

Cyclic Reconfigurable Flow Shop under Different Configurations Modeling and Optimization Based on Timed Event Graph¹⁾

REN Si-Cheng XU De WANG Fang TAN Min

(Institute of Automation, Chinese Academy of Sciences, Beijing 100080)

(E-mail: scren@compsys.ia.ac.cn)

Abstract Based on the idea that modules are independent of machines, different combinations of modules and machines result in different configurations and the system performances differ under different configurations, a kind of cyclic reconfigurable flow shops are proposed for the new manufacturing paradigm—reconfigurable manufacturing system. The cyclic reconfigurable flow shop is modeled as a timed event graph. The optimal configuration is defined as the one under which the cyclic reconfigurable flow shop functions with the minimum cycle time and the minimum number of pallets. The optimal configuration, the minimum cycle time and the minimum number of pallets can be obtained in two steps.

Key words Reconfigurable manufacturing, cyclic flow shop, timed event graph, modeling and optimization

1 Introduction

The increasingly competitive global market and rapid development of manufacturing technologies bring to manufacturing industry opportunities as well as challenges. How to respond to the drastic changes of market demands quickly and cost-effectively becomes the key for companies to survive in this new environment. Traditional manufacturing systems such as dedicated manufacturing system (DMS) and flexible manufacturing system could not meet this requirement and Koren *et al.* proposed a new manufacturing paradigm—reconfigurable manufacturing system (RMS), which was designed at the outset for rapid change in structure, as well as in hardware and software components, in order to quickly adjust production capacity and functionality within a part family in response to sudden changes in market or in regulatory requirements^[1]. RMS is a dynamic, evolving system and the key characteristics of RMS are modularity, integrability, customization, convertibility and diagnosability.

Different aspects of RMS have been extensively studied by many scholars. Zhao *et al.* have proposed that products required by customers can be classified into several product families, each of which is a set of similar products and each family corresponds to one configuration of the RMS^[2]. The products belonging to the same family will be produced by the RMS under the corresponding configuration. Yigit *et al.* have addressed the problem of optimizing modular products in RMS^[3]. The problem is posed as a generalized subset selection problem where the best subsets of modules instances of unknown sizes are determined by minimizing an object function that represents a trade-off between the quality loss due to modularization and the cost of reconfiguration while satisfying the problem constraints. Abdi and Labib have employed analytical hierarchical process (AHP) or fuzzy analytical hierarchical process (FAHP) in the design strategy, grouping and selecting products and feasibility study of the tactical design justification for RMS^[4~6].

In this paper, a kind of cyclic reconfigurable flow shops are proposed for RMS based on the idea that modules are independent of machines and different combinations of modules and machines result in different configurations. The system performances differ under different configurations and the optimal configuration is defined as the one under which the cyclic reconfigurable flow shop functions with the minimum cycle time and the minimum number of pallets. The cyclic reconfigurable flow shop can be modeled as a strongly connected timed event graph and two mixed-integer programs are established to obtain the optimal configuration, the minimum cycle time and the minimum number of pallets.

2 Problem description

The manufacturing process of a part involves a number of sequential steps and each step could be viewed as the part being manufactured by a process module with some specific function. The

1) Supported by National Key Fundamental Research and Development Project of P. R. China (2002CB312200)
Received October 25, 2004; in revised form October 24, 2005

module must be placed on a machine and connected with the machine *via* standard interfaces. The machine considered here is a carrier or general platform that can hold one or several different modules simultaneously. The machine plays the role of supplying power, communicating, coordinating and controlling different modules, *etc.* For example, a robot (machine) can perform the operations of cutting and drilling if it is equipped with the cutting and drilling tools (modules). Other examples are the dedicated machine in DMS and computerized numerically controlled (CNC) machine in FMS. The dedicated machine could be considered as a machine with only one module to perform a single function, while the CNC machine is a machine with multiple modules to achieve functional flexibility. However, the modules are fixed on the dedicated or CNC machine traditionally. In order to quickly adjust system capacity and functionality to meet market changes, one possible solution would be to make modules independent of machines, that is, modules can be removed from one machine and added to another machine freely. Different combinations of modules and machines result in different configurations. The process of the system changing from one configuration to another is called reconfiguration. Generally speaking, the system performances differ under different configurations. One goal of the reconfigurable manufacturing systems is to find a reasonable configuration method (*i.e.*, to distribute modules over machines) to achieve the desired system performance.

Based on the above idea, a kind of cyclic reconfigurable flow shops are proposed. For the convenience of descriptions, some notations are made as follows.

- 1) $J = \{J_k | k = 1, 2, \dots, |J|\}$ is a finite set of jobs.
- 2) $M = \{M_j | j = 1, 2, \dots, |M|\}$ is a finite set of machines.
- 3) $m = \{m_i | i = 1, 2, \dots, |m|\}$ is a finite set of modules.
- 4) $m(J_k)$ denotes the set of modules required to manufacture J_k such that $m = \bigcup_{k=1}^{|J|} m(J_k)$.
- 5) $\sigma(J_k) = \{(m_{i_1}, m_{i_2}) | m_{i_1}, m_{i_2} \in m(J_k) \text{ and } m_{i_1} \text{ precedes } m_{i_2}\}$ denotes the set of module precedences required to manufacture J_k . The ordered pair (m_{i_1}, m_{i_2}) denotes that module m_{i_1} precedes module m_{i_2} when manufacturing J_k .
- 6) $\sigma = \bigcup_{k=1}^{|J|} \sigma(J_k)$ denotes the set of module precedences.

In a cyclic reconfigurable flow shop, jobs (parts) $J_1, J_2, \dots, J_{|J|}$ are carried by each own pallet or pallets and access serial machines $M_1, M_2, \dots, M_{|M|}$ sequentially. The sequences of the jobs on each machine are identical $J_1, J_2, \dots, J_{|J|}$. The job is unloaded from the pallet after being processed by all the machines and the pallet returns immediately to pick up the next job. The cyclic reconfigurable flow shop repetitively produces the set of jobs J , the so-called minimal part set (MPS) which is defined as the smallest set of parts of different types in proportion to a certain production requirement. The combination of modules and machines can be represented as a configuration matrix Y and each entry of Y is defined as

$$y_{i,j} = \begin{cases} 1 & \text{if } m_i \text{ is placed on } M_j \\ 0 & \text{otherwise} \end{cases}$$

Because one module must be placed on only one machine under any configuration, Y should satisfy

$$\sum_{j=1}^{|M|} y_{i,j} = 1, \quad i = 1, 2, \dots, |m| \quad (1)$$

and the index of the machine that m_i is place on can be represented as

$$MI(m_i) = \sum_{j=1}^{|M|} j y_{i,j}, \quad i = 1, 2, \dots, |m|$$

According to the characteristics of flow shops (*i.e.* all the jobs visit the machines in the increasing order of the machine indexes and every machine is visited exactly once by each job), for any ordered pair (m_{i_1}, m_{i_2}) , we have

$$MI(m_{i_1}) \leq MI(m_{i_2}), \quad (m_{i_1}, m_{i_2}) \in \sigma$$

or

$$\sum_{j=1}^{|M|} j y_{i_1,j} \leq \sum_{j=1}^{|M|} j y_{i_2,j}, \quad (m_{i_1}, m_{i_2}) \in \sigma \quad (2)$$

where the equality holds iff m_{i_1}, m_{i_2} are placed on the same machine.

To simplify the analysis, the following assumptions are made:

1) The buffers between consecutive machines are sufficiently large.
 2) The processing time of J_k on m_i , denoted by $z_{k,i}$, is deterministic (a positive constant) if $m_i \in m(J_k)$ or 0 otherwise.

3) Compared with the processing time, the transportation time between machines, the setup time of machines and the changeover time between modules on the same machine can be neglected. The processing time of J_k on M_j , denoted by $w_{k,j}$, equals the sum of the processing time of J_k on all the needed modules placed on M_j , *i.e.*,

$$w_{k,j} = \sum_{m_i \in m(J_k) \wedge MI(m_i)=j} z_{k,i}$$

which can be rewritten as

$$w_{k,j} = \sum_{i=1}^{|m|} z_{k,i} y_{i,j} \quad (3)$$

or

$$W = ZY \quad (4)$$

where $W = (w)_{k,j}$ ($Z = (z)_{k,i}$) denotes the processing time matrix of jobs on machines (respectively on modules).

4) The operations of jobs on machines are non-preemptive.

3 Modeling and optimization

Timed event graphs are a subclass of Petri nets and are defined as a 4-tuple $TEG = (P, T, F, K_0)$ where

$P = \{p_1, p_2, \dots, p_{|P|}\}$ is a finite set of untimed places and each place has exactly one input transition and exactly one output transition;

$T = \{t_1, t_2, \dots, t_{|T|}\}$ is a finite set of timed transitions;

$F \subseteq (P \times T) \cup (T \times P)$ is a set of directed arcs;

$K_0 : P \rightarrow \{0, 1, 2, \dots\}$ is the initial marking;

$P \cap T = \emptyset$ and $P \cup T \neq \emptyset$.

If places and transitions are viewed as nodes and directed arcs as directed edges, a Petri net is essentially a bipartite digraph. The transition starts firing and consumes one token from each of its input places after being enabled; after holding the tokens for certain time (release time) the transition ends firing and generates one token to each of its output places. In [7], Ramamoorthy *et al.* have proved that an event graph is live if and only if each circuit contains at least one token in the initial marking; the total number of tokens in each circuit is constant in all the reachable markings; in a live and strongly connected event graph, all the transitions have the same cycle time and the event graph is periodic and the cycle time λ is given as

$$\lambda = \max_{\gamma} \frac{\mu(\gamma)}{\kappa(\gamma)}$$

where γ denotes any circuit in the event graph; $\mu(\gamma)$ denotes the sum of release time of all the transitions in circuit γ ; $\kappa(\gamma)$ denotes the number of tokens circuit γ contains in the initial marking K_0 . Karp algorithm^[8], Howard algorithm^[9], linear programming method^[10~12] and so on. are available for evaluating λ .

The cyclic reconfigurable flow shop described in Section 2 can be modeled as a timed event graph, $TEG = (P, T, F, K_0)$, where

$P = P^b \cup P^r \cup P^{c1} \cup P^{c2}$ is the set of places where

$P^b = \{p_{k,j}^b | k = 1, 2, \dots, |J|; j = 1, 2, \dots, |M| - 1\}$ is the set of buffer places.

$P^r = \{p_{k,|M|}^r | k = 1, 2, \dots, |J|\}$ is the set of resource places.

$P^{c1} = \{p_{k,j}^{c1} | j = 1, 2, \dots, |M|; k = 1, 2, \dots, |J| - 1\}$ is the set of initially unmarked command places.

$P^{c2} = \{p_{|J|,j}^{c2} | j = 1, 2, \dots, |M|\}$ is the set of initially marked command places.

$T = \{t_{k,j} | k = 1, 2, \dots, |J|; j = 1, 2, \dots, |M|\}$ is the set of transitions. Transition $t_{k,j}$ denotes the operation of job J_k on machine M_j and the release time of $t_{k,j}$ is $w_{k,j}$.

$F = F^b \cup F^r \cup F^{c1} \cup F^{c2}$ is the set of directed arcs where
 $F^b = \{(t_{k,j}, p_{k,j}^b), (p_{k,j}^b, t_{k,j+1}) | k = 1, 2, \dots, |J|; j = 1, 2, \dots, |M| - 1\}$ is the set of directed arcs that go from or to the buffer places.

$F^r = \{(t_{k,|M|}, p_{k,|M|}^r), (p_{k,|M|}^r, t_{k,i}) | k = 1, 2, \dots, |J|\}$ is the set of directed arcs that go from or to the resource places.

$F^{c1} = \{(t_{k,j}, p_{k,j}^{c1}), (p_{k,j}^{c1}, t_{k+1,j}) | j = 1, 2, \dots, |M|; k = 1, 2, \dots, |J| - 1\}$ is the set of directed arcs that go from or to the initially unmarked command places.

$F^{c2} = \{(t_{|J|,j}, p_{|J|,j}^{c2}), (p_{|J|,j}^{c2}, t_{1,j}) | j = 1, 2, \dots, |M|\}$ is the set of directed arcs that go or from to the initially marked command places.

$K_0 : P \rightarrow \{0, 1, 2, \dots\}$ is the initial marking where $K_0(p) = 0$ if $p \in P^b \cup P^{c1}$, $K_0(p) = 1$ if $p \in P^{c2}$ and $K_0(p_{k,|M|}^r)$ equals to the number of pallets carrying J_k if $p_{k,|M|}^r \in P^r$.

The buffer, resource and command places are classified according to the method used in [13]. TEG is shown in Fig. 1 where bars represent transitions, circles represent places and dots represent tokens.

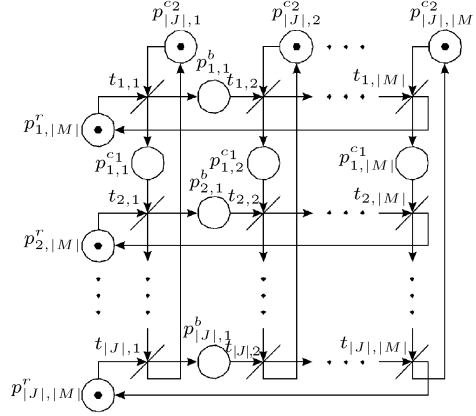


Fig. 1 Timed event graph model TEG

The cyclic reconfigurable flow shop produces one MPS each cycle and the throughput can be represented as the inverse of the cycle time (*i.e.* $1/\lambda$). As described in Section 2, different configurations would result in different system performances. The optimal configuration can be defined as the one under which the cyclic reconfigurable flow shop functions with the minimum cycle time and the minimum number of pallets. The optimal configuration can be obtained in two steps.

Step 1. Suppose that the number of pallets is sufficiently large and let the minimum cycle time be λ^* . Under the assumption that the number of pallets is sufficiently large, the cycle time is totally determined by the command circuits, which consists of only command places, *i.e.*,

$$\lambda = \max_j \frac{\mu(\gamma_j^c)}{\kappa(\gamma_j^c)}, \quad j = 1, 2, \dots, |M|$$

where γ_j^c denotes the j -th command circuit, $\kappa(\gamma_j^c) = 1$ and

$$\mu(\gamma_j^c) = \sum_{k=1}^{|J|} w_{k,j} = \sum_{k=1}^{|J|} \sum_{i=1}^{|m|} z_{k,j} y_{i,j}$$

Combined with the constraints (1) and (2), the following mixed-integer program *MIP1* can be formulated to obtain the minimum cycle time λ^* .

$$\begin{aligned} & \min \quad \lambda \\ & \text{subject to} \quad \lambda - \sum_{k=1}^{|J|} \sum_{i=1}^{|m|} z_{k,i} y_{i,j} \geq 0, \quad j = 1, 2, \dots, |M| \\ & \quad \sum_{j=1}^{|M|} y_{i,j} = 1, \quad i = 1, 2, \dots, |m| \end{aligned} \quad (5)$$

$$\begin{aligned} \sum_{j=1}^{|M|} j y_{i_2, j} - \sum_{j=1}^{|M|} j y_{i_1, j} &\geq 0, \quad (m_{i_1}, m_{i_2}) \in \sigma \\ \lambda &\geq 0 \text{ and } y_{i, j} \in \{0, 1\} \end{aligned}$$

Step 2. Solve the mixed-integer program *MIP2* derived from *TEG* to obtain the optimal configuration and the minimum number of pallets.

For every $p \in P$ and its related directed arcs in *TEG*, there exists one inequality constraint

$$x_{p \bullet} - x_{\bullet p} + K_0(p) \lambda^* \geq w_{\bullet p}$$

where $p \bullet$ ($\bullet p$) denotes the output (resp. input) transition of p , $w_{\bullet p}$ denotes the release time of $\bullet p$ and λ^* is the minimum cycle time obtained in Step 1. By combining with constraints (1), (2) and replacing $w_{k, j}$ with equation (3), *MIP2* can be formulated in the following standard form:

$$\begin{aligned} \min \quad & \sum_{k=1}^{|J|} K_0(p_{k, |M|}^r) \\ \text{subject to} \quad & x_{k, j+1} - x_{k, j} - \sum_{i=1}^{|m|} z_{k, i} y_{i, j} \geq 0, \quad k = 1, 2, \dots, |J|; j = 1, 2, \dots, |M| - 1 \\ & x_{k, 1} - x_{k, |M|} + K_0(p_{k, |M|}^r) \lambda^* - \sum_{i=1}^{|m|} z_{k, i} y_{i, |M|} \geq 0, \quad k = 1, 2, \dots, |J| \\ & x_{k+1, j} - x_{k, j} - \sum_{i=1}^{|m|} z_{k, i} y_{i, j} \geq 0, \quad j = 1, 2, \dots, |M|; k = 1, 2, \dots, |J| - 1 \quad (6) \\ & x_{1, j} - x_{|J|, j} - \sum_{i=1}^{|m|} z_{|J|, i} y_{i, j} \geq -\lambda^*, \quad j = 1, 2, \dots, |M| \\ & \sum_{j=1}^{|M|} y_{i, j} = 1, \quad i = 1, 2, \dots, |m| \\ & \sum_{j=1}^{|M|} j y_{i_2, j} - \sum_{j=1}^{|M|} j y_{i_1, j} \geq 0, \quad (m_{i_1}, m_{i_2}) \in \sigma \end{aligned}$$

$x_{k, j} \geq 0, y_{i, j} \in \{0, 1\}$ and $K_0(p_{k, |M|}^r)$ are positive integers.

4 Case study

In this example, the cyclic reconfigurable flow shop consists of 3 jobs, 3 machines and 4 modules. J_1 is to be processed on modules in the sequence of $m_3 m_4$, J_2 in the sequence of $m_1 m_4$ and J_3 in the sequence of $m_1 m_2 m_3$. The processing time matrix of jobs on modules

$$Z = \begin{pmatrix} 0 & 0 & 45 & 30 \\ 23 & 0 & 0 & 61 \\ 60 & 76 & 5 & 0 \end{pmatrix}$$

We have

$$\begin{aligned} J &= \{J_1, J_2, J_3\}; M = \{M_1, M_2, M_3\}; m(J_1) = \{m_3, m_4\}; m(J_2) = \{m_1, m_4\}; m(J_3) = \{m_1, m_2, m_3\} \\ m &= \bigcup_{k=1}^3 m(J_k) = \{m_1, m_2, m_3, m_4\}; \sigma(J_1) = \{(m_3, m_4)\}; \sigma(J_2) = \{(m_1, m_4)\}; \sigma(J_3) = \{(m_1, m_2), (m_2, m_3)\} \\ \sigma &= \bigcup_{k=1}^3 \sigma(J_k) = \{(m_1, m_2), (m_2, m_3), (m_3, m_4), (m_1, m_4)\} \end{aligned}$$

After solving *MIP1* and *MIP2*, the optimal configuration is

$$\mathbf{Y}^* = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

the minimum cycle time is 126 and the minimum number of pallets carrying J_1, J_2, J_3 are 1, 2 and 2 respectively.

5 Future work

Future work should be concentrated mainly on the following aspects:

1) The optimal configuration is defined as the one under which the cyclic reconfigurable flow shop functions with the minimum cycle time and the minimum number of pallets. More generally, other factors such as the reconfiguration cost should be considered in evaluating the overall system performance.

2) The cyclic reconfigurable flow shop with different scheduling policies and limited buffers between machines should be further studied.

3) Stochastic models should be established for the case that $z_{k,i}$ is random.

References

- 1 Koren Y, Heisel U, Jovane F, Moriwaki T, Pritschow G, Ulsoy G, Van Brussel H. Reconfigurable manufacturing systems. *Annals of the CIRP*, 1999, **48**(2): 527~540
- 2 Zhao X B, Wang J C, Luo Z B. A stochastic model of a reconfigurable manufacturing system-Part 1: A framework. *International Journal of Production Research*, 2000, **38**(10): 2273~2285
- 3 Yigit A S, Allahverdi A. Optimal selection of module instances for modular products in reconfigurable manufacturing systems. *International Journal of Production Research*, 2003, **41**(17): 4063~4074
- 4 Abdi M R, Labib A W. A design strategy for reconfigurable manufacturing systems (RMSs) using analytical hierarchical process (AHP): A case study. *International Journal of Production Research*, 2003, **41**(10): 2273~2299
- 5 Abdi M R, Labib A W. Grouping and selecting products: The design key of reconfigurable manufacturing systems (RMSs). *International Journal of Production Research*, 2004, **42**(3): 521~546
- 6 Abdi M R, Labib A W. Feasibility study of the tactical design justification for reconfigurable manufacturing systems using the fuzzy analytical hierarchical process. *International Journal of Production Research*, 2004, **42**(15): 3055~3076
- 7 Ramamoorthy C V, Ho G S. Performance evaluation of asynchronous concurrent systems using Petri nets. *IEEE Transactions on Software Engineering*, 1980, **6**(5): 440~449
- 8 Karp R M. A characterization of the minimum cycle mean in a digraph. *Discrete Mathematics*, 1978, **23**(3): 309~311
- 9 Cochet-Terrasson J, Cohen G, Gaubert S, Gettrick M M, Quadrat J P. Numerical computation of spectral elements in max-plus algebra. In: Proceedings of the IFAC Conference on System Structure and Control, Nantes, France: 1998
- 10 Campos J, Chiola G, Colom J M, Silva M. Properties and performance bounds for timed marked graphs. *IEEE Transactions on Circuits and Systems I: Fundamental Theory and Applications*, 1992, **39**(5): 386~401
- 11 Magott J. Performance evaluation of concurrent systems using Petri nets. *Information Processing Letters*, 1984, **18**(1): 7~13
- 12 Yamada T, Kataoka S. On some LP problems for performance evaluation of timed marked graphs. *IEEE Transactions on Automatic Control*, 1994, **39**(3): 696~698
- 13 Hillion H, Proth J M. Performance evaluation of job-shop systems using timed event-graphs. *IEEE Transactions on Automatic Control*, 1989, **34**(1): 3~9

REN Si-Cheng Ph.D. candidate at Institute of Automation, Chinese Academy of Sciences. His research interests include reconfigurable manufacturing system modeling and optimization.

XU De Received his bachelor and master degrees from Shandong University of Technology in 1985 and 1990, respectively. He received his Ph. D. degree from Zhejiang University in 2001. He has been with Institute of Automation, Chinese Academy of Sciences (CASIA) since 2001. He is an associate professor with the Laboratory of Complex Systems and Intelligence Science, CASIA. His research interests include robotics, computer vision, and reconfigurable manufacturing system.

WANG Fang Postdoctor at Institute of Automation, Chinese Academy of Sciences. Her research interests include system reliability and advanced manufacturing systems.

TAN Min Professor at Institute of Automation, Chinese Academy of Sciences. His research interests include robotics, theory of complex system, and reconfigurable manufacturing system.