, C_1, \cdots, C_q, G, D] \in \Sigma \tag{2}$$

where Σ is a given convex bounded polyhedral domain

$$\Sigma := \left\{ \Omega(\eta) = \sum_{j=1}^{l} \eta_j \Omega_j;\ 0 \leqslant \eta_j \leqslant 1,\ \sum_{j=1}^{l} \eta_j = 1 \right\}$$

and the l vertices of the polytope are described by

$$\Omega_j := [A_0^{(j)}, A_1^{(j)}, \cdots, A_q^{(j)}, F^{(j)}, B^{(j)}, C_0^{(j)}, C_1^{(j)}, \cdots, C_q^{(j)}, G^{(j)}, D^{(j)}]$$

Throughout this paper, we make the following assumptions.

Assumption 1. System (1) is asymptotically stable.

Assumption 2. There exists a positive real function $\rho(k)$ such that $\omega(k)$ satisfies $\|\omega(k)\| \leqslant \rho(k)$.

Assumption 3. The nonlinearities $f_1(\boldsymbol{x}_k)$, $f_2(\boldsymbol{x}_k)$ satisfy the followings.

1) $f(0, 0, \cdots, 0) = 0$ and $g(0, 0, \cdots, 0) = 0$;

2) (Lipschitz conditions) There exist known real matrices M_j, $N_j (j = 1, 2, \cdots, q)$ with appropriate dimensions such that for all $x_0, x_1, \cdots, x_q \in \mathbb{R}^n$ and $y_0, y_1, \cdots, y_q \in \mathbb{R}^n$ the follwings hold

$$\|f(x_0, x_1, \cdots, x_q) - f(y_0, y_1, \cdots, y_q)\| \leqslant \sum_{j=0}^{q} \|M_j(x_j - y_j)\|$$
$$\|g(x_0, x_1, \cdots, x_q) - g(y_0, y_1, \cdots, y_q)\| \leqslant \sum_{j=0}^{q} \|N_j(x_j - y_j)\|$$

Our objective is to design a full order sliding-mode filter in the following form

$$\hat{\boldsymbol{x}}(k+1) = A_f\hat{\boldsymbol{x}}(k) + H_f(\boldsymbol{y}(k) - \hat{\boldsymbol{y}}(k)) + Ff(\hat{\boldsymbol{x}}_k) + Bv(k), \quad \hat{\boldsymbol{x}}(0) = 0$$
$$\hat{\boldsymbol{y}}(k) = C_0\hat{\boldsymbol{x}}(k) + Gg(\hat{\boldsymbol{x}}_k) \tag{3}$$

where $\hat{\boldsymbol{x}}(k)$ is the state estimate, the real matrices $A_f \in \mathbb{R}^{n\times n}$ and $H_f \in \mathbb{R}^{n\times p}$ are filter parameters to be specified. $\boldsymbol{w}(k) \in \mathbb{R}^l$ is a discontinuous feedback compensation control.

Augmenting the model of (1) to include the states of the filter, we obtain the filtering error system as

$$\zeta(k+1) = \bar{A}_0\zeta(k) + \sum_{j=1}^{q} \bar{A}_j\zeta(k-d_j) + H(h(\boldsymbol{x}_k) - h(\hat{\boldsymbol{x}}_k)) + \bar{H}h(\boldsymbol{x}_k) + \bar{B}\omega(k) - K_nBv(k) \tag{4}$$

where $\boldsymbol{e}(k) := \boldsymbol{x}(k) - \hat{\boldsymbol{x}}(k)$, $\boldsymbol{\zeta}(k) := \mathrm{col}\{\boldsymbol{x}(k), \boldsymbol{e}(k)\}$ and

$$h(\boldsymbol{x}_k) := \begin{bmatrix} f(\boldsymbol{x}_k) \\ g(\boldsymbol{x}_k) \end{bmatrix},\ K_n = \begin{bmatrix} 0 \\ I_n \end{bmatrix},\ H = \begin{bmatrix} 0 & 0 \\ F & -H_fG \end{bmatrix},\ \bar{H} = \begin{bmatrix} F & 0 \\ 0 & 0 \end{bmatrix},\ \bar{B} = \begin{bmatrix} B \\ B - H_fD \end{bmatrix}$$
$$\bar{A}_0 = \begin{bmatrix} A_0 & 0 \\ A_0 - A_f & A_f - H_fC_0 \end{bmatrix},\ \bar{A}_j = \begin{bmatrix} A_j & 0 \\ A_j - H_fC_j & 0 \end{bmatrix}$$

Therefore, the robust filtering design problem to be addressed in this paper is stated as: For system (1) whose noise input $\boldsymbol{\omega}(k)$ is assumed to be an arbitrary signal satisfying Assumption 2, design a full-order sliding-mode filter in the form of (3) such that the filtering error system (4) is robust asymptotically stable.

The following lemma plays a key role in deriving our main results.

Lemma 1[2]**.** For vectors $\boldsymbol{a}, \boldsymbol{b}$ and matrices N, X, Y, Z with compatible dimensions, if $\begin{bmatrix} X & Y \\ Y^{\mathrm{T}} & Z \end{bmatrix} \geqslant 0$ then

$$-2\boldsymbol{a}^{\mathrm{T}}N\boldsymbol{b} \leqslant \begin{bmatrix} \boldsymbol{a} \\ \boldsymbol{b} \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} X & Y-N \\ Y^{\mathrm{T}}-N^{\mathrm{T}} & Z \end{bmatrix} \begin{bmatrix} \boldsymbol{a} \\ \boldsymbol{b} \end{bmatrix} \tag{5}$$

3 Main results

In this section, we shall design the sliding mode filter in the form of (3). First, we design the following discontinuous feedback compensation control

$$\nu(k) = (K_n B)^{+}\left[\alpha\sigma(k) + \rho(k)\|\bar{B}\|\frac{\sigma(k)}{\|\sigma(k)\|}\right] \tag{6}$$

where $\alpha > 0$ is a constant, and $\rho(k)$ is defined in Assumption 2. Correspondingly, we chose the following switching function[9]:

$$\sigma(k) = K_{2n}^{\mathrm{T}}P\psi(k) \tag{7}$$

The following theorem is essential for solving the sliding-mode filtering problem.

Theorem 1. Consider system (1) with Assumptions 1, 2 and 3. The filtering error system (4) is exponentially stable for all the points in the uncertainty polytope, if there exist matrices $0 < P_1 \in \mathbb{R}^{2n\times 2n}$, $P_2 \in \mathbb{R}^{2n\times 2n}$, $P_3 \in \mathbb{R}^{2n\times 2n}$, $P = \begin{bmatrix} P_1 & 0 \\ P_2 & P_3 \end{bmatrix}$, $X_j = \begin{bmatrix} X_{1j} & X_{2j} \\ * & X_{3j} \end{bmatrix}$, $Y_j = \begin{bmatrix} Y_{1j} \\ Y_{2j} \end{bmatrix}$, $X_{ij} \in \mathbb{R}^{2n\times 2n}(i = 1, 2, 3)$, $Y_{lj} \in \mathbb{R}^{2n\times 2n}(l = 1, 2)$, $0 < Q_j \in \mathbb{R}^{2n\times 2n}$, $Z_j \in \mathbb{R}^{2n\times 2n}(j = 1, 2, \cdots, q)$ and scalars $\varepsilon_1 > 0$, $\varepsilon_2 > 0$, such that the following matrix inequalities hold:

$$\begin{bmatrix} \bar{\Pi}_{11} & \bar{\Pi}_{12} & -Y_{1\mathrm{col}}^{\mathrm{T}} & P_2^{\mathrm{T}}H^{\mathrm{T}} & P_2^{\mathrm{T}}\bar{H}^{\mathrm{T}} \\ * & \bar{\Pi}_{22} & -Y_{2\mathrm{col}}^{\mathrm{T}} & P_3^{\mathrm{T}}H^{\mathrm{T}} & P_3^{\mathrm{T}}\bar{H}^{\mathrm{T}} \\ * & * & \varepsilon_{\mathrm{dia}}M_{\mathrm{dia}} - Q_{\mathrm{dia}} & 0 & 0 \\ * & * & * & -\varepsilon_1 I & 0 \\ * & * & * & * & -\varepsilon_2 I \end{bmatrix}^{(i)} < 0, \quad i = 1, 2, \cdots, l \tag{8}$$

$$\begin{bmatrix} X_j & Y_j \\ * & Z_j \end{bmatrix} \geqslant 0, \quad \forall j = 1, 2, \cdots, q \tag{9}$$

where

$$Z_\Sigma := \sum_{j=1}^{q} d_j Z_j,\ Q_\Sigma := \sum_{j=1}^{q} Q_j,\ Q_{\mathrm{dia}} := \mathrm{diag}\{Q_1, Q_2, \cdots, Q_q\},\ M_{\mathrm{dia}} := \mathrm{diag}\{\Psi_1, \Psi_2, \cdots, \Psi_q\}$$

$$\varepsilon_{\mathrm{dia}} := (m+1)\begin{bmatrix} \varepsilon_2 I_n & 0 \\ 0 & \varepsilon_1 I_n \end{bmatrix},\ \Psi_j := \begin{bmatrix} M_j^{\mathrm{T}}M_j + N_j^{\mathrm{T}}N_j & 0 \\ 0 & M_j^{\mathrm{T}}M_j + N_j^{\mathrm{T}}N_j \end{bmatrix} (j = 0, 1, \cdots, q)$$

$$\bar{\Pi}_{11} := 2(\bar{A}_0 + \sum_{j=0}^{q}\bar{A}_j)P_2 + \varepsilon_{\mathrm{dia}}\Psi_0 + Q_\Sigma + \sum_{j=1}^{q}[d_j X_{1j} + (Y_{1j} - P_2^{\mathrm{T}}\bar{A}_j^{\mathrm{T}}) + (Y_{1j}^{\mathrm{T}} - \bar{A}_j P_2)]$$

$$\bar{\Pi}_{12} := P_1 - P_2^{\mathrm{T}} + (\bar{A}_0 + \sum_{j=1}^{q}\bar{A}_j)P_3 + \sum_{j=1}^{q}\bar{d}_j X_{2j} + \sum_{j=1}^{q}(Y_{2j}^{\mathrm{T}} - \bar{A}_j P_3),\ \bar{\Pi}_{22} := -P_3^{\mathrm{T}} - P_3 + \sum_{j=1}^{q}\bar{d}_j X_{3j} + Z_\Sigma$$

$$Y_{1\mathrm{col}} := \mathrm{col}\{(Y_{11}^{\mathrm{T}} - \bar{A}_1^{\mathrm{T}}P_2), (Y_{12}^{\mathrm{T}} - \bar{A}_2^{\mathrm{T}}P_2), \cdots, (Y_{1q}^{\mathrm{T}} - \bar{A}_q^{\mathrm{T}}P_2)\}$$

$$Y_{2\mathrm{col}} := \mathrm{col}\{(Y_{21}^{\mathrm{T}} - \bar{A}_1^{\mathrm{T}}P_3), (Y_{22}^{\mathrm{T}} - \bar{A}_2^{\mathrm{T}}P_3), \cdots, (Y_{2q}^{\mathrm{T}} - \bar{A}_q^{\mathrm{T}}P_3)\}$$

Due to space limitation, the proof of this theorem is omitted.

Now, we are in position to solve the sliding-mode filter synthesis problem based on the condition obtained in Theorem 1. The following theorem provides a sufficient condition for the existence of such a filter for system (1).

Theorem 2. Consider system (1) with Assumptions 1, 2, and 3. An admissible robust sliding mode filter in the form (3) exists, if there exist matrices $0 < W_1 \in \mathbb{R}^{2n\times 2n}$, $W_2, W_3 \in \mathbb{R}^{2n\times 2n}$, $0 < \bar{Q}_j \in$

$\mathbb{R}^{2n\times 2n}$, $\bar{Z}_j \in \mathbb{R}^{2n\times 2n}$, $W_1 = \begin{bmatrix} W_{11} & 0 \\ 0 & W_{12} \end{bmatrix}$, $\bar{X}_j = \begin{bmatrix} \bar{X}_{1j} & \bar{X}_{2j} \\ * & \bar{X}_{3j} \end{bmatrix}$, $Y_j = \begin{bmatrix} \bar{Y}_{1j} \\ \bar{Y}_{2j} \end{bmatrix}$, $\bar{X}_{ij} \in \mathbb{R}^{2n\times 2n} (i=1,2,3)$, $\bar{Y}_{lj} \in \mathbb{R}^{2n\times 2n} (l=1,2)$, $(j=1,2,\cdots,q)$, $\Delta_1 \in \mathbb{R}^{n\times p}$ and scalars $\varepsilon_1 > 0$, $\varepsilon_2 > 0$, such that (10)$\sim$(12) hold

$$\begin{bmatrix} [1,1] & [1,2] & -\bar{Y}_{1\text{col}}^{\mathrm{T}} & 0 & 0 \\ * & [2,2] & -\bar{Y}_{2\text{col}}^{\mathrm{T}} & (W_1 H)^{\mathrm{T}} & \bar{H}^{\mathrm{T}} \\ * & * & \Lambda_{\text{dia}} M_{\text{dia}} - \bar{Q}_{\text{dia}} & 0 & 0 \\ * & * & * & -\varepsilon_1 W_1^{\mathrm{T}} W_1 & 0 \\ * & * & * & * & -\varepsilon_2 I_{2n} \end{bmatrix}^{(i)} < 0, \quad i=1,2,\cdots,l \tag{10}$$

$$\begin{bmatrix} \bar{X}_{1j} & \bar{X}_{2j} & \bar{Y}_{1j} \\ * & \bar{X}_{3j} & \bar{Y}_{2j} \\ * & * & Z_j \end{bmatrix} \geqslant 0, \quad \forall j=1,2,\cdots,q \tag{11}$$

$$W_2 = W_3 \tag{12}$$

Moreover, the filter parameter matrices A_f and H_f are given by

$$A_f = W_{12}^{-1}\Delta_1, \quad H_f = W_{12}^{-1}\Delta_2 \tag{13}$$

where

$$\Lambda_{\text{dia}} := \text{diag}\overbrace{\{\bar{\varepsilon}_{\text{dia}}, \bar{\varepsilon}_{\text{dia}}, \cdots, \bar{\varepsilon}_{\text{dia}}\}}^{q},\ \bar{Q}_\Sigma := \sum_{j=1}^{q} W_1 Q_j W_1 = \sum_{j=1}^{q} \bar{Q}_j,\ \bar{Z}_\Sigma := \sum_{j=1}^{q} d_j W_2^{\mathrm{T}} Z_j W_2 = \sum_{j=1}^{q} d_j \bar{Z}_j$$

$$[1,1] := W_2 + W_2^{\mathrm{T}} + \sum_{j=1}^{q}(d_j \bar{X}_{1j} + \bar{Y}_{1j} + \bar{Y}_{1j}^{\mathrm{T}}) + \bar{\varepsilon}_{\text{dia}} \Psi_0 + \bar{Q}_\Sigma + \bar{Z}_\Sigma,\ [2,2] := -W_3 - W_3^{\mathrm{T}} + \sum_{j=1}^{q} d_j \bar{X}_{3j} + \bar{Z}_\Sigma$$

$$[1,2] := W_3 + W_1\bar{A}_0 - W_2^{\mathrm{T}} + \sum_{j=1}^{q}(d\bar{X}_{2j} + \bar{Y}_{2j}^{\mathrm{T}}) + \bar{Z}_\Sigma,\ \bar{Q}_{\text{dia}} = W_{1\text{dia}} Q_{\text{dia}} W_{1\text{dia}} := \text{diag}\{\bar{Q}_1, \bar{Q}_2, \cdots, \bar{Q}_q\}$$

$$\bar{\varepsilon}_{\text{dia}} := W_1 \varepsilon_{\text{dia}} W_1,\ \bar{Y}_{1\text{col}} := \text{col}\{\bar{Y}_{11}^{\mathrm{T}}, \bar{Y}_{12}^{\mathrm{T}}, \cdots, \bar{Y}_{1q}^{\mathrm{T}}\},\ \bar{Y}_{2\text{col}} := \text{col}\{(\bar{Y}_{21}^{\mathrm{T}} - W_1\bar{A}_1), \cdots, (\bar{Y}_{2q}^{\mathrm{T}} - W_1\bar{A}_q)\}$$

$$W_1\bar{A} = \begin{bmatrix} W_{11}A_0 & 0 \\ W_{12}A_0 - \Delta_1 & \Delta_1 - \Delta_2 C_0 \end{bmatrix}, W_1 H = \begin{bmatrix} 0 & 0 \\ W_{12}F & -\Delta_2 G \end{bmatrix}, W_1\bar{A}_j = \begin{bmatrix} W_{11}A_j & 0 \\ \Delta_1 - \Delta_2 C_j & 0 \end{bmatrix} (j=1,2,\cdots,q)$$

4 Numerical example

Consider the uncertain nonlinear time-delay system (1) with parameters as follows.

$$A_0 = \begin{bmatrix} -4 & 0 \\ 1 & -5+\delta_1 \end{bmatrix},\ A_1 = \begin{bmatrix} -0.2 & 0 \\ 0.1 & -0.3+\delta_2 \end{bmatrix},\ A_2 = \begin{bmatrix} 0 & 0.2 \\ -0.4 & -0.3+\delta_2 \end{bmatrix},\ B = \begin{bmatrix} 0.8 \\ 1+\delta_2 \end{bmatrix}$$

$$F = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},\ \boldsymbol{G} = [0.5 \quad 0.8],\ M_j = \begin{bmatrix} 0.2 & 0 \\ 0.1 & 0.3 \end{bmatrix},\ \boldsymbol{N}_j = [0.5 \quad 0.4],\ (j=0,1,2)$$

$$\boldsymbol{C}_0 = [1 \quad 1],\ \boldsymbol{C}_1 = [0.2 \quad 0.5],\ \boldsymbol{C}_2 = [0.3 \quad 0.4],\ D = 1+\delta_2,\ -0.5 \leqslant \delta_1 \leqslant 0.5,\ -0.1 \leqslant \delta_2 \leqslant 0.1$$

Assuming $d_1 = 1$, $d_2 = 2$ and by solving (10)$\sim$(13) in Theorem 2, we obtain the parameter matrices A_f and H_f as follows:

$$A_f = \begin{bmatrix} 0.2037 & 0.4586 \\ -0.2414 & -0.3308 \end{bmatrix},\ H_f = \begin{bmatrix} -0.3647 \\ 0.1663 \end{bmatrix},\ P = \begin{bmatrix} P_1 & 0 \\ P_2 & P_3 \end{bmatrix} = W^{-1}$$

$$W_1 = \begin{bmatrix} 0.4662 & 0.3260 & 0 & 0 \\ -0.1379 & 0.1257 & 0 & 0 \\ 0 & 0 & 0.3302 & -0.3028 \\ 0 & 0 & -0.1439 & 0.2847 \end{bmatrix},\ W_2 = W_3 = \begin{bmatrix} 0.2337 & 0.1825 & -0.3325 & 0.3379 \\ 0.2729 & -0.5466 & 0.3958 & -0.3641 \\ -0.6590 & -0.8410 & 0.7104 & 0.2535 \\ -0.3322 & 0.6612 & -0.3230 & -0.1251 \end{bmatrix}$$

5 Conclusion

A new robust sliding-mode filter has been designed for a class of uncertain nonlinear discrete-time systems with time-delays. The filter has Luenberger type with a discontinuous feedback compensation control injected into it. A sufficient condition with delay-dependency has been proposed for the existence of such a filter, and desired filter can be found by solving a set of matrix inequalities. A numerical example has been given to demonstrate the effectiveness of the proposed design methods.

References

1 Gao Hui-Jun, Wang Chang-Hong. Delay-dependent robust H_∞ and L_2-L_∞ filtering for a class of uncertain nonlinear time-delay systems. *IEEE Transactions on Automatic Control*, 2003, **48**(9): 1661∼1666

2 Fridman E, Shaked U. Delay-dependent stability and H_∞ control: Constant and time-varying delays. *International Journal of Control*, 2003, **76**(1): 48∼60

3 Fridman E, Shaked U. A new H_∞ filter design for linear time delay systems. *IEEE Transactions on Signal Processing*, 2001, **49**(11): 2839∼2843

4 De Souza C E, Palhares R M, Peres P L D. Robust H_∞ filtering design for uncertain linear systems with multiple time-varying state delays. *IEEE Transactions on Signal Processing*, 2001, **49**(3): 569∼576

5 Alessandri A. Sliding-mode estimators for a class of non-linear system affected by bounded disturbances. *International Journal of Control*, 2003, **76**(3): 226∼236

6 Koshkouei A, Zinober A. Sliding mode state observation for nonlinear system. *International Journal of Control*, 2004, **77**(2): 118∼127

7 Aguilar-Lopez R, Maya-Yescas R. State estimation for nonlinear systems under model uncertainties: A class of sliding-mode observers. *Journal of Process Control*, 2005, **15**(3): 363∼370

8 Gao W, Hung J. Variable structure control of nonlinear systems: A new approach. *IEEE Transactions on Industrial Electronics*, 1993, **40**(1): 45∼55

9 Niu Y, Lam J, Wang X. Observer-based sliding mode control for nonlinear state-delayed systems. *International Journal of Systems Science*, 2004, **35**(2): 139∼150

WU Li-Gang Ph. D. candidate in the Department of Control Science and Engineering at Harbin Institute of Technology, His research interests include time-delay systems, robust control/filter theory, and nonlinear system control.

WANG Chang-Hong Professor in the Department of Control Science and Engineering at Harbin Institute of Technology. His research interests include intelligent control and intelligent systems, inertial technology and its testing equipment, robotics, precision servo systems, and network control.

ZENG Qing-Shuang Professor in the Space Control and Inertial Technology Research Center at Harbin Institute of Technology. His research interests include inertial technology and its testing equipment, satellite control and network control.

GAO Hui-Jun Professor in the Space Control and Inertial Technology Research Center at Harbin Institute of Technology. His research interests include time-delay systems, 2D systems and robust control/filter theory.