A Dual-mode Nonlinear Model Predictive Control with the Enlarged Terminal Constraint Sets[1)]

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Abstract Aiming at a class of nonlinear systems with multiple equilibrium points, we present a dual-mode model predictive control algorithm with extended terminal constraint set combined with control invariant set and gain schedule. Local LQR control laws and the corresponding maximum control invariant sets can be designed for finite equilibrium points. It is guaranteed that control invariant sets are overlapped each other. The union of the control invariant sets is treated as the terminal constraint set of predictive control. The feasibility and stability of the novel dual-mode model predictive control are investigated with both variable and fixed horizon. Because of the introduction of extended terminal constrained set, the feasibility of optimization can be guaranteed with short prediction horizon. In this way, the size of the optimization problem is reduced so it is computationally efficient. Finally, a simulation example illustrating the algorithm is presented.

Key words Invariant set, nonlinear model predictive control, constrained nonlinear systems, gain schedule

1 Introduction

Model predictive control (MPC) or receding horizon control (RHC) is a form of control algorithm based on the system model, which is attractive to industry. Recently, MPC has become an accepted standard for complex constrained multivariable control problems in the process industries. Compared with other control technologies, MPC has powerful abilities to deal with both the constraint conditions and nonlinearity of the system. The early proponents of MPC for process control did not address stability theoretically so that their early versions of MPC are not automatically stabilizing. However, they were obviously aware of its importance. The stability of the early version of MPC could be achieved with the increasing length of the control horizon, indicating the stability property of an infinite horizon predictive control scheme. Since 1990s, the stability theory of MPC (especially in nonlinear model predictive control (NMPC)) has been established[1]. The theoretical framework of NMPC that guarantees feasibility and stability is now well understood. It consists of the optimization of a finite horizon cost with a terminal cost function subject to system constraints in addition to a terminal stability constraint set. Generally, the terminal cost function is a Lyapunov function and the terminal constraint set is a positive invariant set[2].

The big drawback of the NMPC is the relatively formidable on-line computational effort, which limits its applicability to relatively slow and/or small problems. Aiming at this problem, a series of efficient and suboptimal algorithms were presented. For the linear constrained systems, [3] presented a technique to compute the explicit state-feedback solution to both the finite and infinite horizon linear quadratic optimal control problem subject to state and input constraints and showed that this closed form solution is piecewise linear and continuous. [4] developed an off-line robust constrained MPC algorithm with the substantial reduction of the on-line MPC computation. However, the nonlinear system was rarely investigated. Moreover, the online algorithm is indispensable for the real-time processing ability of the controller. Accordingly, it is necessary to develop the algorithm with guaranteed closed loop stability and low online computation burden.

The domain of attraction of the controller depends on the size of the terminal region and the control horizon. By increasing the control horizon, the domain of attraction is enlarged at the expense of a

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greater computational burden, while increasing the terminal region produces an enlargement without an extra cost. In this paper, a modified dual-mode predictive control (DMPC) algorithm[5~7] is presented. For a class of nonlinear systems with continuous equilibrium surface[8], an enlarged terminal constraint set can be computed off-line. We modify the stability constraint by this enlarged terminal constraint. The feasibility of nonlinear optimization can be guaranteed in short predictive horizon due to the enlargement of terminal constraint set, thereby the computational burden is reduced greatly. Similar to the existing DMPC, the presented strategy guarantees closed-loop stability of the system.

2 Problem statement

Consider the nonlinear discrete-time dynamic system

$$\boldsymbol{x}(k+1)=\boldsymbol{f}(\boldsymbol{x}(k),\boldsymbol{u}(k)),\ \boldsymbol{x}(t_0)=\boldsymbol{x}_0,\quad k\geqslant t_0 \tag{1}$$

where k is the discrete time index, and t_0 is the initial time and $\boldsymbol{x}(k)\in\Re^n, \boldsymbol{u}(k)\in\Re^m$. The state and control variables are required to fulfill the following constraints: $\boldsymbol{x}(k)\in X, \boldsymbol{u}(k)\in U$.

Definition 1. Given $\boldsymbol{u}^{eq}\in U$ and $\boldsymbol{x}^{eq}\in X$, $\boldsymbol{x}^{eq}$ is an equilibrium point of system (1) such that $\boldsymbol{x}^{eq}=\boldsymbol{f}(\boldsymbol{x}^{eq},\boldsymbol{u}^{eq})$. The connected set of all the equilibrium points is called the equilibrium surface.

Consider a nonlinear system with several operating points. Linearization of the nonlinear system around these operating points and discretization gives a bank of piecewise linear systems of the form

$$\dot{\boldsymbol{\delta}}\hat{\boldsymbol{x}}=A_i'\boldsymbol{\delta}\hat{\boldsymbol{x}}+B_i'\boldsymbol{\delta}\hat{\boldsymbol{u}},\quad \boldsymbol{y}=C_i'\boldsymbol{\delta}\hat{\boldsymbol{x}}+D_i'\boldsymbol{\delta}\hat{\boldsymbol{u}} \tag{2}$$

where i denotes the linear model at ith equilibrium point, and

$$A_i'=\frac{\partial\boldsymbol{f}(\boldsymbol{x},\boldsymbol{u})}{\partial\boldsymbol{x}}|(\bar{\boldsymbol{x}}_i,\bar{\boldsymbol{u}}_i),\ B_i'=\frac{\partial\boldsymbol{f}(\boldsymbol{x},\boldsymbol{u})}{\partial\boldsymbol{u}}|(\bar{\boldsymbol{x}}_i,\bar{\boldsymbol{u}}_i),\ C_i'=\frac{\partial\boldsymbol{g}(\boldsymbol{x},\boldsymbol{u})}{\partial\boldsymbol{x}}|(\bar{\boldsymbol{x}}_i,\bar{\boldsymbol{u}}_i),\ D_i'=\frac{\partial\boldsymbol{g}(\boldsymbol{x},\boldsymbol{u})}{\partial\boldsymbol{x}}|(\bar{\boldsymbol{x}}_i,\bar{\boldsymbol{u}}_i)$$

$\boldsymbol{\delta x}$ and $\boldsymbol{\delta u}$ are deviation variables and are defined by $\boldsymbol{\delta}\hat{\boldsymbol{x}}=\boldsymbol{x}-\bar{\boldsymbol{x}}_i$ and $\boldsymbol{\delta}\hat{\boldsymbol{u}}=\boldsymbol{u}-\bar{\boldsymbol{u}}_i$, respectively. In order to get a piecewise linear model at each operating point a new set of variables are defined as $\hat{\boldsymbol{x}}=\boldsymbol{x}-\bar{\boldsymbol{x}}_j$ and $\hat{\boldsymbol{u}}=\boldsymbol{u}-\bar{\boldsymbol{u}}_j$ where $(\bar{\boldsymbol{x}}_j,\bar{\boldsymbol{u}}_j)$ are desired set point. We can get the following equation

$$\begin{cases}\hat{\boldsymbol{x}}(k+1)=A_i\hat{\boldsymbol{x}}(k)+B_i\hat{\boldsymbol{u}}(k)+\boldsymbol{b}_i\\ \boldsymbol{y}(k)=C_i\hat{\boldsymbol{x}}(k)+D_i\hat{\boldsymbol{u}}(k)+\boldsymbol{d}_i\end{cases},\quad i=1,\cdots,M \tag{3}$$

It is assumed that:

A1) $X\subset\Re^n$, $U\subset\Re^m$ are compact, convex and contain the origin as interior point.

A2) $\boldsymbol{f}:\Re\times\Re^m\to\Re^n$ is twice continuously differentiable and $\boldsymbol{f}(0,0)=\boldsymbol{0}$. Thus, $\boldsymbol{0}\in\Re^n$ is an equilibrium point of the system with $\boldsymbol{u}=\boldsymbol{0}$.

A3) The nonlinear system (1) has an equilibrium surface.

A4) The pair (A,B) is stabilizable, where $A=(\partial\boldsymbol{f}/\partial\boldsymbol{x})(\boldsymbol{x}^{eq},\boldsymbol{u}^{eq})$, and $B=(\partial\boldsymbol{f}/\partial\boldsymbol{u})(\boldsymbol{x}^{eq},\boldsymbol{u}^{eq})$ lead to the Jacobean linearization at the equilibrium point $(\boldsymbol{x}^{eq},\boldsymbol{u}^{eq})$.

Remark 1. A3 is very stringent for any nonlinear system. However, for most of the practical chemical processes or tidal current of power systems, there are many equilibrium points so we can draw a conclusion that this assumption is representative. Although some Jacobean linearization model (A,B) may not satisfy A4), the unstable linearized system can be stabilized with the terminal zero constraint restricting the terminal state to be zero [9]. It is noted that the control law obtained in this paper is explicit or implicit state feedback and thus it is assumed that all states are measurable.

3 Design of the local controller

In order to design a state feedback control law one can consider the minimization with respect to $u(\cdot)$ of the infinite horizon (IH) cost function

$$J(\boldsymbol{x},\boldsymbol{u}(\cdot))=\sum_{k=t}^{\infty}(\|\boldsymbol{x}(k)\|_Q^2+\|\boldsymbol{u}(k)\|_R^2) \tag{4}$$

subject to

$$(1)\text{ and }\boldsymbol{x}(k)\in X\text{ and }\boldsymbol{u}(k)\in U \tag{5}$$

where $Q \geqslant 0$, $R > 0$ are symmetric weighting matrices and the pair $(A, Q^{1/2})$ is observable. Here, $\|\boldsymbol{x}(i)\|_P^2 = \boldsymbol{x}^{\mathrm{T}} P \boldsymbol{x}$, where P is a symmetric positive matrix. Suppose the solution of the optimization problem (4), (5) exists, denoted by $\boldsymbol{u}_0^{\infty} = \{u(0), u(1), u(2), \cdots\}$.

The IH nonlinear optimal control problem is computationally intractable since it involves an infinite number of decision variables. Nevertheless, it constitutes a touchstone for suboptimal approaches[5], *i.e.*, the two-step control strategy. In general, the suboptimal method can be described as follows

$$\min J(\boldsymbol{x}, \boldsymbol{u}(\cdot)) = \sum_{i=0}^{N-1} (\|\boldsymbol{x}(k+i)\|_Q^2 + \|\boldsymbol{u}(k+i)\|_R^2) + \|\boldsymbol{x}(k+N)\|_P^2 \tag{6}$$

subject to

$$(5), \text{ and } \boldsymbol{x}(k+N) \in \Omega_\alpha \tag{7}$$

The problem can be divided into two steps. First, in the neighborhood of the origin, the control action is generated by a linear feedback control law designed for the linearized system. Second, outside this neighborhood, nonlinear receding horizon control is employed. The two controllers combine together to form an overall solution to the original problem.

Definition 2. The non-empty set $X^g \subset \Re^n$ is called a control invariant set with respect to $\boldsymbol{u}$ for system (1) at time $k = 0$, if there exists a feedback control law $\boldsymbol{u}(k) = \boldsymbol{g}(\boldsymbol{x}(k))$, such that $\boldsymbol{f}(\boldsymbol{x}(k), \boldsymbol{u}(k)) \in X^g$, $\boldsymbol{x}(k) \in X$ and $\boldsymbol{u}(k) \in U$ for $\forall \boldsymbol{x}(k) \in X^g$, $k \geqslant 0$.

When a suboptimal design is performed, it is necessary to introduce an auxiliary linear controller to convert the infinite horizon optimization problem into a finite horizon problem. The simplest way to obtain an easy-to-compute suboptimal solution is to resort to linearization techniques. Letting

$$A = (\partial \boldsymbol{f}(\boldsymbol{x}, \boldsymbol{u})/\partial \boldsymbol{x})|(0,0), \quad B = (\partial \boldsymbol{f}(\boldsymbol{x}, \boldsymbol{u})/\partial \boldsymbol{u})|(0,0) \tag{8}$$

then a linearized model is obtained at the origin.

We have assumed that (A, B) is stabilizable, so there exists a feedback control gain K that can stabilize the closed-loop system $\boldsymbol{x}(k+1) = A\boldsymbol{x}(k) + BK\boldsymbol{x}(k)$. Furthermore, for the given control law

$$\boldsymbol{u} = K\boldsymbol{x} \tag{9}$$

we use $\Omega(K)$ to denote the control invariant set of the closed-loop system $\boldsymbol{x}(k+1) = \boldsymbol{f}(\boldsymbol{x}(k), K\boldsymbol{x}(k))$.

Lemma 1. There exist some positive real number γ that satisfies $\gamma < \lambda_{\min}(Q + K^{\mathrm{T}} RK)$ and a constant $\alpha \in (0, \infty)$ specifying a neighborhood Ω_α of the origin in the form of

$$\Omega_\alpha \triangleq \{\boldsymbol{x} \in R^n | \boldsymbol{x}^{\mathrm{T}} P \boldsymbol{x} \leqslant \alpha\} \tag{10}$$

such that 1) Ω_α is control invariant with respect to the control law (9) for the nonlinear system (1)

2) $\forall \boldsymbol{x}_0 \in \Omega_\alpha$, if $\boldsymbol{u}(k) = K\boldsymbol{x}(k)$, then $\boldsymbol{u}(k) \to 0$, $\boldsymbol{x}(k) \to 0$ as $k \to \infty$. (for proof see [10])

Remark 2. In order to reduce the online computation burden of the nonlinear MPC, we hope to get the maximal terminal constraint set Ω_α. However, it is very hard to get maximal control invariant region for the nonlinear system in aforementioned method, because the size of the terminal constraint set depends on many parameters, for example, (K, γ)[11]. It is known that the size of the terminal constraint set generally depends on the nonlinearity of the system to be controlled. The stronger nonlinear the system is, the smaller terminal constraint set will be. In this paper, we can use the direct optimization method of a Lyapunov function to obtain the maximal terminal constraint set for nonlinear discrete system[5,6].

The idea of this paper is that an enlarged terminal constraint set is designed via the combination of the equilibrium surface, control invariant set and gain schedule[8]. The details are as follows: When the equilibrium surface is derived, a series of equilibrium points can be found. According to the above design method of the locally stabilizing controller, the control invariant region can be estimated offline. The idea can be stated as follows: we can firstly design the local controller $\boldsymbol{u}(k) = K_0 \boldsymbol{x}(k)$ at the origin with its maximal control invariant region Ω^0. As the equilibrium surface is a connected set in Definition 1, the equilibrium points $(\boldsymbol{x}^{(1)}, \boldsymbol{u}^{(1)})$ satisfying $x^{(1)} \in \Omega^0$ can always be found. Then, we can design the local controller $\bar{\boldsymbol{u}}(k) = K_1 \bar{\boldsymbol{x}}(k)$ and the associated control invariant region according to the new equilibrium point. In this way, we can construct a series of local controllers and control invariant regions, and use their union to cover the equilibrium surface. The schedule table of the local controller can be formed as shown in Table 1.

Table 1 Control invariant set schedule table of nonlinear system

Local controller Schedule i	Controller Gain K	Equilibrium points	Control invariant region
0	K_0	$\boldsymbol{x}^{(0)}, \boldsymbol{u}^{(0)}$	Ω^0
1	K_1	$\boldsymbol{x}^{(1)}, \boldsymbol{u}^{(1)}$	Ω^1
$\vdots$	$\vdots$	$\vdots$	$\vdots$
M	K_M	$\boldsymbol{x}^{(M)}, \boldsymbol{u}^{(M)}$	Ω^M

The local control law in the terminal constraint set can be determined by judging which control invariant region the current state belongs to.

Theorem 1. For the nonlinear system (1), if the initial system state $\boldsymbol{x}(0) \in X$ satisfies $\boldsymbol{x}(0) \in \bigcup_{i=0}^{M} \Omega_i$ then the gain schedule algorithm that is used in the interior of the enlarged terminal constraint set can regulate $\boldsymbol{x}(0)$ to the desire equilibrium point $(\boldsymbol{x}^{(0)}, \boldsymbol{u}^{(0)})$.

Proof. When $\boldsymbol{x}(0) \in \Omega^{i+1}\ \Omega^i$, $i \neq 0$, the local stabilizing control law K_{i+1} in the next region Ω^{i+1} can regulate $\boldsymbol{x}(0)$ to the equilibrium point $(\boldsymbol{x}^{(i+1)}, \boldsymbol{u}^{(i+1)})$ asymptotically. Moreover, since $\boldsymbol{x}^{(i+1)} \in \Omega^i$, the state trajectory must intersect with the boundary Ω^i, *i.e.*, $\boldsymbol{x}$ must belong to the intersection set $\Omega^{i+1} \cap \Omega^i$ or Ω^i because the system we investigate is discrete. According to the switching rule of the controller, when $\boldsymbol{x}(k) \in \Omega^i$, the local control law K_i should be implemented. In this manner, as soon as $\boldsymbol{x} \in \Omega^1 \cap \Omega^0$ or $\boldsymbol{x} \in \Omega^0$, the local control law of the system will be switched to K_0. Since Ω^0 is the control invariant set at the origin $\boldsymbol{x}^{(0)}$ for nonlinear system (1), by Lemma 1 the results can converge to $\boldsymbol{x}^{(0)}$ straightforward. □

Remark 3. For some nonlinear systems that satisfy A1∼A4, maybe the nonlinearity is very strong. This will make the control invariant sets around the each equilibrium points very small. Therefore, the terminal constraint set that is constructed using the presented method will not be enlarged remarkably. By now, it is still a hard problem to be solved urgently. The equilibrium surface is a fundamental property of a nonlinear system. Different shape or position will bring some effect on the design of the controller[8].

4 Dual-mode nonlinear predictive control with the enlarged terminal constraint set

There exist two kinds of dual-mode predictive control strategies: variable horizon and fixed horizon. At first, we present the variable horizon one, then the next one.

4.1 The design and analysis of the variable horizon dual-mode NMPC

The variable horizon dual model predictive control algorithm includes the following steps.

Step 1. If the initial state of the system $\boldsymbol{x}(0) \in \bigcup_{i=1}^{M} \Omega^i$ and $N > 0$, given $t > 0$, then solve the following optimization problem:

$$J(\boldsymbol{x}, \boldsymbol{u}(\cdot)) = \sum_{i=0}^{N-1} (\|\boldsymbol{x}(t+i)\|_Q^2 + \|\boldsymbol{u}(t+i)\|_R^2) + \|\boldsymbol{x}(t+N)\|_F^2 \tag{11}$$

$$\text{s.t. (5), and } \boldsymbol{x}(t+N) \in \bigcup_{i=0}^{M} \Omega^i \tag{12}$$

If feasible, implement $\boldsymbol{u}(t)$, go to Step 2; otherwise, let $N = N + 1$, and repeat Step 1.

Step 2. At the time $t+1$, if $\boldsymbol{x}(t+1) \in \bigcup_{i=0}^{M} \Omega^i$, then implement the local switching law $\boldsymbol{h}_L(x)$ in the terminal constraint set; Otherwise, if $N > 1$, then let $N = N - 1$; if $N = 1$, then let $N = N$. Set $t = t + 1$ and repeat Step 1.

Theorem 2. Suppose there are no disturbances in the system and the optimization problem (11) is feasible at the initial time, then the state will be guaranteed to converge to $\bigcup_{i=0}^{M} \Omega^i$ in a finite time.

Proof. Because the optimization problem (11) is feasible at time $t = 0$, the solution sequence $\boldsymbol{u}(t/t) \cdots \boldsymbol{u}(t+N-1/t)$ can satisfy the terminal constraint $\boldsymbol{x}(t+N/t) \in \bigcup_{i=0}^{M} \Omega^i$. According to the principle of the MPC, at the time t, control action $\boldsymbol{u}(t)$ is injected to the process to be controlled, and there are no disturbance, so at time $t+1$, control sequence $\boldsymbol{u}(t+1/t)\cdots\boldsymbol{u}(t+N-1/t)$ are still feasible for the optimization with the control horizon $N-1$. Using the above argument repeatedly, it is always feasible when optimization problem is solved from t to $t+N-1$ and the control horizon is decreasing from N to 1. At the time $t+N-1$, the optimization with control horizon $N=1$ is solved and the current control action is implemented, then the system state will enter $\bigcup_{i=0}^{M}\Omega^i$. □

Definition 3. In the state set X_F, if the optimization problem (11) is feasible, then X_F is the feasible set to the optimization.

Theorem 3. Suppose there are no disturbances in the system and the initial state belongs to X_F, then the variable horizon dual-mode NMPC is asymptotically stable.

Proof. From Theorem 2, we can get that when the initial state $x_0\in X_F$, the state x will converge to $\bigcup_{i=0}^{M}\Omega^i$ in a finite time. After that by Theorem 1, x will converge to the origin asymptotically. □

4.2 The design and analysis of the fixed horizon dual-mode NMPC

In the variable horizon dual-mode NMPC presented in the above context, $\bigcup_{i=0}^{M}\Omega^i$ is a constraint condition on stability. Namely, once the system state enters $\bigcup_{i=0}^{M}\Omega^i$, we can consider the control objective has been reached and let $\|\boldsymbol{x}(t+N)\|_F^2=0$. In this way, the cost function of MPC can be modified as follows with the simplification $L(\boldsymbol{x}_t,\boldsymbol{u}_t)=\|\boldsymbol{x}(t)\|_Q^2+\|\boldsymbol{u}(t)\|_R^2$

$$J(\boldsymbol{x},\boldsymbol{u}(\cdot))=\sum_{k=t}^{t+N-1}L(\boldsymbol{x}_k,\boldsymbol{u}_k) \tag{13}$$

The fixed horizon dual-mode NMPC is designed in following two steps.

Step 1. It is the same as the Step 1 of the variable horizon dual-mode NMPC except for substituting (13) for the cost function.

Step 2. At time $t+1$, if $\boldsymbol{x}(t+1)\in\bigcup_{i=0}^{M}\Omega^i$, then implement the local controller in the terminal constraint set; otherwise, according to which the terminal set of the equilibrium point it belongs to at the previous time, convert the open-loop optimization into the form of the fixed dual-mode NMPC in [6]. If $\boldsymbol{x}(N/t)\in\Omega^j$, then let Ω^j be the terminal constraint set; if $\boldsymbol{x}(t+N)$ not only belongs to Ω^j but also belongs to Ω^{j+1}, then we deem $\boldsymbol{x}(t+N/t)\in\Omega^j$, *i.e.*, the one that has a small index number. At time $t=t+1$, repeat Step 1.

When the equilibrium point $(\boldsymbol{x}^{(j)},\boldsymbol{u}^{(j)})$ in Ω^j is not the origin of the system, the coordination transform is needed to convert the origin into $(\boldsymbol{x}^{(j)},\boldsymbol{u}^{(j)})$. Suppose the new system dynamics equation is $\tilde{\boldsymbol{x}}(k+1)=\boldsymbol{f}'(\tilde{\boldsymbol{x}}(k),\tilde{\boldsymbol{u}}(k))$, where $\tilde{\boldsymbol{x}}=\boldsymbol{x}-\boldsymbol{x}^{(j)}$ and $\tilde{\boldsymbol{u}}=\boldsymbol{u}-u^{(j)}$. The terminal constraint set after transformation is $\Omega'^{(j)}$, meanwhile the optimization problem is also modified accordingly.

Lemma 2. For the nominal system with no state measurement error and no disturbance, the feasibility of optimization problem (13) of fixed horizon dual mode NMPC at time $t=0$ implies its feasibility for all $t>0$.

The proof of Lemma 2 is a straightforward consequence of Lemma 2 in [11] and is not given here.

Theorem 4. Suppose there is no disturbance and the initial state belong to X_F. Then the fixed dual-mode NMPC is asymptotically stable.

Proof. It has been proven that the local control law can stabilize the system when the state enters $\boldsymbol{x}\in\bigcup_{i=0}^{M}\Omega^i$. Now we only need to show that the state in X_F can enter $\boldsymbol{x}\in\bigcup_{i=0}^{M}\Omega^i$ in a finite time.

Suppose that the initial state is $\boldsymbol{x}_0\notin\bigcup_{i=0}^{M}\Omega^i$ at t time ($t=0$) and satisfies $\boldsymbol{x}_0\in X_F$; then the optimization problem (13) is feasible. There must exist an optimal control sequence $\boldsymbol{u}_t^{N-1}=\{\boldsymbol{u}(0/t),\boldsymbol{u}(1/t),\cdots,\boldsymbol{u}(N-1/t)\}$ that satisfies the terminal constraint condition $\boldsymbol{x}(N/t)\in\bigcup_{i=0}^{M}\Omega^i$. Implement the control action $\boldsymbol{u}(0/t)$ at time t. At time $t+1$, if $\boldsymbol{x}(t+1)\in\bigcup_{i=0}^{M}\Omega^i$, then use the local control to regulate the state to the origin with the asymptotic stability; otherwise, if $\boldsymbol{x}(t+1)\notin\bigcup_{i=0}^{M}\Omega^i$, then convert the problem into the fixed horizon dual-mode NMPC in [6] with one control invariant set of some equilibrium point as the terminal constraint set, *i.e.*, suppose $\boldsymbol{x}(N/t)\in\Omega^j$. According to the feasibility Lemma 2, the new optimization problem after coordination transformation is still feasible. By now, we have converted the NMPC with the union of multiple control invariant set as terminal constraint set into an NMPC with only one control invariant set. This class of the dual-model NMPC has already been proved that when $\boldsymbol{x}_0\in X_F$, the state can enter Ω^j in a finite time[6]. □

Remark 4. From the design and analysis of the presented dual-mode predictive control algorithm, the closed-loop stability does not depend on the optimality of the solution but on its feasibility.

Remark 5 (Robustness). Because the model is only an approximation of the real process, the robustness of MPC to the uncertainty of the model is very important. Robustness was achieved by employing conservative state and stability constraint set, a margin of error was permitted in [5]. This robust stability strategy can still be applied in the proposed NMPC.

5 Simulation example

Consider a two-tank system[12] and the model of this nonlinear system is as follows:

$$\rho S_1\dot{h}_1 = -\rho A_1\sqrt{2gh_1} + u, \quad \rho S_2\dot{h}_2 = \rho A_1\sqrt{2gh_1} - \rho A_2\sqrt{2gh_2}$$

The parameters are as in [12]. According to the system model, the equilibrium surface equation can be easily obtained just as the solid line in Fig. 1. Take $([2.963 \quad 15]^{\mathrm{T}}, 0.686)$ as the desired equilibrium point (origin) and the linearized model at the point is obtained as

$$A = \begin{bmatrix} 0.9769 & 0 \\ 0.0362 & 0.9929 \end{bmatrix}, \quad \boldsymbol{B} = \begin{bmatrix} 0.2 \\ 0 \end{bmatrix}$$

Solving the LQR problem with the weighting matrices $Q = I, R = 10$ for the above linearized system (A, B), we get a linear locally stabilizing feedback control law $\boldsymbol{K} = [-0.3228 \quad -0.2259]$.

In the simulation process, 10 more equilibrium points are selected, which are (2.9037,14.7), (2.8049, 14.2), (2.7062, 13.7), (2.6272, 13.3), (2.5284, 12.8), (2.4296, 12.3), (2.3309, 11.8), (2.2519, 11.4), (2.1728, 11), (2.0938, 10.6). By the linearization at the equilibrium points and selection of the same LQR coefficients, *i.e.*, $Q = I$, $R = 10$, the LQR control law and control invariant set of each equilibrium point are calculated off-line. Here, we will use the fixed horizon dual-mode NMPC with $\bigcup_{i=0}^{10} \Omega^i$ as terminal constraint set and $Q = I, R = 1$ as the controller parameters. Suppose there are three initial states, (2, 13.5), (4.5, 14), and (4, 11.5), respectively. With the same selection of the controller parameters, to the initial state (4, 11.5) if we use the algorithm presented in [5] the optimization will not be feasible until the control horizon N increases to 25. However, using the algorithm presented by this paper, it becomes feasible when $N = 10$. Fig. 1 and 2 show the system state and control input trajectories. In Fig. 2, the dash dotted line, solid line and dotted line represent the control input trajectories for the initial states (2, 13.5), (4.5, 14), and (4, 11.5), respectively. It is obvious that the constraints are satisfied.

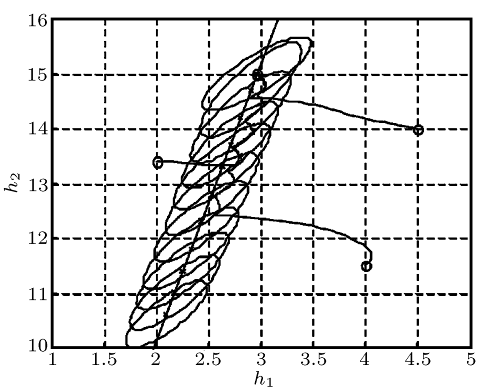

Fig. 1 Closed-loop state trajectory of nonlinear system

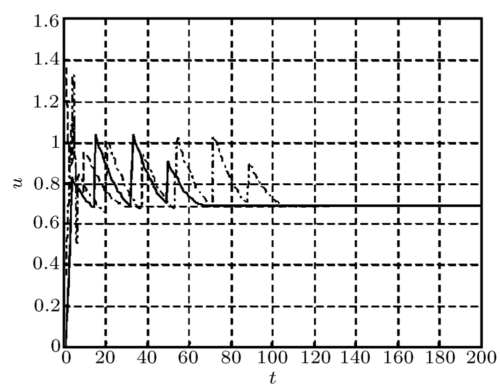

Fig. 2 Control input profile of nonlinear system

6 Conclusion

For a class of nonlinear system with a continuous equilibrium surface, a dual-mode predictive control algorithm is presented, which possesses the merits of both the control invariant set and gain schedule. From the design of the local controller, an enlarged terminal constraint set can be obtained off-line. Variable horizon and fixed horizon strategies in dual-mode predictive control are investigated and the feasibility and stability are also analyzed. Because of the introduction of extended terminal constrained set, the feasibility of optimization can be guaranteed with short prediction horizon. In this way, the size of the optimization problem is reduced so it is computationally efficient.

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