

# Expectation-maximization (EM) Algorithm Based on IMM Filtering with Adaptive Noise Covariance<sup>1)</sup>

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**Abstract** A novel method under the interactive multiple model (IMM) filtering framework is presented in this paper, in which the expectation-maximization (EM) algorithm is used to identify the process noise covariance  $Q$  online. For the existing IMM filtering theory, the matrix  $Q$  is determined by means of design experience, but  $Q$  is actually changed with the state of the maneuvering target. Meanwhile it is severely influenced by the environment around the target, *i.e.*, it is a variable of time. Therefore, the experiential covariance  $Q$  can not represent the influence of state noise in the maneuvering process exactly. Firstly, it is assumed that the evolved state and the initial conditions of the system can be modeled by using Gaussian distribution, although the dynamic system is of a nonlinear measurement equation, and furthermore the EM algorithm based on IMM filtering with the  $Q$  identification online is proposed. Secondly, the truncated error analysis is performed. Finally, the Monte Carlo simulation results are given to show that the proposed algorithm outperforms the existing algorithms and the tracking precision for the maneuvering targets is improved efficiently.

**Key words** Interactive multiple model (IMM) filter, EM algorithm, noise covariance identification, online parameter estimation

## 1 Introduction

The adaptive estimation methods for maneuvering target have been proposed<sup>[1,2]</sup>, such as variable state dimension (VSD) filter, input estimation (IE) filter, multiple-layer process noise filter and multiple model (MM) mixed estimator. Currently, the mainstream method of the mixed estimation is the interactive multiple model (IMM) algorithm<sup>[1,3]</sup>, a great deal of research for IMM focuses on the following aspects. 1) Optimal design of model set and development of so-called variable structure model set (VSM). The estimation performance of the IMM algorithm depends on the model set seriously, and there is a trend to use more models in order to improve the estimation precision. Unfortunately, the research result shows that more models do not mean higher estimation performance, and too more models merely increase the computational load and decreased the precision<sup>[4]</sup>. Hence, many work on the variable structure IMM (VS-IMM)<sup>[5~7]</sup> should be completed. 2) Theoretic analysis on the IMM estimator. Although the IMM filtering has been employed in many cases successfully, the theoretic analysis on its performance and characteristic remains absent yet<sup>[8]</sup>. 3) Some artificial intelligence methods such as artificial neural network, fuzzy logic algorithm, *etc.* are combined with the IMM. 4) New methods to deal with the nonlinear problems such as the particle filtering are merged into the IMM. 5) The online estimation of the IMM parameters. For example, the model probability transition matrix is frequently used to update the model probability under the Bayesian framework, the online estimation of the model probability transition matrix outperforms that of prior determined<sup>[4,9,10]</sup>. Under the current IMM framework, the general processing of state noise and measurement noise is assumed to be zero-mean Gaussian with the independent covariance, which can be determined by means of experience design. But in practice, the measurement noise covariance can be pre-determined generally, and can keep constant with time. On the contrary, the state noise covariance is variable with the maneuvering target, meanwhile, it is heavily influenced by other factors such as circumstances of the target, therefore, the state noise covariance should be determined online.

In this paper, by using the expectation maximum (EM) algorithm<sup>[11~14]</sup>, and under the assumption that the state transition obeys the Gaussian distribution and the Markovian chain theory, the

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estimation equation about the state noise covariance and the measurement noise covariance are obtained. The estimation errors are analyzed, and furthermore, Monte-Carlo simulations are performed. The simulation results show that the IMM estimation precision (root mean square error-RMSE) with the adaptive measurement noise covariance is better than that with pre-determined one.

## 2 Nonlinear system model

In order to use the EM algorithm on parameters learning, the discrete-time multiple-model dynamic system can be formulated as

$$\mathbf{x}_k = F_r \mathbf{x}_{k-1} + D_r \mathbf{w}_{k-1} \quad (1)$$

$$\mathbf{z}_k = \mathbf{h}(\mathbf{x}) + \mathbf{v}_k, \quad r = 1, \dots, M, \quad k = 1, \dots, L \quad (2)$$

where  $M$  denotes the number of the models in the model set,  $L$  the total time step number of the simulation,  $\mathbf{x}_k \in \mathbb{R}^n$  the state vector at time step  $k$ , and  $\mathbf{w}_k$  the process noise vector. The state transition matrix  $F_r$  and noise input matrix  $D_r$  are assumed to be constant matrixes<sup>[1]</sup>,  $\mathbf{z}_k, \mathbf{v}_k \in \mathbb{R}^m$  the measurement vector and measurement noise in time step  $k$ , respectively, and  $\mathbf{z}_k$  can be observed in every time step  $k$ .  $\mathbf{h} : \mathbb{R}^n \rightarrow \mathbb{R}^m$  is a nonlinear measurement mapping<sup>[2]</sup>,  $\mathbf{w}_k$  and  $\mathbf{v}_k$  are the process and measurement noises, respectively, which are assumed to be independent of each other and obey zero-mean Gaussian distribution, *i.e.*,  $\mathbf{w}_k \sim \mathcal{N}(0, Q_0)$  and  $\mathbf{v}_k \sim \mathcal{N}(0, R)$ . Hence, the covariance of  $D_r \mathbf{w}_k$  is

$$Q = E[D_r \mathbf{w}_k (D_r \mathbf{w}_k)'] = D_r Q_0 D_r' \quad (3)$$

where  $(\cdot)'$  stands for the transposition of matrix. In the general situation,  $R$  can be determined a priori, but it is impossible for  $Q_0$ . Obviously, the online estimation of state noise covariance is reasonably necessary.

## 3 EM algorithm for the dynamic system model with nonlinear measurement equation

To derive the EM algorithm for nonlinear state space models, the likelihood of the complete data should be obtained firstly. It is assumed that the initial conditions, the evolution of the states and the likelihood of the measurements data can be represented by the Gaussian distribution. The unknown parameter set is denoted as  $\varphi = \{\boldsymbol{\mu}, \Pi, Q, R\}$ , where  $\boldsymbol{\mu}$  and  $\Pi$  are the mean and covariance of the initial states, respectively,  $Q$  and  $R$  are the covariance matrices of the state and measurement noises respectively, *i.e.*,

$$p(\mathbf{x}_0 | \varphi) = \frac{1}{(2\pi)^{n/2} |\Pi|^{1/2}} \exp\left[-\frac{1}{2}(\mathbf{x}_0 - \boldsymbol{\mu})' \Pi^{-1} (\mathbf{x}_0 - \boldsymbol{\mu})\right] \quad (4)$$

$$p(\mathbf{x}_k | \mathbf{x}_{k-1}, \varphi) = \frac{1}{(2\pi)^{n/2} |Q|^{1/2}} \exp\left[-\frac{1}{2}(\mathbf{x}_k - F_r \mathbf{x}_{k-1})' Q^{-1} (\mathbf{x}_k - F_r \mathbf{x}_{k-1})\right] \quad (5)$$

$$p(\mathbf{z}_k | \mathbf{x}_k, \varphi) = \frac{1}{(2\pi)^{m/2} |R|^{1/2}} \exp\left[-\frac{1}{2}(\mathbf{z}_k - \mathbf{h}(\mathbf{x}_k))' R^{-1} (\mathbf{z}_k - \mathbf{h}(\mathbf{x}_k))\right] \quad (6)$$

Under the assumptions of uncorrelated noise and state evolution according to the Markovian chain, the likelihood of the complete data can be represented by

$$p(\mathbf{x}^k, \mathbf{z}^k | \varphi) = p(\mathbf{x}_0 | \varphi) \prod_{s=1}^k p(\mathbf{x}_s | \mathbf{x}_{s-1}, \varphi) \prod_{s=1}^k p(\mathbf{z}_s | \mathbf{x}_s, \varphi)$$

where  $\mathbf{x}^k \triangleq \{\mathbf{x}_1, \dots, \mathbf{x}_k\}$ ,  $\mathbf{z}^k \triangleq \{\mathbf{z}_1, \dots, \mathbf{z}_k\}$ . Actually, the so-called "slide-window" with limited length  $N$  is used here to rewrite the above equation. At time step  $k$ , it is assumed that the measurement  $\mathbf{z}_k$  has been obtained, and the starting  $p(\mathbf{x}_{k-N} | \varphi)$  is the same as that in (4) by replacing  $\mathbf{x}_0$  with  $\mathbf{x}_{k-N}$ . Define  $\mathbf{x}_{k-N:k} \triangleq \{\mathbf{x}_{k-N}, \dots, \mathbf{x}_k\}$ ,  $\mathbf{z}_{k-N+1:k} \triangleq \{\mathbf{z}_{k-N+1}, \dots, \mathbf{z}_k\}$ . So the likelihood of the complete data is obtained as

$$p(\mathbf{x}_{k-N:k}, \mathbf{z}_{k-N+1:k} | \varphi, \mathbf{z}^k) = p(\mathbf{x}_{k-N} | \varphi) \prod_{s=k-N+1}^k p(\mathbf{x}_s | \mathbf{x}_{s-1}, \varphi) \prod_{s=k-N+1}^k p(\mathbf{z}_s | \mathbf{x}_s, \varphi) \quad (7)$$

Hence, by making logarithm operation on both sides of (7), and then substituting the (4)~(6) into it, the log-likelihood of the complete data can be derived as

$$\begin{aligned} \ln p(\mathbf{x}_{k-N:k}, \mathbf{z}_{k-N+1:k} | \varphi, \mathbf{z}^k) = & - \sum_{s=k-N+1}^k \left[ \frac{1}{2} (\mathbf{z}_s - \mathbf{h}(\mathbf{x}_s))' \mathbf{R}^{-1} (\mathbf{z}_s - \mathbf{h}(\mathbf{x}_s)) \right] - \\ & \frac{N}{2} \ln |R| - \sum_{s=k-N+1}^k \left[ \frac{1}{2} (\mathbf{x}_s - F_r \mathbf{x}_{s-1})' Q^{-1} (\mathbf{x}_s - F_r \mathbf{x}_{s-1}) \right] - \frac{N}{2} \ln |Q| - \\ & \left[ \frac{1}{2} (\mathbf{x}_{k-N} - \boldsymbol{\mu})' \Pi^{-1} (\mathbf{x}_{k-N} - \boldsymbol{\mu}) \right] - \frac{1}{2} \ln |\Pi| - \frac{(N+1)n + Nm}{2} \ln(2\pi) \end{aligned} \quad (8)$$

### 3.1 The expectation of the log-likelihood

Taking the expectations on both sides of the log-likelihood for the complete data leads to

$$\begin{aligned} E[\ln p(\mathbf{x}_{k-N:k}, \mathbf{z}_{k-N+1:k} | \varphi, \mathbf{z}^k)] = & - \frac{N}{2} \ln |R| - \frac{N}{2} \ln |Q| - \frac{1}{2} \ln |\Pi| - \frac{(N+1)n + Nm}{2} \ln(2\pi) - \\ & \sum_{s=k-N+1}^k \frac{1}{2} E[(\mathbf{z}'_s R^{-1} \mathbf{z}_s - \mathbf{z}'_s R^{-1} \mathbf{h}(\mathbf{x}_s) - \mathbf{h}(\mathbf{x}_s)' R^{-1} \mathbf{z}_s + \mathbf{h}(\mathbf{x}_s)' R^{-1} \mathbf{h}(\mathbf{x}_s)) | \varphi, \mathbf{z}^k] - \\ & \sum_{s=k-N+1}^k \frac{1}{2} E[(\mathbf{x}'_s Q^{-1} \mathbf{x}_s - \mathbf{x}'_s Q^{-1} F_r \mathbf{x}_{s-1} - \mathbf{x}'_{s-1} F_r' Q^{-1} \mathbf{x}_s + \mathbf{x}'_{s-1} F_r' Q^{-1} F_r \mathbf{x}_{s-1}) | \varphi, \mathbf{z}^k] - \\ & \frac{1}{2} E[(\mathbf{x}'_{k-N} \Pi^{-1} \mathbf{x}_{k-N} - \mathbf{x}'_{k-N} \Pi^{-1} \boldsymbol{\mu} - \boldsymbol{\mu}' \Pi^{-1} \mathbf{x}_{k-N} + \boldsymbol{\mu}' \Pi^{-1} \boldsymbol{\mu}) | \varphi, \mathbf{z}^k] \end{aligned} \quad (9)$$

In order to compute the expectation of the measurement mapping  $\mathbf{h}(\mathbf{x}_k)$ , it is assumed that

$$E[\mathbf{x}_k | \varphi, \mathbf{z}^{k-1}] = \hat{\mathbf{x}}_{k|N} \quad (10)$$

The Taylor series extension of the nonlinear measurement mapping  $\mathbf{h}(\mathbf{x}_k)$  at the state point  $\hat{\mathbf{x}}_{k|N}$  can be obtained as

$$\mathbf{h}(\mathbf{x}_k) \approx \mathbf{h}(\hat{\mathbf{x}}_{k|N}) + G_{k|N} \tilde{\mathbf{x}}_{k|N} + \frac{1}{2} \sum_{i=1}^m \mathbf{e}_i \tilde{\mathbf{x}}'_{k|N} S_{k|N,i} \tilde{\mathbf{x}}_{k|N} \quad (11)$$

where  $\mathbf{e}_i \in \mathbb{R}^m$  denotes the  $i$ th unit normal vector,  $\tilde{\mathbf{x}}_{k|N} \triangleq \mathbf{x}_k - \hat{\mathbf{x}}_{k|N}$  is the prediction error (also called residual in some papers), and  $G_{k|N} \triangleq \left. \frac{\partial \mathbf{h}(\mathbf{x}_k)}{\partial \mathbf{x}_k} \right|_{\mathbf{x}_k = \hat{\mathbf{x}}_{k|N}} = \{\partial h_i / \partial x_j\}_{m \times n}$  is the gradient, and  $S_{k|N,i} \triangleq \left. \frac{\partial}{\partial \mathbf{x}_k} \left[ \frac{\partial h_i(\mathbf{x}_k)}{\partial \mathbf{x}_k} \right]' \right|_{\mathbf{x}_k = \hat{\mathbf{x}}_{k|N}}$  is the Jacobian matrix of the  $i$ th element  $h_i(\mathbf{x}_k)$  of  $\mathbf{h}(\mathbf{x}_k)$ . Taking expectation on both sides of the above equation yields

$$\begin{aligned} E[\mathbf{h}(\mathbf{x}_k) | \varphi, \mathbf{z}^{k-1}] & \triangleq \mathbf{h}(\mathbf{x}_k) - E[\mathbf{h}(\mathbf{x}_k) | \varphi, \mathbf{z}^{k-1}] \approx \\ & G_{k|N} \tilde{\mathbf{x}}_{k|N} - \frac{1}{2} \sum_{i=1}^m \mathbf{e}_i \text{tr}\{S_{k|N,i} P_{k|N}\} + \frac{1}{2} \sum_{i=1}^m \mathbf{e}_i \tilde{\mathbf{x}}_{k|N} S_{k|N,i} \tilde{\mathbf{x}}_{k|N} \end{aligned} \quad (12)$$

where  $\text{tr}(\cdot)$  denotes the matrix trace operator,  $P_{k|N} \triangleq E[\tilde{\mathbf{x}}_{k|N} \tilde{\mathbf{x}}'_{k|N} | \varphi, \mathbf{z}^{k-1}]$  the covariance matrix of state prediction error. Define

$$\begin{aligned} \tilde{\mathbf{h}}(\mathbf{x}_k) & \triangleq \mathbf{h}(\mathbf{x}_k) - E[\mathbf{h}(\mathbf{x}_k) | \varphi, \mathbf{z}^{k-1}] \approx G_{k|N} \tilde{\mathbf{x}}_{k|N} + \frac{1}{2} \sum_{i=1}^m \mathbf{e}_i \tilde{\mathbf{x}}'_{k|N} S_{k|N,i} \tilde{\mathbf{x}}_{k|N} - \\ & \frac{1}{2} \sum_{i=1}^m \mathbf{e}_i \text{tr}\{S_{k|N,i} P_{k|N}\} \end{aligned}$$

and  $E[\tilde{\mathbf{h}}(\mathbf{x}_k) | \varphi, \mathbf{z}^{k-1}] = 0$  is obvious. It is assumed that the conditional probability density function of  $\mathbf{x}_k$  is almost symmetry about  $\hat{\mathbf{x}}_{k|N}$ , and hence the central-moments of more than the third order case can be approximated by zero. Consequently, it can be written as

$$\text{cov}[\tilde{\mathbf{h}}(\mathbf{x}_k), \tilde{\mathbf{h}}(\mathbf{x}_k) | \varphi, \mathbf{z}^{k-1}] = E[\tilde{\mathbf{h}}(\mathbf{x}_k) \tilde{\mathbf{h}}'(\mathbf{x}_k) | \varphi, \mathbf{z}^{k-1}] \approx G_{k|N} P_{k|N} G'_{k|N} -$$

$$\begin{aligned}
& \frac{1}{4} \sum_{i=1}^m \sum_{j=1}^m \mathbf{e}_i \mathbf{e}'_j E[\tilde{\mathbf{x}}'_{k|N} S_{k|N,i} \tilde{\mathbf{x}}_{k|N} | \varphi, \mathbf{z}^{k-1}] \text{tr}\{S_{k|N,j} P_{k|N}\} - \\
& \frac{1}{4} \sum_{i=1}^m \sum_{j=1}^m \mathbf{e}_i \mathbf{e}'_j \text{tr}\{S_{k|N,i} P_{k|N}\} E[\tilde{\mathbf{x}}'_{k|N} S'_{k|N,j} \tilde{\mathbf{x}}_{k|N} | \varphi, \mathbf{z}^{k-1}] + \\
& \frac{1}{4} \sum_{i=1}^m \sum_{j=1}^m \mathbf{e}_i \mathbf{e}'_j \text{tr}\{S_{k|N,i} P_{k|N}\} \text{tr}\{S_{k|N,j} P_{k|N}\} = \\
& G_{k|N} P_{k|N} G'_{k|N} - \frac{1}{4} \sum_{i=1}^m \sum_{j=1}^m \mathbf{e}_i \mathbf{e}'_j \text{tr}\{S_{k|N,i} P_{k|N}\} \text{tr}\{S_{k|N,j} P_{k|N}\} \quad (13)
\end{aligned}$$

and therefore,

$$\begin{aligned}
E[\mathbf{h}(\mathbf{x}_k) \mathbf{h}'(\mathbf{x}_k) | \varphi, \mathbf{z}^{k-1}] &= G_{k|N} P_{k|N} G'_{k|N} + \mathbf{h}(\hat{\mathbf{x}}_{k|N}) \mathbf{h}'(\hat{\mathbf{x}}_{k|N}) + \\
\mathbf{h}'(\hat{\mathbf{x}}_{k|N}) \sum_{i=1}^m \mathbf{e}_i \text{tr}\{S_{k|N,i} P_{k|N}\} &- \frac{1}{4} \sum_{i=1}^m \sum_{j=1}^m \mathbf{e}_i \mathbf{e}'_j \text{tr}\{S_{k|N,i} P_{k|N}\} \text{tr}\{S_{k|N,j} P_{k|N}\} \quad (14)
\end{aligned}$$

Similarly,

$$E[\mathbf{x}_k \mathbf{x}'_k | \varphi, \mathbf{z}^{k-1}] = P_{k|N} + \hat{\mathbf{x}}_{k|N} \hat{\mathbf{x}}'_{k|N} \quad (15)$$

$$E[\mathbf{x}_{k-1} \mathbf{x}'_{k-1} | \varphi, \mathbf{z}^{k-1}] = P_{k-1|N} + \hat{\mathbf{x}}_{k-1|N} \hat{\mathbf{x}}'_{k-1|N} \quad (16)$$

$$E[\mathbf{x}_k \mathbf{x}'_{k-1} | \varphi, \mathbf{z}^{k-1}] = P_{k,k-1|N} + \hat{\mathbf{x}}_{k|N} \hat{\mathbf{x}}'_{k-1|N} \quad (17)$$

Then by substituting (14)~(17) into (9), using the fact that the trace and expectation are linear operators, and  $F_r$  not being the symmetrical matrix in general situation, the expected log-likelihood can be rewritten as

$$\begin{aligned}
E[p(\mathbf{x}_{k-N:k}, \mathbf{z}_{k-N+1:k} | \varphi, \mathbf{z}^k)] &= -\frac{N}{2} \ln|R| - \frac{N}{2} \ln|Q| - \frac{1}{2} \ln|\Pi| - \frac{(N+1)n + Nm}{2} \ln(2\pi) - \\
& \sum_{s=k-N+1}^k \frac{1}{2} \text{tr}[R^{-1} \left( \begin{array}{l} \mathbf{z}_s \mathbf{z}'_s - \mathbf{h}(\hat{\mathbf{x}}_{s|N}) \mathbf{z}'_s - \mathbf{z}_s \mathbf{h}'(\hat{\mathbf{x}}_{s|N}) + G_{s|N} P_{s|N} G'_{s|N} + \mathbf{h}(\hat{\mathbf{x}}_{s|N}) \mathbf{h}'(\hat{\mathbf{x}}_{s|N}) + \\ \mathbf{h}'(\hat{\mathbf{x}}_{s|N}) \sum_{i=1}^m \mathbf{e}_i \text{tr}\{S_{s|N,i} P_{s|N}\} - \frac{1}{4} \sum_{i=1}^m \sum_{j=1}^m \mathbf{e}_i \mathbf{e}'_j \text{tr}\{S_{s|N,i} P_{s|N}\} \text{tr}\{S_{s|N,j} P_{s|N}\} \end{array} \right)] - \\
& \sum_{s=k-N+1}^k \frac{1}{2} \text{tr}\{Q^{-1} [(\hat{\mathbf{x}}_{s|N} \hat{\mathbf{x}}'_{s|N} + P_{s|N})' - 2F_r(\hat{\mathbf{x}}_{s|N} \hat{\mathbf{x}}'_{s-1|N} + P_{s,s-1|N})' + F_r(\hat{\mathbf{x}}_{s-1|N} \hat{\mathbf{x}}'_{s-1|N} + \\
& P_{s-1|N}) F']\} - \frac{1}{2} \text{tr}\{\Pi^{-1} (\hat{\mathbf{x}}_{k-N|N} \hat{\mathbf{x}}'_{k-N|N} + P_{k-N|N} - \hat{\mathbf{x}}_{k-N|N} \mathbf{u}' - \boldsymbol{\mu} \hat{\mathbf{x}}'_{k-N|N} + \boldsymbol{\mu} \boldsymbol{\mu}')\} \quad (18)
\end{aligned}$$

Using the following abbreviations

$$\left\{ \begin{array}{l} \boldsymbol{\Theta} \triangleq \mathbf{h}'(\hat{\mathbf{x}}_{s|N}) \sum_{i=1}^m \mathbf{e}_i \text{tr}\{S_{s|N,i} P_{s|N}\} - \frac{1}{4} \sum_{i=1}^m \sum_{j=1}^m \mathbf{e}_i \mathbf{e}'_j \text{tr}\{S_{s|N,i} P_{s|N}\} \text{tr}\{S_{s|N,j} P_{s|N}\} \\ \boldsymbol{\Psi} \triangleq \sum_{s=k-N+1}^k (\hat{\mathbf{x}}_{s|N} \hat{\mathbf{x}}'_{s|N} + P_{s|N}), \quad \boldsymbol{\Omega} \triangleq \sum_{s=k-N+1}^k (\hat{\mathbf{x}}_{s|N} \hat{\mathbf{x}}'_{s-1|N} + P_{s,s-1|N}) \\ \boldsymbol{\Phi} \triangleq \sum_{s=k-N+1}^k (\hat{\mathbf{x}}_{s-1|N} \hat{\mathbf{x}}'_{s-1|N} + P_{s-1|N}) \end{array} \right. \quad (19)$$

obviously  $\boldsymbol{\Psi}$ ,  $\boldsymbol{\Omega}$  and  $\boldsymbol{\Phi}$  are the symmetrical matrixes, then the final expression can be obtained as

$$\begin{aligned}
EL_k \triangleq E[\ln p(\mathbf{x}_{k-N:k}, \mathbf{z}_{k-N+1:k} | \varphi, \mathbf{z}^k)] &= -\frac{N}{2} \ln|R| - \frac{N}{2} \ln|Q| - \frac{1}{2} \ln|\Pi| - \frac{(N+1)n + Nm}{2} \ln(2\pi) - \\
& \sum_{s=k-N+1}^k \frac{1}{2} \text{tr}\{R^{-1} [(\mathbf{z}_s - \mathbf{h}(\hat{\mathbf{x}}_{s|N})) (\mathbf{z}_s - \mathbf{h}(\hat{\mathbf{x}}_{s|N}))' + G_{s|N} P_{s|N} G'_{s|N} + \boldsymbol{\Theta}]\} - \quad (20)
\end{aligned}$$

$$\frac{1}{2}\text{tr}\{Q^{-1}(\Psi - 2F_r\Omega' + F_r\Phi'F_r')\} - \frac{1}{2}\text{tr}\{\Pi^{-1}[(\hat{\mathbf{x}}_{k-N|N} - \boldsymbol{\mu})(\hat{\mathbf{x}}_{k-N|N} - \boldsymbol{\mu})' + P_{k-N|N}]\}$$

### 3.2 Maximizing the expected log-likelihood by differentiation

In order to maximize the expected log-likelihood expression with respect to the unknown parameter set  $\varphi$ , the derivative computation of matrix differentiation with respect to each parameter individually should be performed.

Differentiating the expected log-likelihood with respect to the initial state  $\boldsymbol{\mu}$  yields

$$\begin{aligned} \frac{\partial}{\partial \boldsymbol{\mu}} E[\ln p(\mathbf{x}_{k-N:k}, \mathbf{z}_{k-N+1:k} | \varphi, \mathbf{z}^k)] &= \frac{\partial}{\partial \boldsymbol{\mu}} \left[ -\frac{1}{2} \text{tr}\{\Pi^{-1}(\hat{\mathbf{x}}_{k-N|N} \hat{\mathbf{x}}'_{k-N|N} + P_{k-N|N} \right. \\ &\quad \left. - \hat{\mathbf{x}}_{k-N|N} \boldsymbol{\mu}' - \boldsymbol{\mu} \hat{\mathbf{x}}'_{k-N|N} + \boldsymbol{\mu} \boldsymbol{\mu}')\} \right] = -\frac{1}{2} \Pi^{-1}(-2\hat{\mathbf{x}}_{k-N|N} + 2\boldsymbol{\mu}) \end{aligned} \quad (21)$$

Let this result be zero to yield the value of  $\boldsymbol{\mu}$  that maximizes the log-likelihood approximatively. So the estimation value of the initial state should be

$$\boldsymbol{\mu} = \hat{\mathbf{x}}_{k-N|N} \quad (22)$$

Differentiating the expected log-likelihood with respect to the inverse of the initial covariance  $\Pi^{-1}$  gives

$$\begin{aligned} \frac{\partial}{\partial \Pi^{-1}} E[\ln p(\mathbf{x}_{k-N:k}, \mathbf{z}_{k-N+1:k} | \varphi, \mathbf{z}^k)] &= \frac{\partial}{\partial \Pi^{-1}} \left[ -\frac{1}{2} \text{tr}\{\Pi\} - \frac{1}{2} \text{tr}\{\Pi^{-1}((\hat{\mathbf{x}}_{k-N|N} - \boldsymbol{\mu})(\hat{\mathbf{x}}_{k-N|N} - \boldsymbol{\mu})' \right. \\ &\quad \left. + P_{k-N|N})\} \right] = \frac{1}{2} \Pi - \frac{1}{2} [(\hat{\mathbf{x}}_{k-N|N} - \boldsymbol{\mu})(\hat{\mathbf{x}}_{k-N|N} - \boldsymbol{\mu})' + P_{k-N|N}] \end{aligned} \quad (23)$$

Similarly, by equating the above expression to zero, the initial covariance should be obtained as

$$\Pi = P_{k-N|N} \quad (24)$$

Differentiating the expected log-likelihood with respect to  $R^{-1}$  gives that

$$\begin{aligned} \frac{\partial}{\partial R^{-1}} E[\ln p(\mathbf{x}_{k-N:k}, \mathbf{z}_{k-N+1:k} | \varphi, \mathbf{z}^k)] &= \frac{\partial}{\partial R^{-1}} \left[ \frac{N}{2} \ln |R^{-1}| - \right. \\ &\quad \left. \sum_{s=k-N+1}^k \frac{1}{2} \text{tr}\{R^{-1}((\mathbf{z}_s - \mathbf{h}(\hat{\mathbf{x}}_{s|N}))(\mathbf{z}_s - \mathbf{h}(\hat{\mathbf{x}}_{s|N}))' + G_{s|N}P_{s|N}G'_{s|N} + \Theta)\} \right] = \\ &\quad \frac{N}{2} R - \sum_{s=k-N+1}^k \frac{1}{2} [(\mathbf{z}_s - \mathbf{h}(\hat{\mathbf{x}}_{s|N}))(\mathbf{z}_s - \mathbf{h}(\hat{\mathbf{x}}_{s|N}))' + G_{s|N}P_{s|N}G'_{s|N} + \Theta] \end{aligned} \quad (25)$$

Equating the above expression to zero yields the value of  $R$  that maximizes the log-likelihood, hence, the measurement noise covariance should be updated by

$$R = \frac{1}{N} \sum_{s=k-N+1}^k [(\mathbf{z}_s - \mathbf{h}(\hat{\mathbf{x}}_{s|N}))(\mathbf{z}_s - \mathbf{h}(\hat{\mathbf{x}}_{s|N}))' + G_{s|N}P_{s|N}G'_{s|N} + \Theta] \quad (26)$$

Similarly, differentiating the expected log-likelihood with respect to the inverse of the  $Q$  yields

$$\begin{aligned} \frac{\partial}{\partial Q^{-1}} E[\ln p(\mathbf{x}_{k-N:k}, \mathbf{z}_{k-N+1:k} | \varphi, \mathbf{z}^k)] &= \frac{\partial}{\partial Q^{-1}} \left[ \frac{N}{2} \ln |Q^{-1}| - \frac{1}{2} \text{tr}\{Q^{-1}(\Psi - 2F_r\Omega' + F_r\Phi'F_r')\} \right] = \\ &\quad \frac{N}{2} Q - \frac{1}{2} (\Psi - 2F_r\Omega' + F_r\Phi'F_r') \end{aligned} \quad (27)$$

Equating the above expression to zero, and based on the fact that the  $\Omega$  and  $\Phi$  in (19) are symmetrical matrixes, the evolved state noise covariance should be updated by  $Q = (\Psi - 2F_r\Omega' + F_r\Phi'F_r')/N$ . Furthermore, considering (3), it follows that

$$Q_0 = \frac{1}{N} D_r^+ (\Psi - 2F_r\Omega' + F_r\Phi'F_r') (D_r^+)^+ \quad (28)$$

where  $(\cdot)^+$  denotes the pseudo-inverse of matrix. The noise input-matrix  $D_r$  is not square-matrix generally.

### 3.3 The IMM filtering with adaptive noise covariance

Firstly, at the discrete time  $k = 0$ , the parameters should be initialized: given the guess value  $\varphi$ , including state covariance  $\{Q_0^{(r)}\}_{r=1}^M$  with respect to the model set  $\{F_r\}_{r=1}^M$ ,  $M$  denotes the model number. Measurement noise covariance  $R_0$ , the mean and covariance of the initial state  $\boldsymbol{\mu}$  and  $\Pi$ , the relative error upper boundary  $\varepsilon$  and the iterative number  $C$ .

Secondly, the  $E$  and  $M$  steps for the one cycle of the IMM (the  $k$ th step filtering recycle,  $N \leq k \leq L$ ,  $L$  denotes numbers of the simulation time step) can be prescribed as follows<sup>[1,3]</sup>:

1) Model-conditional re-initialization (models interacting stage) (for model  $i$ ,  $i = 1, 2, \dots, M$ )

Calculating predicted model probability  $\mu_{k|N}^{(i)}$ , mixing model probability (mixing weight)  $\mu_{k-1}^{(j|i)}$ , mixing state estimation  $\hat{\boldsymbol{x}}_{k-1|N}^{(i)}$  and mixing state covariance  $P_{k-1|N}^{(i)}$ .

2) The EM based iterative process for noise covariance (for iteration  $j$ ,  $j = 1, 2, \dots, C$ )

E-step: determine the expected values  $\hat{\boldsymbol{x}}_{j,k|N}^{(i)}$ ,  $P_{j,k|N}^{(i)}$ ,  $P_{j,k-1|N}^{(i)}$ , and  $P_{j,k,k-1|N}^{(i)}$ , given the last iteration's estimation  $R_{j-1,k}^{(i)}$  and  $Q_{j-1,k}^{(i)}$ , using the extend Kalman filtering algorithm.

M-step: determine the current iteration  $R_{j,k}^{(i)}$  and  $Q_{j,k}^{(i)}$ , using equations described in (26), (28).

Expected-log-likelihood computation step: calculating expected log-likelihood  $EL_{j,k}$ , and judging

$$|EL_{j,k} - EL_{j-1,k}|/|EL_{j,k}| \leq \varepsilon$$

if true, the iteration of  $j$  should be terminated. Finally, using the  $R_{j,k}^{(i)}$  and  $Q_{j,k}^{(i)}$  as the approximations of the true  $R_k^{(i)}$  and  $Q_k^{(i)}$ , respectively.

3) Model-conditional filtering (for model  $i$ , for  $i = 1, 2, \dots, M$ )

Calculating predicted state  $\hat{\boldsymbol{x}}_{k|N}^{(i)}$  and predicted error covariance  $P_{k|N}^{(i)}$ , measurement predicted error covariance  $T_k^{(i)}$ , filtering gain  $K_k^{(i)}$ , updated state  $\hat{\boldsymbol{x}}_{k|k}^{(i)}$  and updated state covariance  $P_{k|k}^{(i)}$ .

4) Model probability update (for model  $i$ ,  $i = 1, 2, \dots, M$ )

Calculating the model likelihood  $L_k^{(i)} \triangleq p[\tilde{\boldsymbol{z}}_{k|N}^{(i)} | m_k^{(i)}, \boldsymbol{z}^{k-1}]$ ,  $m_k^{(i)}$  denotes the  $i$ th model at time step  $k$ , model probability  $\mu_k^{(i)}$ .

5) Estimation fusion

Calculating the overall state estimation  $\hat{\boldsymbol{x}}_{k|k}$  and overall state residual covariance  $P_{k|k}$ .

## 4 Error analysis

From section 3, we easily know that the main error stems from (11). Hence, we can quantify the influence of the truncated error as follows.

The Taylor series expansion of the nonlinear measurement function  $\boldsymbol{h}(\boldsymbol{x}_k)$  at  $\hat{\boldsymbol{x}}_{k|N}$  is expressed by

$$\boldsymbol{h}(\boldsymbol{x}_k) = \boldsymbol{h}(\hat{\boldsymbol{x}}_{k|N}) + \boldsymbol{G}_{k|N} \tilde{\boldsymbol{x}}_{k|N} + \frac{1}{2} \sum_{i=1}^m \boldsymbol{e}_i \tilde{\boldsymbol{x}}_{k|N}' S_{k|N,i} \tilde{\boldsymbol{x}}_{k|N} + \sum_{i=3}^{\infty} C_{k|N,i} \quad (29)$$

where  $C_{k|N,i}$  denotes the  $i$ th-order term about the state prediction error  $\tilde{\boldsymbol{x}}_{k|N}$ . The meaning of the  $\boldsymbol{G}_{k|N}$ ,  $S_{k|N,i}$  and  $\boldsymbol{e}_i$  is the same as that in (11). Furthermore,

$$\begin{aligned} \tilde{\boldsymbol{h}}(\boldsymbol{x}_k) &\triangleq \boldsymbol{h}(\boldsymbol{x}_k) - E[\boldsymbol{h}(\boldsymbol{x}_k) | \varphi, \boldsymbol{z}^{k-1}] = \boldsymbol{G}_{k|N} \tilde{\boldsymbol{x}}_{k|N} + \frac{1}{2} \sum_{i=1}^m \boldsymbol{e}_i \tilde{\boldsymbol{x}}_{k|N}' S_{k|N,i} \tilde{\boldsymbol{x}}_{k|N} \\ &- \frac{1}{2} \sum_{i=1}^m \boldsymbol{e}_i \text{tr}\{S_{k|N,i} P_{k|N}\} + \sum_{i=3}^{\infty} C_{k|N,i} - \sum_{i=3}^{\infty} E[C_{k|N,i} | \varphi, \boldsymbol{z}^{k-1}] \end{aligned}$$

again let  $E[\tilde{\boldsymbol{h}}(\boldsymbol{x}_k) | \varphi, \boldsymbol{z}^{k-1}] = 0$ . From the supposition in Section 3.1 that the conditional probability density function of  $\boldsymbol{x}_k$  is almost symmetry about its conditional expectation, hence, the odd exponential terms of the third-order as well as those above the third-order central-moment can be approximated by

zero; and the even exponential terms those of above the fourth-order central-moment are so small that can be ignored. So from (29), (13) we can get

$$\begin{aligned} \text{cov}[\tilde{\mathbf{h}}(\mathbf{x}_k), \tilde{\mathbf{h}}(\mathbf{x}_k)|\varphi, \mathbf{z}^{k-1}] &\approx G_{k|N}P_{k|N}G'_{k|N} - \\ &\frac{1}{4} \sum_{i=1}^m \sum_{j=1}^m \mathbf{e}_i \mathbf{e}'_j \text{tr}\{S_{k|k-i,i}P_{k|N}\} \text{tr}\{S_{k|N,j}P_{k|N}\} + \Xi_k \end{aligned} \quad (30)$$

where

$$\begin{aligned} \Xi_k &\triangleq \frac{1}{4} \sum_{i=1}^m \sum_{j=1}^m \mathbf{e}_i E[\tilde{\mathbf{x}}'_{k|N} S_{k|N,i} \tilde{\mathbf{x}}_{k|N} \tilde{\mathbf{x}}'_{k|N} S'_{k|N,j} \tilde{\mathbf{x}}_{k|N} | \varphi, \mathbf{z}^{k-1}] \mathbf{e}'_j - \\ &\sum_{j=1}^{\infty} E[C_{k|N,2j+2} | \varphi, \mathbf{z}^{k-1}] \sum_{i=1}^m \mathbf{e}_i \text{tr}\{S_{k|N,i}P_{k|N}\} \end{aligned}$$

From (29), (30) and (14), we have

$$\begin{aligned} E[\mathbf{h}(\mathbf{x}_k)\mathbf{h}'(\mathbf{x}_k)|\varphi, \mathbf{z}^{k-1}] &= \text{cov}[\tilde{\mathbf{h}}(\mathbf{x}_k), \tilde{\mathbf{h}}(\mathbf{x}_k)|\varphi, \mathbf{z}^{k-1}] + 2E[\mathbf{h}(\mathbf{x}_k)\mathbf{h}'(\hat{\mathbf{x}}_{k|N})|\varphi, \mathbf{z}^{k-1}] - \\ E[\mathbf{h}(\hat{\mathbf{x}}_{k|N})\mathbf{h}'(\hat{\mathbf{x}}_{k|N})|\varphi, \mathbf{z}^{k-1}] &\approx G_{k|N}P_{k|N}G'_{k|N} + \mathbf{h}(\hat{\mathbf{x}}_{k|N})\mathbf{h}'(\hat{\mathbf{x}}_{k|N}) + \\ \mathbf{h}'(\hat{\mathbf{x}}_{k|N}) \sum_{i=1}^m \mathbf{e}_i \text{tr}\{S_{k|N,i}P_{k|N}\} - \frac{1}{4} \sum_{i=1}^m \sum_{j=1}^m \mathbf{e}_i \mathbf{e}'_j \text{tr}\{S_{k|N,i}P_{k|N}\} \text{tr}\{S_{k|N,j}P_{k|N}\} &+ \Xi_k \end{aligned} \quad (31)$$

Consequently, from (9), (29), (31), we can get the error of the expected log-likelihood  $\Delta_{EL}$  (see (20)) as

$$\begin{aligned} \Delta_{EL} &= \sum_{s=k-N+1}^k \frac{1}{2} \text{tr}\{R^{-1}((\mathbf{z}_s - \mathbf{h}(\hat{\mathbf{x}}_{s|N}))(\mathbf{z}_s - \mathbf{h}(\hat{\mathbf{x}}_{s|N}))' + G_{s|N}P_{s|N}G'_{s|N} + \Theta + \Xi_s)\} - \\ &\sum_{s=k-N+1}^k \frac{1}{2} \text{tr}\{R^{-1}((\mathbf{z}_s - \mathbf{h}(\hat{\mathbf{x}}_{s|N}))(\mathbf{z}_s - \mathbf{h}(\hat{\mathbf{x}}_{s|N}))' + G_{s|N}P_{s|N}G'_{s|N} + \Theta)\} \end{aligned} \quad (32)$$

Finally, from (32), (21), (23), (25), and (27) the estimated errors of  $\mathbf{u}$ ,  $\Pi$ ,  $Q$  and  $R$  at time step  $k$  are

$$\frac{\partial \Delta_{EL}}{\partial \mathbf{u}} = \frac{\partial \Delta_{EL}}{\partial \Pi^{-1}} = \frac{\partial \Delta_{EL}}{\partial Q^{-1}} = 0, \quad \frac{\partial \Delta_{EL}}{\partial R^{-1}} \approx \frac{1}{2} \sum_{s=k-N+1}^k \Xi_s$$

That is,

$$\Delta_{k,\mu} = \Delta_{k,\Pi} = \Delta_{k,Q} = 0, \quad \Delta_{k,R} = \frac{2}{N} \frac{\partial \Delta_{EL}}{\partial R^{-1}} = \frac{1}{N} \sum_{s=k-N+1}^k \Xi_s \quad (33)$$

So we can see from (33) that the sum of the truncated error  $\sum_{i=3}^{\infty} C_{k|k-1,i}$  has no influence on the initial state  $\boldsymbol{\mu}$ , the initial state covariance  $\Pi$ , and the state noise covariance  $Q$ , but affects the measurement noise covariance  $R$  quantitated by  $\Delta_{k,R}$ . Obviously, the  $\Delta_{k,R}$  has relations with the  $\hat{\mathbf{x}}_{k|N}$ ,  $G_{k|N}$  and  $S_{k|N,i}$  merely, so the effective method to decrease the  $\Delta_{k,R}$  is to improve the state estimation  $\hat{\mathbf{x}}_{k|N}$  indeed.

## 5 Simulation

### 5.1 Design of the model set

Under the IMM framework, the evolved state equation can be formulated as  $\mathbf{x}_k = F_r \mathbf{x}_{k-1} + D_r \mathbf{w}_{k-1}$ , the model set  $\{F_r\}_{r=1}^M$  includes three classical models only, that is, constant acceleration (CA) model, the constant turn coordinate (CT) model, and constant velocity (CV) model [1,3,15~17]. For two dimensioned plane motion, the state vector is defined as

$$\mathbf{x}_k = [x(k) \dot{x}(k) \ddot{x}(k) y(k) \dot{y}(k) \ddot{y}(k)]' \quad (34)$$

The evolved state matrix  $F_r$  in  $\{F_r\}_{r=1}^3$  can be employed as follows

$$F_{CT} = \begin{bmatrix} F_{CT}^1 & F_{CT}^2 \\ -F_{CT}^2 & F_{CT}^1 \end{bmatrix}, F_{CT}^1 = \begin{bmatrix} 1 & \sin(\omega(k)T)/\omega(k) & 0 \\ 0 & \cos(\omega(k)T) & 0 \\ 0 & 0 & 0 \end{bmatrix}, F_{CT}^2 = \begin{bmatrix} 0 & [\cos(\omega(k)T) - 1]/\omega(k) & 0 \\ 0 & -\sin(\omega(k)T) & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$F_{CA} = \begin{bmatrix} F_1 & 0 \\ 0 & F_1 \end{bmatrix}, F_1 = \begin{bmatrix} 1 & T & T^2/2 \\ 0 & 1 & T \\ 0 & 0 & 1 \end{bmatrix}, F_{CV} = \begin{bmatrix} F_2 & 0 \\ 0 & F_2 \end{bmatrix}, F_2 = \begin{bmatrix} 1 & T & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

The uniform expression of the state noise input-matrix for the above evolving matrix is

$$D_{CT} = D_{CA} = D_{CV} = \begin{bmatrix} D & 0 \\ 0 & D \end{bmatrix}, \mathbf{D} = [T^2/2 \quad T \quad 1]'$$

## 5.2 Measurement equation

The measurement equation with the range and the azimuth angle can be formulized as follows<sup>[1,18]</sup>:

$$\mathbf{z}_k = \mathbf{h}(\mathbf{x}_k) + \mathbf{v}_k = [h_1(\mathbf{x}_k) \quad h_2(\mathbf{x}_k)]' + \mathbf{v}_k \quad (35)$$

where the range measurement is  $h_1(\mathbf{x}_k) = (\mathbf{x}(k)^2 + y(k)^2)^{1/2}$ , the azimuth angle measurement is  $h_2(\mathbf{x}_k) = \tan^{-1}(y(k)/x(k))$ .

## 5.3 Selection of the trajectory parameters and simulation results

Suppose that the trajectory consists of seven segments: 150 seconds CA motion with noise covariance  $Q_1 = \text{diag}(0.2, 0.2)$ ; 170 seconds clock-wise CT motion with noise covariance  $Q_2 = \text{diag}(1.0, 1.0)$  and turning rate  $\omega_1 = 0.015\text{rad/s}$ ; 180 seconds CV motion with covariance  $Q_3 = \text{diag}(0.5, 0.5)$ ; 180 seconds clock-wise CT motion with noise covariance  $Q_4 = Q_2$  and turning rate  $\omega_1$ ; 200 seconds CA motion with noise covariance  $Q_5 = Q_1$ ; 200 seconds clock-wise CT motion with noise covariance  $Q_6 = Q_2$  and  $\omega_1$ ; 180 seconds CV motion with noise  $Q_7 = Q_3$ . The initial covariance and state are  $P_0 = \text{diag}(100\text{m}^2, 10(\text{m/s})^2, 1(\text{m/s}^2)^2, 100\text{m}^2, 10(\text{m/s}^2)^2, 1)$  and  $\mathbf{x}_0 = [0\text{m}, 100\text{m/s}, 20\text{m/s}^2, 0\text{m}, 100\text{m/s}, 20\text{m/s}^2]'$  respectively. The measurement noise covariance  $R = \text{diag}(10, 1)$ . The state noise covariance used in the common IMM filtering can be designed as  $Q = \text{diag}(0.5, 0.5)$ , contrarily, in the adaptive IMM filtering (IMM+EM) it can be estimated by the EM iteration. The root mean square (RMS) error is used as the evaluating criterion of the simulation results.

200 runs of the Monte-Carlo simulation are performed and the results are showed in Fig. 1~Fig. 3. From the figures we can see that the state covariance error has slight influence on the RMS error of the position estimation, but heavy influence on the RMS error of the velocity and the acceleration estimation. Comparing with the common IMM filtering, the adaptive state covariance IMM improves the performance to some extent indeed. The main reason is that the online estimated state covariance is employed in the filtering stage, this can greatly reduce the additional filtering error brought by the mismatch of the variable state covariance.

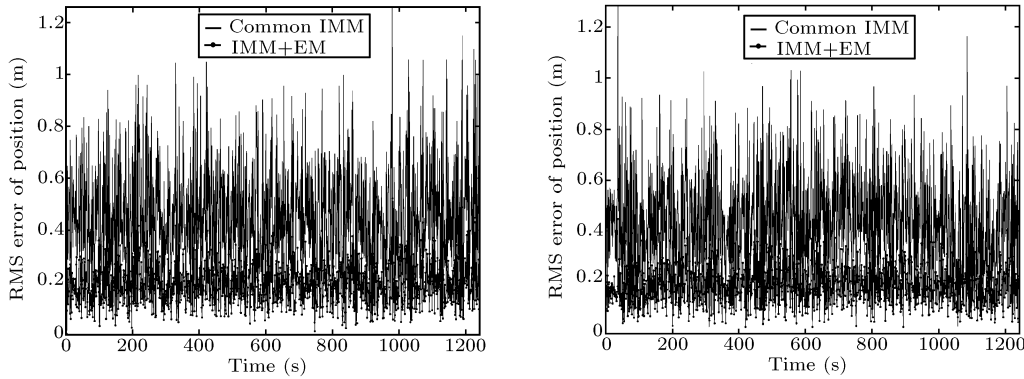


Fig. 1 The comparing of estimated position RMS error in  $x$  and  $y$  directions with respect to the left and right Figs



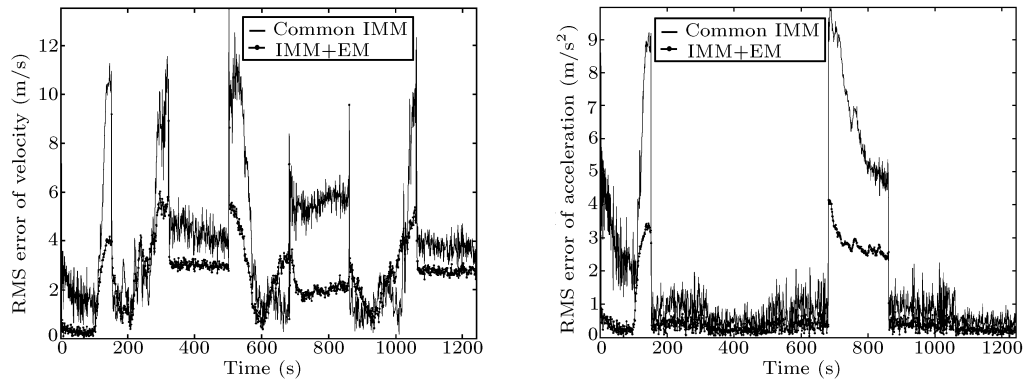


Fig. 2 The comparing of the estimated velocity and acceleration RMS error in  $x$  direction with respect to the left and right Figs

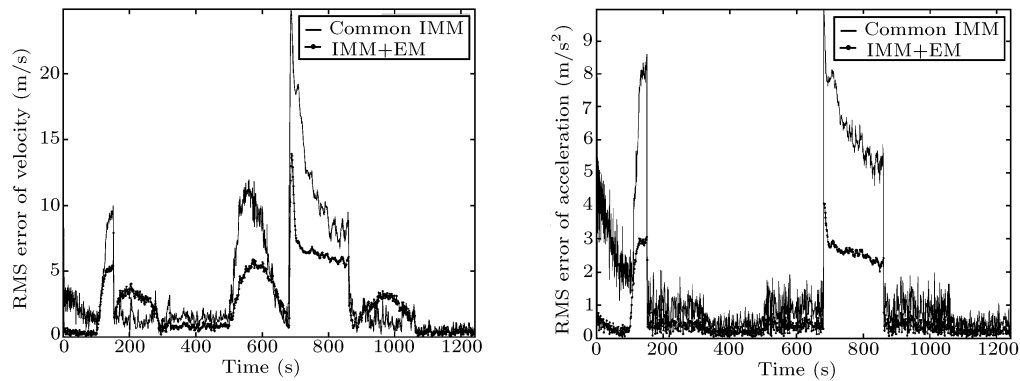


Fig. 3 The comparing of the estimated velocity and acceleration RMS error in  $y$  direction with respect to the left and right Figs

## 6 Conclusion

For the maneuvering target tracking system, generally, measurement noise variance  $R$  can be pre-determined by means of experiments at a radar receiver, but the quantity of state noise variance  $Q$  can be variable with the maneuvering state and can not be prior determined. Under the framework of IMM, the quantity of  $Q$  is variable with model coefficient  $F_r$ , that is, different  $F_r$  with different  $Q_0^{(r)}$ . Hence, online estimating state noise variance (level) can be one of the primary problems waiting to be solved eagerly. In this paper, based on the dynamic system with nonlinear measurement equation and EM algorithm, the online estimation equation of  $Q$  is derived, the schedule of the IMM with adaptive  $Q$  is prescribed, and the analysis of the estimation error is performed quantitatively. The new algorithm theoretically improves the performance of the IMM by online adaptive  $Q$ . Meanwhile, Monte-Carlo simulations also show that the filtering precision can be improved to some extent.

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