An Improved Cooperative Tracking Model Used for Large-scale Social Foraging Swarm[1)]

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Abstract An improved cooperative tracking model is proposed, which is based on the local information between mutually observable individuals with global object information, and this model is used for scalable social foraging swarm. In this model, the "follower" individuals in the swarm take the center of the minimal circumcircle decided by the neighbors in the positive visual set of individual as its local object position. We study the stability properties of cooperative tracking behavior of social foraging swarm based on Lyapunov stability theory. Simulations show that the stable cooperative tracking behavior of the global social foraging swarm can be achieved easily, and beautiful scalability emerge from the proposed model for social foraging swarm.

Key words Foraging swarm, cooperative tracking behavior, stability, scalability

1 Introduction

Biologists have been working on understanding and modeling the swarm behavior for a long time. Such collective behavior has certain advantage such as increasing the chances of finding food and avoiding predators, and the operational principles include modeling, coordination strategy specification, *etc*., which are from such biological systems, can be used for developing distributed cooperative control and coordination for multi-agent systems such as autonomous multi-robot applications, and unmanned undersea, land or air vehicles.

Breder is one of the early scientists who used mathematical equation to study the behavior of fish schools, and he proposed a simple model composed of a constant attraction and a repulsion inversely proportional to the square of the inter-individual distance[1]. In [2] Warburton and Lazarus formulated the effect on cohesion of a family of attraction/repulsion functions. [3] and [4] made good contributions to the stability analysis of swarm behavior. As for the study about the cooperative tracking behavior of the social foraging swarm, many important results have been accomplished, *e.g.*, Reynolds model was firstly proposed in [5], Vicsek model was independently proposed by Vicsek and his coworkers in [6], *etc*., Vicsek model can be looked as a special form of Reynolds model, and the difference between the two models is that all individuals move at the same speed in Vicsek model. [7] provided the mathematical proof of convergence of Vicsek model, and [8] proposed an individual-based swarm model. One drawback of all the models is that all the individuals need to know the exact relative positions of all other individuals. This is not biologically realistic, and although in engineering it may be overcome by technology like the global positioning system, the production cost will increase exponentially along with the scale of the swarm. In order to overcome this problem, we propose a motion equation for every individual based on the minimal circumcircle method in [9], which is used for static aggregating behavior of swarm. To continue the work in [9], we will study the dynamic tracking model of foraging swarm based on the local individual information with global object information, and provide the corresponding stability conclusion of cooperative tracking behavior and provide the corresponding stability conclusion of cooperative tracking behavior when the interaction between the individuals is composed of a bounded attraction function and a bounded or unbounded repulsion function in this paper.

2 Cooperative tracking model of foraging swarm

Similar to the initial distribution of swarm in [9], we will only consider the situation that there exists a mutually observable chain between two arbitrary individuals in the initial distribution of the swarm.

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Consider a swarm of m individuals in an n-dimensional Euclidean space. We model the individuals as points and ignore their dimensions. It is assumed that the global object position is invariable and all individuals have the same observation ability. That is, individual i can find the exact position of individual j if individual j can find the exact position of individual i.

Added to the definitions provided in [9], the definition of positive visual set of an individual is as follows.

Definition 1. $\boldsymbol{S} = \{1, 2, \cdots, m\}$ is composed of all individuals in the swarm. The positive visual set of individual i can be defined as

$$\boldsymbol{A}_i^+ = \{j : \frac{-\pi}{2} < [\angle(\boldsymbol{x}^i, \boldsymbol{x}^j) - \angle(\boldsymbol{x}^i, \boldsymbol{x}_{goal})] < \frac{\pi}{2}, j \in \boldsymbol{A}_i\}$$

where $\boldsymbol{A}_i = \{\boldsymbol{x}^j : \|\boldsymbol{x}^i - \boldsymbol{x}^j\| \leqslant \varepsilon, \forall j \in S\}$, ε denotes the visual range of all individuals, and $\angle(\boldsymbol{x}^i, \boldsymbol{x}^j)$ denotes the angle between the vectors $\boldsymbol{x}^i$ and $\boldsymbol{x}^j$. We select the point $\boldsymbol{x}^i$ as the origin of the coordinate system, the vertical axis is in the direction from $\boldsymbol{x}^i$ to $\boldsymbol{x}_{goal}$ and the horizontal axis is in the direction as turned $90°$ clockwise from the vertical axis. And we use 2-norm $\|\boldsymbol{x}^i - \boldsymbol{x}^j\| = \sqrt{(\boldsymbol{x}^i - \boldsymbol{x}^j)^{\mathrm{T}}(\boldsymbol{x}^i - \boldsymbol{x}^j)}$ to describe the distance between the individual i and individual j.

The cooperative tracking model of a social foraging swarm is given in this part when the above assumption is satisfied. It is assumed that individual i discretely calculates the center of the minimal circumcircle composed by all individuals in $\boldsymbol{A}_i^+$ at every sample time, and the center is invariable between every two sample times. The motion equation for individual i is given by

$$\dot{\boldsymbol{x}}^i(t) = g_a(\boldsymbol{x}_{goal}, \bar{\boldsymbol{x}}_{io+}(t)) + g_r(\min_{j \in R_i} \|\boldsymbol{x}^i(t) - \boldsymbol{x}^j(t)\|), \quad i = 1, 2, \cdots, m \tag{1}$$

where the attractive function $g_a(\boldsymbol{x}_{goal}, \bar{\boldsymbol{x}}_{io+}(t))$ determines the attraction to the local or global object position and takes the following form

$$g_a(\boldsymbol{x}_{goal}, \bar{\boldsymbol{x}}_{io+}(t)) = -\sigma_1 a(\boldsymbol{x}^i(t) - \bar{\boldsymbol{x}}_{io+}(t)) - \sigma_2 A_\sigma(\boldsymbol{x}^i(t) - \boldsymbol{x}_{goal}), \quad a > 0, A_\sigma > 0 \tag{2}$$

$$\sigma_1 = \begin{cases} 1, & \|\boldsymbol{x}^i(t) - \boldsymbol{x}_{goal}\| > \varepsilon \\ 0, & \|\boldsymbol{x}^i(t) - \boldsymbol{x}_{goal}\| \leqslant \varepsilon \end{cases} \tag{3}$$

$$\sigma_2 = \begin{cases} 1, & \|\boldsymbol{x}^i(t) - \boldsymbol{x}_{goal}\| \leqslant \varepsilon \\ 0, & \|\boldsymbol{x}^i(t) - \boldsymbol{x}_{goal}\| > \varepsilon \end{cases} \tag{4}$$

the repulsive function $g_r(\boldsymbol{x}^i(t), \boldsymbol{x}^j(t))$ determines repulsion from other individuals on short distances and takes the following form[9]

$$g_r(\boldsymbol{x}^i(t) - \boldsymbol{x}^j(t)) = \begin{cases} b\dfrac{(r - \|\boldsymbol{x}^i(t) - \boldsymbol{x}^j(t)\|)}{(\|\boldsymbol{x}^i(t) - \boldsymbol{x}^j(t)\| - \rho)}(\boldsymbol{x}^i(t) - \boldsymbol{x}^j(t)), b > 0; & \rho < \|\boldsymbol{x}^i(t) - \boldsymbol{x}^j(t)\| < r \\ 0, & r \leqslant \|\boldsymbol{x}^i(t) - \boldsymbol{x}^j(t)\| \\ \infty, & \|\boldsymbol{x}^i(t) - \boldsymbol{x}^j(t)\| \leqslant \rho \end{cases} \tag{5}$$

where $\bar{\boldsymbol{x}}_{io+} \in R^n$ denotes the center of the minimal circumcircle composed by all individuals in $\boldsymbol{A}_i^+$; r denotes the farthest distance within which the repulsion function between individuals can effect, and the minimal safe distance between individuals is described by ρ; the model is based on the local individual information because all terms in the model are either predefined or decided by the neighbors in the visual range of individual i. $\sigma_1 = 1$ means that the global object position is not in the visual range of individual i, and here individual i follows the leader in $\boldsymbol{A}_i^+$ to maintain the swarm aggregated. If $\boldsymbol{A}_i^+ = \{i\}$ exists, we can look the global object as a virtual neighbor of individual i and add it into the set $\boldsymbol{A}_i^+$; $\sigma_2 = 1$ means that the global object position is in the visual range of individual i, *i.e.*, individual i will tow the swarm to the object position to finish foraging behavior as the leader of swarm. The constant b in the repulsive function is used to adjust the equilibrium of individual i when attractive and repulsive functions effect at the same time, and its influence on equilibrium will be analyzed later. As to the situation of $\sigma_2 = 1$, the "leader" behavior of individual i can be looked as a kind of tracking behavior to the global object. So the motion equation of an individual can be simplified as follows

$$\dot{\boldsymbol{x}}^i(t) = g_a(\bar{\boldsymbol{x}}_{io+}(t)) + g_r(\min_{j \in R_i} \|\boldsymbol{x}^i(t) - \boldsymbol{x}^j(t)\|), \quad i = 1, 2, \cdots, m \tag{6}$$

$$g_a(\bar{\boldsymbol{x}}_{io+}(t)) = -a(\boldsymbol{x}^i(t) - \bar{\boldsymbol{x}}_{io+}(t)), \quad a > 0 \tag{7}$$

Further more, (5) can be transformed as following bounded form if we consider the fact that the repulsion between two real physical particles is finite.

$$g_r(\boldsymbol{x}^i(t), \boldsymbol{x}^j(t)) = \begin{cases} 0, & r \leqslant \|\boldsymbol{x}^i(t) - \boldsymbol{x}^j(t)\| \\ b\dfrac{r - \|\boldsymbol{x}^i(t) - \boldsymbol{x}^j(t)\|}{(r-\rho)\|\boldsymbol{x}^i(t) - \boldsymbol{x}^j(t)\|}(\boldsymbol{x}^i(t) - \boldsymbol{x}^j(t)), b > 0, & \rho < \|\boldsymbol{x}^i(t) - \boldsymbol{x}^j(t)\| < r \\ b\dfrac{\boldsymbol{x}^i(t) - \boldsymbol{x}^j(t)}{\|\boldsymbol{x}^i(t) - \boldsymbol{x}^j(t)\|}, & \|\boldsymbol{x}^i(t) - \boldsymbol{x}^j(t)\| \leqslant \rho \end{cases} \tag{8}$$

3 Stability analysis of cooperative tracking behavior

Our first result is about a follower individual which does not have any neighbors in its repulsive range. We call it a free agent.

Lemma 1. If individual i described by the model in (6) with an attractive function given in (7) is a free agent at time t, then it moves toward the object position and as time progresses it will converge to a hyperball

$$B_\eta(\bar{\boldsymbol{x}}_{io+}) = \{\boldsymbol{x}^i : \|\boldsymbol{x}^i - \bar{\boldsymbol{x}}_{io+}\| \leqslant \eta\}$$

where η is an arbitrary positive number.

Proof. Let $\boldsymbol{e}^i(t) = \boldsymbol{x}^i(t) - \bar{\boldsymbol{x}}_{io+}(t)$ be the error function.

$\dot{\bar{\boldsymbol{x}}}_{io+}(t) = 0$ because the center of the minimal circumcircle composed by all individuals in $\boldsymbol{A}_i^+$ is invariable between every two sample times due to the assumption.

Let $\boldsymbol{V}_i(t) = \dfrac{1}{2}\boldsymbol{e}_i^{\mathrm{T}}(t)\boldsymbol{e}_i(t)$ be the corresponding Lyapunov function.

$$\begin{aligned} \dot{v}_i(t) = (\dot{\boldsymbol{e}}_i(t))^{\mathrm{T}}\boldsymbol{e}_i(t) = (\dot{\boldsymbol{x}}^i(t) - \dot{\bar{\boldsymbol{x}}}_{io+}(t))^{\mathrm{T}}(\boldsymbol{x}^i(t) - \bar{\boldsymbol{x}}_{io+}(t)) = \\ (-a(\boldsymbol{x}^i(t) - \bar{\boldsymbol{x}}_{io+}(t)))^{\mathrm{T}}(\boldsymbol{x}^i(t) - \bar{\boldsymbol{x}}_{io+}(t)) = -a\|\boldsymbol{x}^i(t) - \bar{\boldsymbol{x}}_{io+}\|^2 \leqslant 0 \end{aligned}$$

Therefore, for any positive number η, if

$$\|\boldsymbol{e}^i(t)\| = \|\boldsymbol{x}^i(t) - \bar{\boldsymbol{x}}_{io+}(t)\| \geqslant \eta$$

then we have $\dot{V}_i(t) < 0$. □

Intuitively, once individual j gets to the vicinity of individual i, then the repulsive force will be in effect and the condition of Lemma 1 will not be satisfied anymore. To prove this we need to analyze the cooperative tracking motion of the members which are not necessarily free agents. When the repulsive force of individual j is in the same direction as the attractive force imposed by $\bar{\boldsymbol{x}}_{io+}$, stability of individual i is the same as the stability of free agents with the attractive constant a increased. When individual j is in the line between $\boldsymbol{x}^i$ and $\bar{\boldsymbol{x}}_{io+}$, stability analysis of individual i will be specified in Theorem 1. When individual j is in the line extended from $\boldsymbol{x}^i$ to $\bar{\boldsymbol{x}}_{io+}$, the tracking stability of the individual can refer to Theorem 1 in which the corresponding repulsive constant b decreases. When the repulsion force is not pointing to the same or opposite direction as the attractive force imposed by the object position, the component of the repulsion force along $\boldsymbol{e}^i(t)$ will be in effect associated with the attractive force. Individual i will move along the combined direction of attractive and repulsive forces except that individual j is replaced by other nearest neighbor (individual k) or disappear in the repulsive range of individual i. The latter individual will be a free individual and classification of the former individual k is similar to individual j.

Theorem 1. Consider individual i described by the model in (6). Individual j is in the line between $\boldsymbol{x}^i$ and $\bar{\boldsymbol{x}}_{io+}$. Then

1) when the attractive/repulsive function is given in (5) and (7), the individual will converge to a hyperball

$$B_\varphi(\boldsymbol{x}^i, \boldsymbol{x}^j) = \{\|\boldsymbol{x}^i - \boldsymbol{x}^j\| \leqslant \varphi, \forall i \in S\}$$

where $\varphi = (a\rho + br)/(a+b)$, individual j is the nearest neighbor in the repulsive range of individual i.

2) when the attractive/repulsive function is given in (7), (8), the individual will converge to a hyperball

$$B_\theta(\boldsymbol{x}^i, \bar{\boldsymbol{x}}_{io+}) = \{\|\boldsymbol{x}^i - \bar{\boldsymbol{x}}_{io+}\| \leqslant \theta, \forall i \in S\}$$

where $\theta = b/a$.

Proof.

1) From the definition of the swarm model, we know that $\dot{\bar{\boldsymbol{x}}}_{io+} = 0$.

Let $\boldsymbol{e}^i(t) = \boldsymbol{x}^i(t) - \bar{\boldsymbol{x}}_{io+}(t)$ and $V_i(t) = \dfrac{1}{2}\boldsymbol{e}_i^{\mathrm{T}}(t)\boldsymbol{e}_i(t)$.

For any two n-dimensional vectors: $\boldsymbol{x} = (x_1, x_2, \cdots, x_n)$ and $\boldsymbol{y} = (y_1, y_2, \cdots, y_n)$, we have $\boldsymbol{x}^{\mathrm{T}}\boldsymbol{y} = \sum\limits_{i=1}^{n} x_i y_i$, $\|\boldsymbol{x}\| = \sqrt{\sum\limits_{i=1}^{n} x_i^2}$ and $\|\boldsymbol{y}\| = \sqrt{\sum\limits_{i=1}^{n} y_i^2}$.

From the following inequation

$$(\|\boldsymbol{x}\|\|\boldsymbol{y}\|)^2 - (\boldsymbol{x}^{\mathrm{T}}\boldsymbol{y})^2 = \sum_{i=1}^{n} x_i^2 \sum_{i=1}^{n} y_i^2 - (\sum_{i=1}^{n} x_i y_i)^2 = \sum_{i=1}^{n-1}\sum_{j=i+1}^{n}(x_i y_j - x_j y_i)^2 \geqslant 0$$

we have $\boldsymbol{x}^{\mathrm{T}}\boldsymbol{y} \leqslant \|\boldsymbol{x}\|\|\boldsymbol{y}\|$, and

$$(\boldsymbol{x}^i(t) - \boldsymbol{x}^j(t))^{\mathrm{T}}\boldsymbol{e}_i(t) \leqslant \|\boldsymbol{x}^i(t) - \boldsymbol{x}^j(t)\|\|\boldsymbol{e}_i(t)\|$$

Therefore, we have

$$\begin{aligned}\dot{\boldsymbol{V}}_i(t) = (\dot{\boldsymbol{e}}_i(t))^{\mathrm{T}}\boldsymbol{e}_i(t) = -a\|\boldsymbol{e}_i(t)\|^2 + b(r - \|\boldsymbol{x}^i(t) - \boldsymbol{x}^j(t)\|)(\boldsymbol{x}^i(t) - \boldsymbol{x}^j(t))^{\mathrm{T}}\boldsymbol{e}_i(t)/(\|\boldsymbol{x}^i(t) -\\ \boldsymbol{x}^j(t)\| - \rho) \leqslant -a\|\boldsymbol{e}_i(t)\|^2 + b(r - \|\boldsymbol{x}^i(t) - \boldsymbol{x}^j(t)\|)\|\boldsymbol{x}^i(t) - \boldsymbol{x}^j(t)\|\|\boldsymbol{e}_i(t)\|/(\|\boldsymbol{x}^i(t) - \boldsymbol{x}^j(t)\| - \rho)\end{aligned}$$

From the precondition that individual j is in the line between $\boldsymbol{x}^i$ and $\bar{\boldsymbol{x}}_{io+}$, so we have $\|\boldsymbol{e}_i(t)\| = \|\boldsymbol{x}^i(t) - \bar{\boldsymbol{x}}_{io+}\| \geqslant \|\boldsymbol{x}^i(t) - \boldsymbol{x}^j(t)\|$. And then

$$\dot{\boldsymbol{V}}_i(t) \leqslant (-a + b(r - \|\boldsymbol{x}^i(t) - \boldsymbol{x}^j(t)\|)/(\|\boldsymbol{x}^i(t) - \boldsymbol{x}^j(t)\| - \rho))\|\boldsymbol{x}^i(t) - \boldsymbol{x}^j(t)\|\|\boldsymbol{e}_i(t)\|$$

Then if $\|\boldsymbol{x}^i(t) - \boldsymbol{x}^j(t)\| > (a\rho + br)/(a + b)$, we will have $\dot{V}_i(t) < 0$.

2) Let $\boldsymbol{e}^i(t) = \boldsymbol{x}^i(t) - \bar{\boldsymbol{x}}_{io+}(t)$ and $\boldsymbol{V}_i(t) = \frac{1}{2}\boldsymbol{e}_i^{\mathrm{T}}(t)\boldsymbol{e}_i(t)$

Therefore, we have

$$\begin{aligned}\dot{\boldsymbol{V}}_i(t) = (\dot{\boldsymbol{e}}_i(t))^{\mathrm{T}}\boldsymbol{e}_i(t) = (\dot{\boldsymbol{x}}^i(t) - \dot{\bar{\boldsymbol{x}}}_{io+}(t))^{\mathrm{T}}(\boldsymbol{x}^i(t) - \bar{\boldsymbol{x}}_{io+}(t)) =\\ -a(\boldsymbol{x}^i(t) - \bar{\boldsymbol{x}}_{io+}(t))^{\mathrm{T}}(\boldsymbol{x}^i(t) - \bar{\boldsymbol{x}}_{io+}(t)) + (g_r(\boldsymbol{x}^i, \boldsymbol{x}^j))^{\mathrm{T}}(\boldsymbol{x}^i(t) - \bar{\boldsymbol{x}}_{io+}(t))\end{aligned}$$

From the precondition that individual j is in the line between $\boldsymbol{x}^i$ and $\bar{\boldsymbol{x}}_{io+}$, $\dfrac{\boldsymbol{x}^i - \bar{\boldsymbol{x}}_{io+}}{\|\boldsymbol{x}^i - \bar{\boldsymbol{x}}_{io+}\|}$ and $\dfrac{\boldsymbol{x}^i(t) - \boldsymbol{x}^j(t)}{\|\boldsymbol{x}^i(t) - \boldsymbol{x}^j(t)\|}$ denote the same unit vector. So the repulsion force defined in (8) can be bounded as

$$g_r(\boldsymbol{x}^i(t), \boldsymbol{x}^j(t)) \leqslant b\frac{\boldsymbol{x}^i(t) - \boldsymbol{x}^j(t)}{\|\boldsymbol{x}^i(t) - \boldsymbol{x}^j(t)\|} = b\frac{(\boldsymbol{x}^i - \bar{\boldsymbol{x}}_{io+})}{\|\boldsymbol{x}^i - \bar{\boldsymbol{x}}_{io+}\|}$$

Thus we have

$$\dot{\boldsymbol{V}}_i(t) \leqslant -a\|\boldsymbol{e}^i(t)\|^2 + \left(b\frac{\boldsymbol{e}^i(t)}{\|\boldsymbol{e}^i(t)\|}\right)^{\mathrm{T}}\boldsymbol{e}^i(t) = -a\|\boldsymbol{e}^i(t)\|(\|\boldsymbol{e}^i(t)\| - \frac{b}{a})$$

Therefore, if

$$\|\boldsymbol{e}^i(t)\| > \frac{b}{a}$$

then we have $\dot{\boldsymbol{V}}_i(t) < 0$.

From the item $\theta = b/a$ or $(a\rho + br)/(a + b)$, we know that the repulsive constant b decides the dimension of the swarm whether we consider bounded repulsion force or not. The swarm enlarges when b increases and vice versa. □

4 Simulation examples

Simulation examples are provided in this section to illustrate the previous results. We chose an $n = 2$ dimensional space for ease of visualization of the results.

The result shown in Fig. 1 to Fig. 3 is for 100 individuals to track an global object position at (58cm, 58cm) through the region of 40cm×40cm in the space. For parameters we use $r = 1$cm, $\rho = 0.5$cm, $a = 0.2$, $b = 0.4$, $\varepsilon = 10$cm. From the result shown in Fig. 3 we can find that the swarm can track respective "leader" (local object position $\bar{\boldsymbol{x}}_{io+}$) and arrive at the object position while maintaining cohesive.

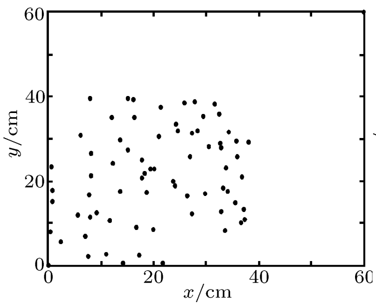

Fig. 1 Initial distribution of swarm ($m = 100$)

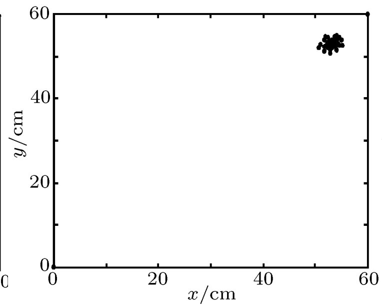

Fig. 2 Final distribution of swarm ($m = 100$)

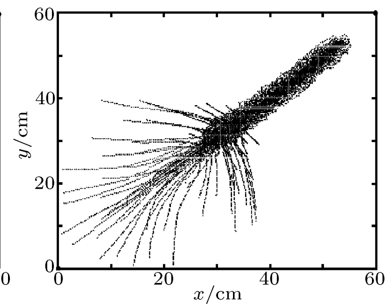

Fig. 3 Trace of individual′s tracking behavior ($m = 100$)

5 Conclusion

The cooperative tracking model and stability analysis of large-scale social foraging swarm have been studied in this paper. The scale of the swarm will not be limited because only local individual information is used for modeling. The swarm can maintain cohesiveness while responding to the stimulation in environment such as the position information of the global object because the mutual observability of the individuals can be held due to the characteristics of the minimal circumcircle method. Simulations illustrated high scalability and stability of the proposed model and motion equation.

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