On the Control of Plants with Hysteresis: Overview and a Prandtl-Ishlinskii Hysteresis Based Control Approach[1)]

Qingqing Wang[1] Chun-Yi Su[1] Yonghong Tan[2]

[1](*Deptartment of Mechanical and Industrial Engineering, Concordia University, Montreal, Quebec, H3G 1M8 Canada*)

[2](*Laboratory of Intelligent Systems and Control Engineering, Guilin University of Electronic Technology, Guilin* 541000 *P.R.China*)

(E-mail: cysu@me.concordia.ca)

Abstract The development of control techniques to mitigate the effects of unknown hysteresis preceding with plants has recently re-attracted significant attention. In this paper, we first give a brief review of presently developed hysteresis models and hysteresis compensating control methods. Then, with the use of the Prandtl-Ishlinskii hysteresis model, we propose a robust adaptive control scheme. The novelty is that the model of hysteresis nonlinearities is firstly fused with the available control techniques without necessarily constructing a hysteresis inverse. The global stability of the adaptive system and tracking a desired trajectory to a certain precision are achieved. Simulations performed on a nonlinear system illustrate and clarify the approach.

Key words Hysteresis model, hysteresis compensating control method

1 Introduction

Hysteresis usually refers to a nonlinear relation between two time-dependent variables that is multi-valued, and often takes the form of non-smooth loops, as shown in Fig. 1. It occurs in a wide range of physical systems. In particular, smart material-based actuators which have been found a variety of applications such as in high-precision positioning devices exhibit hysteresis phenomena[1,2]. It was shown that errors caused by the hysteresis effects can lead to undesirable inaccuracies or oscillations and even instability[3,4]. The development of control techniques to mitigate the effects of unknown hysteresis has been studied for decades and has recently re-attracted significant attention. Much of this renewed interest is a direct consequence of the importance of hysteresis in current applications. Interest in studying dynamic systems with actuator hysteresis is also motivated by the fact that they are nonlinear systems with *non-smooth* nonlinearities for which traditional control methods are insufficient[5]. It is typically challenging in developing new approaches to control a system in the presence of unknown hysteresis nonlinearities.

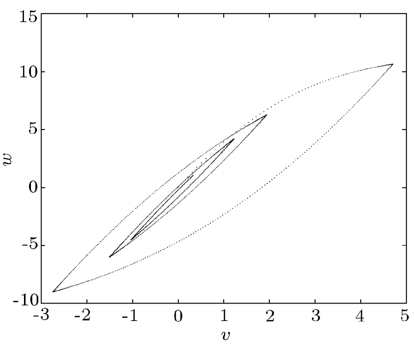

Fig. 1 Hysteresis curves

To address such a challenge, it is necessary to develop suitable mathematical models that are sufficiently accurate, amenable to controller design for nonlinearity compensation and efficient enough to use in real-time applications. In this paper, we first give a brief review of hysteresis models and presently developed hysteresis compensating control methods. Then, by using Prandtl-Ishlinskii model, we propose an integrated robust adaptive control scheme to fuse the model of hysteresis with the available control techniques without necessarily constructing a hysteresis inverse. The global stability of the adaptive system and tracking a desired trajectory to a certain precision are achieved. Simulations performed on a nonlinear system illustrate and further validate the effectiveness of the proposed approach.

1) Supported by the Concordia University under Research Chair Grant
Received February 12, 2004; in revised form October 18, 2004

2 Literature review

2.1 Models of hysteresis

To develop general models that can represent diverse hysteresis behaviors has been a subject of interest since the end of the 19th century. Several physics-based hysteresis models, which built on the first principles of physics, have been proposed and each has been found applications in certain areas. Since the early 1970's, from purely phenomenological viewpoint, systematic mathematical investigations into the hysteresis as a general nonlinear behavior have been carried out. The basic idea is to model the real complex hysteresis nonlinearity by the weighted aggregate effect of all possible so-called elementary hysteresis operators, which are non-complex hysteresis with a simple mathematical structure. Phenomenological models, such as Preisach models and Prandtl-Ishlinskii models, are used to produce behaviors similar to those physical systems without necessarily providing physical insight of the problems. In the following development, we give a brief overview of currently popular hysteresis models. The reader may also refer to [6,7] for recent review. The detailed discussion on this subject can be found in the monographs[8~11].

The Duhem Models. The Duhem model focuses on the fact that the output can only change its character when the input changes direction. In general, for suitable functions f_1 and f_2, the hysteresis is given by two families of curves in the (v, w) plane defined as the solution to the differential equation

$$\dot{w}(t) = f_1(w, v)\dot{v}_+(t) + f_2(w, v)\dot{v}_-(t)$$

with $\dot{v}_+(t) = \max[0, \dot{v}(t)]$, $\dot{v}_-(t) = \min[0, \dot{v}(t)]$. Coleman and Hodgdon[12,13] extensively studied this model, using the equation

$$\frac{\mathrm{d}B}{\mathrm{d}t} = \alpha|\frac{\mathrm{d}H}{\mathrm{d}t}|[f(H) - B] + \frac{\mathrm{d}H}{\mathrm{d}t}g(H) \tag{1}$$

where $\alpha > 0$ is a constant, H is the applied magnetic field and B is the level of magnetization of the medium. They proved that the following conditions for f and g are necessary and sufficient for Equation (1) to give a hysteresis diagram,

1) $f(\cdot)$ is piecewise smooth, monotone increasing, odd, with $\lim_{H\to\infty} f'(H)$ finite;

2) $g(\cdot)$ is piecewise continuous, even, with $\lim_{H\to\infty} g(H) = \lim_{H\to\infty} f'(H)$;

3) $f'(H) > g(H) > \alpha e^{\alpha H} \int_H^\infty |f'(\eta) - g(\eta)|e^{-\alpha\eta}\mathrm{d}\eta$ for all $H > 0$.

And the solution can be explicitly expressed as

$$B = f(H) + [B_0 - f(H_0)]e^{-\alpha(H-H_0)sgn\dot{H}} + e^{-\alpha H sgn\dot{H}} \int_{H_0}^{H} [g(\eta) - f'(\eta)]e^{-\alpha\eta sgn\dot{H}}\mathrm{d}\eta \tag{2}$$

for H piecewise monotone and $\dot{H}$ constant. They showed that functions and parameters in (1) can be fine-tuned to match experimental results for rate-independent hysteresis in ferromagnetic soft materials. A modification based on exchanging the positions of B and H in the differential Equation (1) was also studied by Hodgdon[14,15].

The Bouc-Wen model. Suppose that x is the position of a oscillator system given by

$$\ddot{x} = f(x, \dot{x}, z, u) \tag{3}$$

where z is the hysteretic variable proportional to the restoring force acting on the oscillator described by the first order differential equation

$$\dot{z} = A\dot{x} - \beta\dot{x}|z|^n - \gamma|\dot{x}||z|^{n-1}z \tag{4}$$

the parameters n, A, β, and γ are shape parameters of the hysteresis curves which can also be functions of time. Note that in this model $\dot{x}$ acts as an input, and the equation is not involved in x although the hysteresis phenomenon is observed between x and z. When $n = 1$, (4) becomes a linear ordinary differential equation which can be solved according to the signs of $\dot{x}$ and z. As n increases to ∞, the hysteresis loop will converge to a bilinear curve defined by $\dot{z} = \dot{x}[sgn(z + A) - sgn(z - A)]/2$. The model has been applied to describe hysteresis in a single degree of freedom oscillator[16] and a magnetorheological damper attached to a scaled three-degree of freedom building[7].

The Jiles-Atherton model. This model is widely used in modelling ferromagnetic hysteresis[17~19]. In its original form[18], magnetization $M = M_{rev} + M_{irr}$ was decomposed into its reversible component M_{rev} and irreversible component M_{irr}. The differential equations with respect to the frequency of the imposed magnetic field $H(t)$ are represented as

$$\frac{\mathrm{d}M_{irr}}{\mathrm{d}H} = \frac{M_{an} - M_{irr}}{\delta k - \alpha(M_{an} - M_{irr})} \tag{5}$$

$$\frac{\mathrm{d}M_{rev}}{\mathrm{d}H} = c(\frac{\mathrm{d}M_{an}}{\mathrm{d}H} - \frac{\mathrm{d}M_{irr}}{\mathrm{d}H}) \tag{6}$$

where M_{an} is the anhysteretic magnetization

$$M_{an} = M_s\left\{coth\left(\frac{H+\alpha M}{a}\right) - \left(\frac{a}{H+\alpha M}\right)\right\} \tag{7}$$

δ is a directional parameter. It takes the value +1 for $\mathrm{d}H/\mathrm{d}t > 0$ and -1 for $\mathrm{d}H/\mathrm{d}t < 0$. a, α, c, k, and the saturation magnetization M_s are the parameters to be determined from experimental measurements of the hysteresis loops, see [18,20,21]. The Jiles-Atherton model and the Preisach model are two models often used in magneto-dynamic field. Philips[22] compared the computation results from both models with experimental measurements. It was found that the identification of the parameters in the Jiles-Atherton model requires less measurements, while the Preisach model fits the hysteresis loops better.

The Preisach model. The most popular hysteresis model is certainly the Preisach model. It was initially proposed by Weiss and de Freundenreichin in 1916[23]. In 1935, Preisach suggested the geometrical interpretation, which is one of the main features of the model[24]. The success of the model, however, has to be ascribed to Krasnosel'skii and other Russian mathematicians having elucidated the phenomenological character of the Preisach' model. They showed that this algorithm can be considered as a superposition of elementary hysteretic "relay" operators

$$W(t) = \int_0^{+\infty}\int_{-\infty}^{+\infty} \mu(\alpha,\beta)\gamma_{\alpha,\beta}[v](t)\mathrm{d}\alpha\mathrm{d}\beta \tag{8}$$

where $\gamma_{\alpha,\beta}[v](t)$ is a relay hysteresis defined as

$$\gamma_{\alpha,\beta}[v](t) = \begin{cases} +1, & \text{if } v(t) > \alpha \\ -1, & \text{if } v(t) < \beta \\ \textit{remains unchanged}, & \text{if } \beta < v(t) < \alpha \end{cases} \tag{9}$$

An extensive review on the Preisach model, its modified forms, and model identification methods can be found in monographs[8,10] and papers[25,26]. There are many experimental setups to show that this model can describe the hysteresis behavior in smart material-based actuators and sensors, such as magnetostrictive[27], piezoceramic in a stacked form[28], and shape memory alloys[29] actuators.

The Prandtl-Ishlinskii model. There are other types of hysteresis operators such as "play" and "stop" operators. The models set up by composition of play or stop operators are referred to as Prandtl-Ishlinskii models. Suppose $E_r[v]$ are basic elastic-plastic elements or stop operators for all $r \in [0, R]$, then the model can be expressed as

$$w(t) = \int_0^R p(r)E_r[v](t)\mathrm{d}r \tag{10}$$

where $p(r)$ is a given density function. Although the model itself was introduced much earlier[30,31], the reader may refer to [8,9,32] for recent development. In our research the Prandtl-Ishlinskii model is applied to define the hysteresis nonlinearity presented as an input of the plant. The detailed discussion about this model will be given in section 4.

The Krasnosel'skii-Pokrovskii hysteron. In 1970's, Krasnosel'skii and Pokrovskii systematically investigated the hysteresis phenomenon from mathematical view of point. They geometrically defined the basic model of hysteresis, referred as hysteron, see the monograph[9]. The definition is general and can cover various forms of hysteresis loops. A simple example is a play operator. Banks, Kurdila and Webb[33,34] developed a model by use of generalized play operators, called Krasnosel'skii-Pokrovskii (KP) operators. The model represents hysteresis as the cumulative effect of weighted KP operators distributed over a domain in R^2. Galinaitis investigated the KP model focusing on the properties of inverse and approximation[35].

2.2 Control methods

Having given a brief review of presently developed hysteresis models, we now turn our attention to the control techniques when a hysteresis is preceding with plants. In the literature, the most common approach to mitigate the effects of hysteresis is to construct an inverse operator, which was pioneered by Tao and Kokotovic[4]. For hysteresis with major and minor loops, they used a simplified linear parameterized model to develop an adaptive hysteresis inverse model with parameters updated on line by adaptive laws. Following this concept, many research papers have been proposed for the compensation of the hysteresis with different hysteresis models. The main issue is how to find the inverse of the hysteresis.

The control architectures based on Preisach model has been studied by many researchers. Ge and Jouaneh[36] proposed a static approach to reduce the hysteresis effects in the problem of tracking control of a piezoceramic actuator for desired sinusoidal trajectory. The relationship between the input and the output of actuator was first initialized by a liner approximation model of a specific hysteresis. The Preisach model of the hysteresis was then used to redefine the corresponding input signals for the desired output of the actuator displacements. PID feedback controller was used to adjust the tracking errors. Galinaitis[35] analytically investigated the inverse properties of the Preisach model and proved that a Preisach operator can only be locally invertible. He gave a closed form inverse formula when the weight function of Preisach model was taking a specific form. Mittal and Meng[37] developed a method of hysteresis compensation in electromagnetic actuator through inversion of numerically expressed Preisach model in terms of the first-order reversal curves and the input history. Instead of modelling the forward hysteresis and then finding the inverse, Croft *et al.* and Bernard[2,38] directly formulated the inverse hysteresis effect using Preisach model.

Control designs based on the inverse of KP model can be found in [35,39]. Webb defined a parameterized discrete inverse KP model, combined with adaptive laws to adjust the parameters on-line to compensate hysteresis effects[39]. Galinaitis mathematically investigated the properties and the discrete approximation method of the KP operators[35]. Recently, a feed-forward control design based on the inverse of Prandtl-Ishlinskii model was also applied to reduce hysteresis effects in piezoelectric actuators[32].

In addition to the above mentioned model-based inverse methods, neural networks and fuzzy logic models were also developed. It is well known that the universal approximation property is one of the most important properties of neural networks and fuzzy systems. However, this property is generally proven for continuous and one-to-one functions. Wei and Sun[57] studied the rate-independent memory property. After analysis multi-layer feed-forward , recurrent and reinforcement learning networks, they found that networks with only computational nodes and links cannot function as hysteresis simulators. They proposed a propulsive neural unit to construct hysteretic memory. Several propulsive neural units with distinct sensible ranges were used to form a model. It can be trained to follow the loops given by the Preisach model. Selmic[40] gave a neural network structure to approximate piecewise continuous functions appearing in friction, or functions with jumps. Hwang[41,42] developed a neuro-adaptive control method for unknown piezoelectric actuator systems. The proposed neural network included two different nonlinear gains according to change rate of input signal and a linear dynamic system to learn the dynamics of the piezoelectric actuators. A forward control based on the inverse of learned model was used to achieve an acceptable tracking result. Because the tracking performance by a control could not be guaranteed as the system was subject to uncertainties, a discrete-time variable-structure control was synthesized to improve the performance. Readers can also refer to [43,44].

Essentially, the inversion methods usually treat hysteresis and structure response function separately, that is, use the inverse model in the forward loop to cancel hysteresis behavior, and then design a feedback controller to compensate the structural dynamic effects. However, it is difficult to decouple the effects from the hysteresis and the structural dynamics from experimental measurements. It would be better to develop an approach that can consider both effects simultaneously[45]. Due to the multi-valued and non-smoothness feature of hysteresis, methods using inverse models are complicated, computationally costly and possess the strong sensitivity of the model parameters to unknown measurement errors. These issues are directly linked to the difficulty of stability analysis of the systems except for certain special cases[4].

Passivity-based stability and control of hysteresis in smart actuators were attempted by Pare and Gorbet[29,46]. In [29] energy properties of the Preisach hysteresis model were investigated, and passivity was demonstrated for the relationship between the input and the derivative of the output. The result only leads to stability of rate control of hysteresis systems.

The differential models of hysteresis were used for control purposes[7,47~49]. The Bouc-Wen model was applied to develop a semi-active structural control model for a magnetorheological damper attached to a three-story scaled building, see [48]. Su *et al.* used Duhem model investigated by Coleman and Hodgdon[49]. He combined the solution properties of the model with adaptive control techniques and developed a robust adaptive control algorithm. This method integrated the hysteresis compensation with control techniques without constructing an inverse of hysteresis. The dynamic characteristic of this type of models can be implemented in state-space. The main challenge is resulted from the high nonlinearity and lack of knowledge about mathematical properties of the differential models when they are applied to system control.

From the above overview, it is clear that the most common approach is to construct an inverse operator. The discussions on the fusion of the available hysteresis models with the available control techniques is still spare in the literature. With all the developed hysteresis models, it is by nature to seek the way to fuse those hysteresis models with available robust control techniques to mitigate the effects of hysteresis. Therefore, the challenge addressed here is to fuse the available hysteresis models with available control techniques to have the basic requirement of stability of the system without constructing inverse hysteresis nonlinearity.

As an illustration, in this paper we show such a possibility by fusing the Prandtl-Ishlinskii models with the adaptive robust control approach to mitigate the effects of the hysteresis. The proposed control law ensures the global stability of the adaptive system and achieves both stabilization and strict tracking precision. Simulations performed on a nonlinear system illustrate and further validate the effectiveness of the proposed approach. The proposed method can be observed as an initial step to to fuse the available hysteresis models with available control techniques.

3 Problem statement

Consider a controlled system consists of a nonlinear plant preceded by an actuator with hysteresis nonlinearity, that is, the hysteresis is presented as an input of the nonlinear plant. A hysteresis denoted as an operator

$$w(t) = P[v](t) \tag{11}$$

with $v(t)$ as input and $w(t)$ as output. The operator $P[v]$ will be discussed in detail in the forthcoming section. The nonlinear dynamic system being preceded by the above hysteresis is described in the canonical form,

$$x^{(n)}(t) + \sum_{i=1}^{k} a_i Y_i(x(t), \dot{x}(t), \dots, x^{(n-1)}(t)) = bw(t) \tag{12}$$

where Y_i are known continuous, linear or nonlinear functions. Parameters a_i and control gain b are constants. It is a common assumption that the sign of b is known. Without losing generality, we assume $b > 0$. It should be noted that more general classes of nonlinear systems can be transformed into this structure[50]. The control objective is to design a control law for $v(t)$ in (11), to force the plant state vector, $\boldsymbol{x} = [x, \dot{x}, \dots, x^{(n-1)}]^{\mathrm{T}}$, to follow a specified desired trajectory, $\boldsymbol{x}_d = [x_d, \dot{x}_d, \dots, x_d^{(n-1)}]^{\mathrm{T}}$, *i.e.*, $\boldsymbol{x} \to \boldsymbol{x}_d$ as $t \to \infty$.

4 Prandtl-Ishlinskii hysteresis models

4.1 Stop and play operators

We list below some basic hysteresis operators. A detailed discussion on this subject can be found in the monographs[8,9,11]. The first operator to be introduced is the stop operator, $w(t) = E_r[v](t)$, with threshold r.

Analytically, suppose $C_m[0, t_E]$ is the space of piece-wise monotone continues functions, for any input $v(t) \in C_m[0, t_E]$, let

$$e_r(v) = \min(r, \max(-r, v)) \tag{13}$$

Then, for any initial value $w_{-1} \in R$ (the initial state before $v(0)$ is applied at time $t = 0$)[8] and $r > 0$, the stop operator E_r can be given by the inductive definition

$$\begin{aligned} &E_r[v; w_{-1}](0) = e_r(v(0) - w_{-1}) \\ &E_r[v; w_{-1}](t) = e_r(v(t) - v(t_i) + E_r[v; w_{-1}](t_i)), \text{ for } t_i < t \leqslant t_{i+1} \text{ and } 0 \leqslant i \leqslant N-1 \end{aligned} \tag{14}$$

where $0 = t_0 < t_1 < \cdots < t_N = t_E$ is a partition of $[0, t_E]$ such that the function v is monotone on each of the sub-intervals $[t_i, t_{i+1}]$. The argument of the operator is written in square brackets to indicate the functional dependence, since it maps a function to a function. The stop operator however is mainly characterized by its threshold parameter r which determines the hight of the hysteresis region in the (v, w) plane.

There is another basic hysteresis nonlinearity operator, **play**. For a given input $v(t) \in C_m[0, t_E]$, the play operator F_r, with threshold $r \geq 0$ and the initial value $w_{-1} \in R$, is defined by

$$\begin{aligned} &F_r[v; w_{-1}](0) = f_r(v(0), w_{-1}) \\ &F_r[v; w_{-1}](t) = f_r(v(t), F_r[v; w_{-1}](t_i)), \text{ for } t_i < t \leqslant t_{i+1} \text{ and } 0 \leqslant i \leqslant N-1 \end{aligned} \tag{15}$$

with

$$f_r(v, w) = \max(v - r, \min(v + r, w)) \tag{16}$$

where the partition $0 = t_0 < t_1 < \cdots < t_N = t_E$ is the same as defined for the stop operator. From the definitions given in (14) and (15), it has been proved[9] that the operator F_r is the complement of E_r, *i.e.*, they are closely related through the equation

$$E_r + F_r = I_d \tag{17}$$

for any piece-wise monotone input function v and $r \geqslant 0$, where I_d is an identity mapping.

In the sequel, we will simply write $E_r[v]$ or $F_r[v]$ to denote $E_r[v; w_{-1}]$ or $F_r[v; w_{-1}]$ so long as doing so does not affect the proof. Due to the nature of the play and stop operators, above discussions are defined on the space $C_m[0, t_E]$ of continuous and piecewise monotone functions; however, they can also be extended to the space $C[0, t_E]$ of continuous functions.

4.2 Prandtl-ishlinskii model

The Prandtl-Ishlinskii model was introduced to formulate the elastic-plastic behavior through a weighted superposition of basic elastic-plastic elements $E_r[v]$, or **stop** as following

$$w(t) = \int_0^R p(r) E_r[v](t) \mathrm{d}r \tag{18}$$

where $p(r)$ is a given density function, satisfying $p(r) \geqslant 0$ with $\int_0^\infty r p(r) \mathrm{d}r < \infty$, which is supposed to be identified from experimental data. With thus defined density function, this operator maps $C[t_0, \infty)$ into $C[t_0, \infty)$,*i.e.*, Lipschitz continuous inputs will yield Lipschitz continuous outputs[9]. Since the density function $p(r)$ vanishes for large value of r, in the literature the choice of $R = \infty$ as upper limit of integration is just a matter of convenience[8]. As an illustration, Fig. 1 shows $w(t)$ generated by model (18), with $p(r) = e^{-0.067(r-1)^2}$, $r \in [0, 10]$, and input $v(t) = 7\sin(3t)/(1 + t)$, $t \in [0, 2\pi]$ with $\psi = 0$. Since the operator F_r is the complement of E_r, the Prandtl-Ishlinskii model can also be expressed through **play** operator. Using Equation (17) and substituting E_r in (18) by F_r, the Prandtl-Ishlinskii model defined by the play hysteresis operator is expressed as

$$w(t) = p_0 v(t) - \int_0^R p(r) F_r[v](t) \mathrm{d}r \tag{19}$$

where $p_0 = \int_0^R p(r)\mathrm{d}r$ is a constant, depending on the density function. Noticing that Equation (19) decomposes the hysteresis behavior into two terms. The first term describes the linear reversible part and the second term gives the hysteresis. This decomposition is crucial for design of the controller, because the currently available robust adaptive control techniques can be utilized for the controller design.

5 Controller design

In this section, we propose an adaptive controller for plants in the form of (12), preceded by the hysteresis described by the Prandtl-Ishlinskii model. The proposed controller will lead to global stability and yields tracking to within a desired precision. Rewrite (19) as following

$$w(t) = p_0 v(t) - d[v](t) \tag{20}$$

where

$$d[v](t) = \int_0^R p(r) F_r[v](t)\mathrm{d}r \tag{21}$$

with $p_0 = \int_0^R p(r)\mathrm{d}r$. If the hysteresis in the system is known, that is, $p(r)$ and the hysteresis internal state (refer to [8]) are given or can be accurately estimated, for any continuous input function $v(t)$ at a time instant t, $F_r[v](t)$ is a fixed set of line segments decided by some extreme values of $v(t)$. The integration of $d[v]$ can be calculated online, we use $d[v]$ as a feed forward compensator to cancel the second non-linear partition. However, in most cases, it is difficult or impossible to accurately decide the hysteresis in the system. In this paper we attempt to develop a direct control method by using currently available robust adaptive control techniques.

Substitute the hysteresis model of (20) into (12), we have

$$x^{(n)}(t) + \sum_{i=1}^{k} a_i Y_i(x(t), \dot{x}(t), \ldots, x^{(n-1)}(t)) = b\{p_0 v(t) - d[v](t)\} \tag{22}$$

which results in a linear relation to the input signal $v(t)$ plus a shifting term $bd[v]$. In the robust control contents, $bd[v](t)$ is normally treated as a disturbance function, which is assumed to be bounded or bounded by a known function. Here, since $bd[v](t)$ is bounded by the control signal v to be decided[8] we cannot make an assumption on its boundedness.

Define the tracking error vector $\tilde{\boldsymbol{x}} = \boldsymbol{x} - \boldsymbol{x}_d$, and a filtered tracking error as

$$s(t) = (\frac{\mathrm{d}}{\mathrm{d}t} + \lambda)^{(n-1)} \tilde{x}(t), \quad \lambda > 0 \tag{23}$$

$s(t)$ can be rewritten as $s(t) = \Lambda^{\mathrm{T}} \tilde{\boldsymbol{x}}(t)$ with $\Lambda^{\mathrm{T}} = [\lambda^{(n-1)}, (n-1)\lambda^{(n-2)}, \ldots, 1]$. Rather than driving the adaptive law with the filtered error $s(t)$, we introduce a tuning error, s_ϵ, as follows:

$$s_\epsilon = s - \epsilon sat(\frac{s}{\epsilon}) \tag{24}$$

where ϵ is an arbitrary positive constant and $sat(\cdot)$ is the saturation function. The tuning error, s_ϵ, disappears when s is less than ϵ. For the development of a robust adaptive control law, the following assumptions regarding the plant and the hysteresis are made.

Assumption 1. The desired trajectory $\boldsymbol{x}_d = [x_d, \dot{x}_d, \ldots, x_d^{(n-1)}]^{\mathrm{T}}$ is continuous and available. Furthermore $[\boldsymbol{x}_d^{\mathrm{T}}, x_d^{(n)}]^{\mathrm{T}} \in \varOmega_d \subset R^{n+1}$ with $\varOmega_d$ a compact set.

Assumption 2. There exist known constants $0 < b_{\min} \leqslant b_{\max}$ such that the control gain b in (12) satisfies $b \in [b_{\min}, b_{\max}]$.

Assumption 3. Define $\theta \triangleq [\frac{a_1}{bp_0}, \ldots, \frac{a_r}{bp_0}]^{\mathrm{T}} \in R^k$, then

$$\theta \in \varOmega_\theta \triangleq \{\theta : \theta_{i\min} \leqslant \theta_i \leqslant \theta_{i\max}, \forall i \in \{1, \ldots, k\}\}$$

where $\theta_{i\min}$ and $\theta_{i\max}$ are some known real numbers.

Assumption 4. There exist known constants $p_{0\min}$ and $p_{\max}$, such that $p_0 > p_{0\min}$, and $p(r) \leqslant p_{\max}$ for all $r \in [0, R]$.

Remark. Assumption 1 is generally adopted for the design of tracking controller. Assumption 2 is common for the nonlinear controller designs. Basically, Assumption 3 implies that the ranges of the plant parameters, $a_i, i = 1...k$, are known in advance. This is a reasonable assumption on the prior knowledge of the system. As for Assumption 4, since $p(r)$ is the density function, it is reasonable to set an upper bound $p_{\max}$ for $p(r)$. Here $p_{0\min} > 0$ must be satisfied.

In presenting the developed robust adaptive control law, the following definitions are required:

$$\tilde{\theta}(t) = \hat{\theta}(t) - \theta \tag{25}$$

$$\tilde{\phi}(t) = \hat{\phi}(t) - \phi, \quad \phi \triangleq (bp_0)^{-1} \tag{26}$$

$$\tilde{p}(t,r) = \hat{p}(t,r) - p(r), \text{ for all } r \in [0, R] \tag{27}$$

where $\hat{\theta}$, $\hat{\phi}$ and $\hat{p}(t,r)$ are the estimates of θ, ϕ, and $p(r)$. Let

$$B(v(t)) \triangleq \int_0^R \frac{p(r)}{p_{0\min}} |F_r[v](t)| \mathrm{d}r \tag{28}$$

and the estimation $\hat{B}(t)$ is given by $\int_0^R \frac{\hat{p}(t,r)}{p_{0\min}} |F_r[v](t)| \mathrm{d}r$, which leads to

$$\tilde{B}(t) = \int_0^R \frac{(\hat{p}(t,r) - p(r))}{p_{0\min}} |F_r[v](t)| \mathrm{d}r \tag{29}$$

Given the plant and hysteresis model subject to the assumptions described above, and noticing that the term $d[v](t)$ in (20) is in the form of integration with the kernel $F_r[v](t)$, we propose the following control law:

$$v(t) = -k_d s(t) + \hat{\phi} u_{fd}(t) + Y^{\mathrm{T}}(\boldsymbol{x})\hat{\theta} + u_N(t) \tag{30}$$

with

$$u_{fd}(t) = x_d^{(n)}(t) - \Lambda_v^{\mathrm{T}} \tilde{\boldsymbol{x}}(t) \tag{31}$$

$$u_N(t) = -sat(\frac{s}{\epsilon})\hat{B}(t) \tag{32}$$

where $k_d > 0$; $Y \triangleq [Y_1, \ldots, Y_k]^{\mathrm{T}} \in R^k$; $\Lambda_v^{\mathrm{T}} = [0, \lambda^{(n-1)}, (n-1)\lambda^{(n-2)}, \ldots, (n-1)\lambda]$. The parameters $\hat{\phi}$, $\hat{\theta}$, and function $\hat{B}(t)$ are updated by the following adaptation laws

$$\dot{\hat{\theta}} = Proj(\hat{\theta}, -\gamma Y(\boldsymbol{x}) s_\epsilon) \tag{33}$$

$$\dot{\hat{\phi}} = Proj(\hat{\phi}, -\eta u_{fd} s_\epsilon) \tag{34}$$

$$\frac{\partial}{\partial t}\hat{p}(t,r) = Proj(\hat{p}(t,r), q\frac{|F_r[v](t)|}{p_{0\min}}|s_\epsilon|), \quad \text{for } r \in [0, R] \tag{35}$$

where parameters γ, η and q are positive constants determining the rates of the adaptations, and $Proj(\cdot,\cdot)$ is a projection operator formulated as follows:

$$\{Proj(\hat{\theta}, -\gamma Y s_\epsilon)\}_i = \begin{cases} 0, & \text{if } \hat{\theta}_i = \theta_{i\max} \text{ and } \gamma(Ys_\epsilon)_i < 0 \\ -\gamma(Ys_\epsilon)_i, & \text{if } [\theta_{i\min} < \hat{\theta}_i < \theta_{i\max}], \\ & \text{or } [\hat{\theta}_i = \theta_{i\max} \text{ and } \gamma(Ys_\epsilon)_i \geqslant 0], \\ & \text{or } [\hat{\theta}_i = \theta_{i\min} \text{ and } \gamma(Ys_\epsilon)_i \leqslant 0] \\ 0, & \text{if } \hat{\theta}_i = \theta_{i\min} \text{ and } \gamma(Ys_\epsilon)_i > 0 \end{cases} \tag{36}$$

$$Proj(\hat{\phi}, -\eta u_{fd} s_\epsilon) = \begin{cases} 0, & \text{if } \hat{\phi} = \phi_{\max} \text{ and } \eta u_{fd} s_\epsilon < 0 \\ -\eta u_{fd} s_\epsilon, & \text{if } [\phi_{\min} < \hat{\phi} < \phi_{\max}], \\ & \text{or } [\hat{\phi} = \phi_{\max} \text{ and } \eta u_{fd} s_\epsilon \geqslant 0], \\ & \text{or } [\hat{\phi} = \phi_{\min} \text{ and } \eta u_{fd} s_\epsilon \leqslant 0] \\ 0, & \text{if } \hat{\phi} = \phi_{\min} \text{ and } \eta u_{fd} s_\epsilon > 0 \end{cases} \tag{37}$$

$$Proj(\hat{p}(t,r), q\frac{|F_r[v](t)|}{p_{0\min}}|s_\epsilon|) = \begin{cases} 0, & \text{if } \hat{p}(t,r) = p_{\max} \\ q\frac{|F_r[v](t)|}{p_{0\min}}|s_\epsilon|, & \text{if } 0 \leqslant \hat{p}(t,r) < p_{\max} \end{cases} \tag{38}$$

The stability of the closed-loop system described in (22), (30) and (33)~(35) is established in the following theorem.

Theorem: For the plant in Equation (12) with the hysteresis (19) at the input subject to Assumptions 1)~4), the robust adaptive controller specified by Equations (30) and (33)~(35) ensures that: if

$$\begin{aligned} &\hat{\theta}(t_0) \in \Omega_\theta = \{\theta : \theta_{i\min} \leqslant \theta_i \leqslant \theta_{i\max}, \forall i \in \{1,\dots,k\}\} \\ &\hat{\phi}(t_0) \in \Omega_\phi = \{\phi = (bp_0)^{-1} : (b_{\max}p_{0\max})^{-1} \leqslant \phi \leqslant (b_{\min}p_{0\min})^{-1}\} \\ &\hat{p}(t_0,r) \in \Omega_p = \{\hat{p}(t,r) : 0 \leqslant \hat{p}(t,r) \leqslant p_{\max}\}, \quad \forall r \in [0,R] \end{aligned} \tag{39}$$

then all the closed-loop signals are bounded and the state vector $\boldsymbol{x}(t)$ converges to $\Omega_\epsilon = \{\boldsymbol{x}(t) || \tilde{x}_i| \leqslant 2^{i-1}\lambda^{i-n}\epsilon, i = 1,\dots,n\}$, $\forall t \geqslant t_0$.

Proof: Using the expression (22), the time derivative of the filtered error (23) can be written as:

$$\dot{s}(t) = -u_{fd}(t) - \sum_{i=1}^{k} a_i Y_i(\boldsymbol{x}(t)) + b\{p_0 v(t) - d[v](t)\} \tag{40}$$

Using the control law (30)~(32), the above equation can be rewritten as

$$\dot{s}(t) = -u_{fd}(t) - \sum_{i=1}^{k} a_i Y_i(\boldsymbol{x}(t)) - bd[v](t) + bp_0[-k_d s(t) + \hat{\phi}u_{fd}(t) + Y^{\mathrm{T}}(\boldsymbol{x})\hat{\theta} + u_N(t)] \tag{41}$$

To establish global boundedness, we define the following Lyapunov function candidate

$$V(t) = \frac{1}{2}[\frac{1}{bp_0}s_\epsilon^2 + \frac{1}{\gamma}(\hat{\theta}-\theta)^{\mathrm{T}}(\hat{\theta}-\theta) + \frac{1}{\eta}(\hat{\phi}-\phi)^2 + \frac{1}{q}\int_0^R \tilde{p}^2(t,r)\mathrm{d}r] \tag{42}$$

Since the discontinuity at $|s| = \epsilon$ is of the first kind and since $s_\epsilon = 0$ when $|s| \leqslant \epsilon$, the derivative $\dot{V}$ exists for all s, with

$$\dot{V}(t) = 0, \quad \text{for } |s| \leqslant \epsilon \tag{43}$$

When $|s| > \epsilon$, using (41) and the fact of $s_\epsilon \dot{s}_\epsilon = s_\epsilon \dot{s}$, we have

$$\begin{aligned} \dot{V}(t) = &- k_d s_\epsilon s + s_\epsilon[\hat{\phi}u_{fd}(t) + Y^{\mathrm{T}}(\boldsymbol{x})\hat{\theta} + u_N(t) - \frac{1}{p}d[v](t)] + s_\epsilon[-\phi u_{fd}(t) - Y^{\mathrm{T}}\theta] + \\ &\frac{1}{\gamma}(\hat{\theta}-\theta)^{\mathrm{T}}\dot{\hat{\theta}} + \frac{1}{\eta}(\hat{\phi}-\phi)\dot{\hat{\phi}} + \frac{1}{q}\int_0^R \tilde{p}(t,r)\frac{\partial}{\partial t}\tilde{p}(t,r)\mathrm{d}r \end{aligned} \tag{44}$$

The above equation can be simplified, by the choice of s_ϵ, to

$$\begin{aligned} \dot{V}(t) \leqslant &- k_d s_\epsilon^2 + s_\epsilon[\hat{\phi}u_{fd}(t) + Y^{\mathrm{T}}(\boldsymbol{x})\hat{\theta} + u_N(t)] + s_\epsilon[-\phi u_{fd}(t) - Y^{\mathrm{T}}\theta - \frac{1}{p_0}d[v](t)] + \\ &\frac{1}{\gamma}(\hat{\theta}-\theta)^{\mathrm{T}}\dot{\hat{\theta}} + \frac{1}{\eta}(\hat{\phi}-\phi)\dot{\hat{\phi}} + \frac{1}{q}\int_0^R \tilde{p}(t,r)\frac{\partial}{\partial t}\tilde{p}(t,r)\mathrm{d}r \end{aligned} \tag{45}$$

By using the adaptive laws given in (33)~(35) and the properties $\frac{1}{\gamma}(\hat{\theta}-\theta)^{\mathrm{T}}Proj(\hat{\theta}, -\gamma Y s_\epsilon) \leqslant -(\hat{\theta}-\theta)^{\mathrm{T}}Y s_\epsilon$, and $\frac{1}{\eta}(\hat{\phi}-\phi)Proj(\hat{\phi}, -\eta u_{fd}s_\epsilon) \leqslant -(\hat{\phi}-\phi)u_{fd}s_\epsilon$, we obtain

$$\dot{V}(t) \leqslant -k_d s_\epsilon^2 + u_N(t)s_\epsilon - \frac{1}{p_0}d[v](t)s_\epsilon + \frac{1}{q}\int_0^R \tilde{p}(t,r)Proj(\hat{p}(t,r), q\frac{|F_r[v](t)|}{p_{0\min}}|s_\epsilon|)\mathrm{d}r \tag{46}$$

Now, we show that $\dot{V}(t) \leqslant -k_d s_\epsilon^2$. Since

$$-\frac{1}{p_0}d[v](t)s_\epsilon + u_N(t)s_\epsilon \leqslant + \frac{|s_\epsilon|}{p_0}\int_0^R p(r)|F_r[v](t)|\mathrm{d}r - \frac{|s_\epsilon|}{p_{0\min}}\int_0^R \hat{p}(t,r)|F_r[v](t)|\mathrm{d}r \leqslant$$

$$-\frac{|s_\epsilon|}{p_{0\min}}\int_0^R \tilde{p}(t,r)|F_r[v](t)|\mathrm{d}r \tag{47}$$

from (38) we have $\tilde{p}(t,r) \geqslant 0$, if r is in a subset $R_{\max} \subset [0,R]$, $R_{\max} = \{r : \hat{p}(t,r) = p_{\max}\}$, and according to adaptation law (35)

$$-\frac{|s_\epsilon|}{p_{0\min}}\int_{R_{\max}} \tilde{p}(t,r)|F_r[v](t)|\mathrm{d}r + \frac{1}{q}\int_{R_{\max}} \tilde{p}(t,r)Proj(\hat{p}(t,r), q\frac{|F_r[v](t)s_\epsilon|}{p_{0\min}})\mathrm{d}r \leqslant 0$$

otherwise, we have $0 \leqslant \hat{p}(t,r) < p_{\max}$ for $r \in R^c_{\max}$ ($R^c_{\max}$ is $R_{\max}$ complement in $[0,R]$), by (38),

$$-\frac{|s_\epsilon|}{p_{0\min}}\int_{R^c_{\max}} \tilde{p}(t,r)|F_r[v](t)|\mathrm{d}r + \frac{1}{q}\int_{R^c_{\max}} \tilde{p}(t,r)q\frac{|F_r[v](t)|}{p_{0\min}}|s_\epsilon|\mathrm{d}r = 0$$

That is

$$\begin{aligned}\dot{V}(t) \leqslant & -k_d s_\epsilon^2 + u_N(t)s_\epsilon - \frac{1}{p_0}d[v](t)s_\epsilon + \frac{1}{q}\int_0^R \tilde{p}(t,r)Proj(\hat{p}(t,r), q\frac{|F_r[v](t)|}{p_{0\min}}|s_\epsilon|)\mathrm{d}r \leqslant \\ & -k_d s_\epsilon^2 - \frac{|s_\epsilon|}{p_{0\min}}\int_0^R \tilde{p}(t,r)|F_r[v](t)|\mathrm{d}r + \frac{1}{q}\int_0^R \tilde{p}(t,r)Proj(\hat{p}(t,r), q\frac{|F_r[v](t)|}{p_{0\min}}|s_\epsilon|)\, dr \leqslant \\ & -k_d s_\epsilon^2\end{aligned} \tag{48}$$

Equations (42), (43) and (48) imply that V is a Lyapunov function leading to global boundedness of variables s_ϵ, $(\hat{\theta}-\theta)$, $(\hat{\phi}-\phi)$, and $\hat{p}(t,r)-p(r)$. From the definition of s_ϵ, $s(t)$ is bounded. It can be shown that if $\tilde{\boldsymbol{x}}(0)$ is bounded, then $\tilde{\boldsymbol{x}}(t)$ is also bounded for all $t \in [0, t_E]$. Since $\boldsymbol{x}_d(t)$ is bounded by design, $\boldsymbol{x}(t)$ must also be bounded. To complete the proof and establish an asymptotic convergence of the tracking error, it is necessary to show that $s_\epsilon \to 0$ as $t \to \infty$. This is accomplished by applying Barbalat's Lemma (popov) to the continuous, non-negative function:

$$\begin{aligned}&V_1(t) = V(t) - \int_0^t (\dot{V}(\tau) + k_d s_\epsilon^2(\tau))\mathrm{d}\tau \text{ with}\\ &\dot{V}_1(t) = -k_d s_\epsilon^2(t)\end{aligned} \tag{49}$$

It can easily be shown that (40) is bounded. We should mention that $\{p_0 v(t) - d[v](t)\}$ is the Prandtl-Ishlinskii model defined by the play operator, which is equivalent to (18). It can be proved that $|\{p_0 v(t) - d[v](t)\}| \leqslant K$, with $K = \int_0^R p(r)r\mathrm{d}r < \infty$. Hence $\dot{s}$ and $\dot{s}_\epsilon$ are bounded. This implies that $\dot{V}_1(t)$ is a uniformly continuous function of time. Since V_1 is bounded below by 0, and $\dot{V}_1(t) \leqslant 0$ for all t, use of Barbalat's lemma proves that $\dot{V}_1(t) \to 0$. Therefore, from (49) it can be shown that $s_\epsilon(t) \to 0$ as $t \to \infty$. The remark following Equation (23) indicates that $\tilde{\boldsymbol{x}}(t)$ will converge to Ω_ϵ.

6 Simulation studies

In this section, we illustrate the methodology presented in the previous sections using a simple nonlinear system described by

$$\dot{x} = a\frac{1-e^{-x(t)}}{1+e^{-x(t)}} + bw(t) \tag{50}$$

where $w(t)$ represents the output of the hysteresis. The actual parameter values are $b = 1$ and $a = 1$. Without control, *i.e.*, $v(t) = 0$, so $w(t) = 0$, using basic analysis method, it can be proved that the system is unstable as $t \to \infty$. The objective is to control the system state x to follow the desired trajectory $x_d = 5\sin(2t) + \cos(3.2t)$. The hysteresis is given by (19) with $p(r) = \alpha e^{-\beta(r-\sigma)^2}$ for $r \in [0, 100]$, parameters $\alpha = 0.5, \beta = 0.0014$, and $\sigma = 1$.

In the simulation, the robust adaptive control law (33)$\sim$(35) were used, taking $k_d = 0.4368$ and $p_{0\min} = 4.91$. In the adaptation laws, we choose $\gamma = 0.23$, $\eta = 0.15$, $q = 0.0098$, and the initial parameters $\hat{\theta}(0) = 1/4.41$, $\hat{\phi}(0) = 1/2.32$, and $\hat{p}(0,r) = \max\{0.3 - 0.0075r, 0\}$. The initial state is chosen as $x(0) = 2.05$, sample time is 0.001, and $\epsilon = 0.025$. We also assume that the hysteresis initial state was $w_{-1} = 0.07$ for $r \in [0,R]$ before $v(0)$ was applied and $v(0) = 5.9$. For the calculation of $\hat{B}(t)$, we replace the integration by the sum $\sum_0^N$. In the simulation, we choose $N = 4000$.

To illustrate the effectiveness of the proposed control scheme, the simulation has also been conducted without controlling the effects of hysteresis, which is implemented by setting $u_N(t)=0$ in the controller $v(t)$. This implies that the control compensation for the hysteresis nonlinearity is ignored. Fig. 2 shows the state trajectories and tracking errors for the desired trajectory, where the solid lines are the results with $u_N(t)\neq 0$ and the dotted lines are with $u_N(t)=0$. This example illustrates that the proposed robust controller clearly demonstrates excellent tracking performance and the developed control algorithm can overcome the effects of the hysteresis.

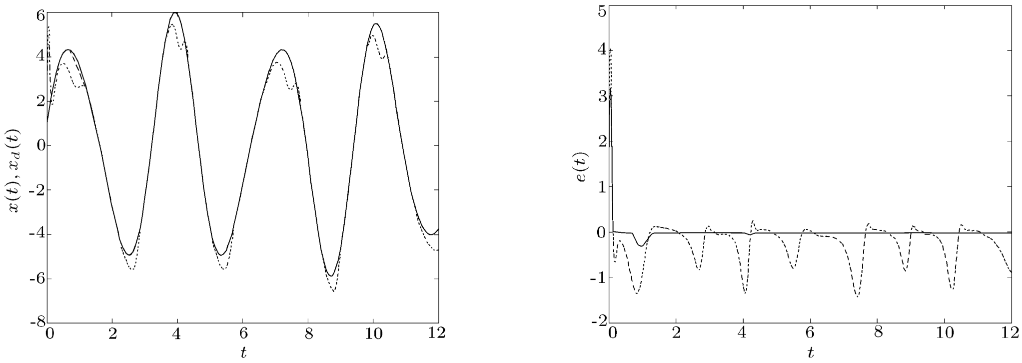

Fig. 2 Left: Desired trajectory $x_d(t)=5\sin(2t)+\cos(3.2t)$, system outputs $x(t)$ with control term u_N (-.) and $u_N=0$ (dotted line). Right: Tracking errors of the state with control term u_N and $u_N=0$ (dotted line)

7 Conclusion

In practical control systems, hysteresis nonlinearity with unknown parameters in physical components may severely limit the performance of control. By using the Prandtl-Ishlinskii model with play operator, a robust adaptive control scheme without constructing a hysteresis inverse is developed for a class of continuous-time nonlinear dynamic systems preceded by a hysteresis nonlinearity. The control law ensures global stability of the entire system and achieves both stabilization and tracking within a desired precision. Simulations performed on an unstable nonlinear system illustrate and further validate the effectiveness of the proposed approach. The primary purpose of exploring new avenues to fuse the model of hysteresis nonlinearities with the available adaptive controller design methodology is achieved with highly promising results.

References

1 Banks H T, Smith R C. Hysteresis modeling in smart material systems, *Journal of Applied Mechanics and Engineering*, 2000, **5**: 31∼45
2 Croft D, Shed G, Devasia S. Creep, hysteresis, and vibration compensation for piezoactuators: atomic force microscopy application *Journal of Dynamic Systems, Measurement, and Control*, 2001,**123**: 35∼43
3 Dickinson C A, Hughes D, Wen J T. Hysteresis in shape memory alloy actuators: The control issues, *Proceeding SPIE 2715, V. V. Varandan and J. Chandra, Eds.*, 1996, 494∼506
4 Tao G, Kokotovic P V. Adaptive control of plants with unknown hystereses, *IEEE Transactions on Automatic Control*, 1995, **40**: 200∼212
5 Tao G, lewis F L (Eds). *Adaptive Control of Nonsmooth Dynamic Systems*, Springer, 2001
6 Macki J W, Nistri P, Zecca P. Mathematical models for hysteresis, *SIAM Review*, 1993, **35**: 94∼123
7 Sain P M, Sain M K, Spencer B F. Models for hysteresis and application to structural control, In: Proceedings American control Confol, 1997, 16∼20
8 Brokate M, Sprekels J. *Hysteresis and Phase Transitions* , Springer-Verlag, 1996
9 Krasnosekl'skii M A, Pokrovskii A V. *Systems with Hysteresis*, Nauka, Moscow, 1983
10 Mayergoyz I D. *The Preisach Model for Hysteresis*, Berlin: Springer-Verlag, 1991
11 Visintin A (ed). *Phase Transitions and Hysteresis. Lecture Notes in Mathematics*, Berlin: Springer-Verlag, 1994, **1584**
12 Coleman B D, Hodgdon M L. A constitutive relation for rate-independent hysteresis in ferromagnetically soft materials, *International J. Engrg. Sci.*, 1986, **24**: 897∼919

13 Coleman B D, Hodgdon M L. On a class of constitutive relations for ferromagnetic hysteresis, *Arch. Rational Mech. Anal.*, 1989, **402**: 375∼396

14 Hodgdon M L. Applications of a theory of ferromagnetic hysteresis, *IEEE Transactions Magn.*, MAG-24 1988, 218∼221

15 Hodgdon M L. Mathematical theory and calculations of magnetic hysteresis curves, *IEEE Transactions Magn.*, MAG-24 1988, 3120∼3122

16 Spencer B P Jr. *Reliability of randomly excited hysteresis structures. Lecture Notes in Engineering*, New York: Springer-Verlag, 1986, **21**

17 Jiles D C, Atherton D L. Theory of ferromagnetic hysteresis, J. Magn. Magn. Mater., 1986, **61**: 48∼60

18 Jiles D C, Thoelke J B. Theory of ferromagnetic hysteresis: determination of model parameters from experimental hysteresis loops, *IEEE Transactions Magn.*, MAG-25 1989, 3928∼3930

19 Jiles D C. Modelling the effects of eddy current losses on frequency dependent hysteresis in electrically conducting media, *IEEE Transactions Magn.*, MAG-30 1994, 4326∼4328

20 Leite J, Sadowski N, Kuo-Peng P, Batistela N, Bastos J. The inverse Jiles-Atherton model parameters identification, *IEEE Transactions Magn.*, MAG-39 2003, 1397∼1400

21 Salvini A, Fulginei F R. Genetic algorithms and neural networks generalizing the Jiles-Atherton model of static hysteresis for dynamic loops, *IEEE Transactions Magn.*, MAG-38 2002, 873∼876

22 Philips D A, Dupre L R, Melkebeek J A. Comparison of Jiles and Preisach hysteresis models in magnetodynamics, *IEEE Transactions Magn.*, MAG-31, 1995, 3551∼3553

23 Weiss P, De Freundereich J. Etude de l'aimantation initiale en function de la temperature, *Arch. Sci. Phys. Nat.*, (geneve), 1916, **42**: 449

24 Preisach P. Uber die magnetische Nachwirkung, *Zeit. fur Physik*, 1935, **94**: 277∼302

25 Ragusa C. An analytical method for the identification of the Preisach distribution function, *J. Magnetism and Magnetic Materials*, 2003, 254∼255 and 259∼261

26 Yu Y, Xiao Z, Lin E-B, Naganathan N. Analytic and experinemtal studies of a wavelet identification of Preisach model of hysteresis,*J. Magnetism and Magnetic Materials*, 2000, **208**: 255∼263

27 Xiaobo Tan, John S. Baras. Modeling and control of a magnetostrictive actuator, In: Proceedings of the 41st IEEE Confernce on Decision and Control, Las Vegas, Nevada, USA, Dec. 2002

28 Hu H, Mrad R B. On the classical Preisach model for hysteresis in piezoceramic actuators, *Mechatronics*, 2003, **13**: 85∼94

29 Gorbet R B, Morris K A, Wang D W L. Passivity-based stability and control of hysteresis in smart actuators, *IEEE Transactions on Control Systems Technology*, 2001, **9**(1):

30 Prandtl L. Ein gedankenmodell zur kinetischen theorie der festen Korper, *ZAMM*, 1928, **8**: 85∼106 (German)

31 Ishlinskii A Yu. Some applications of statistical methods to describing deformations of bodies,*Izv. AN SSSR, Techn. Ser.,*, 1944, (9): 580∼590 (Russian)

32 Krejci P, Kuhnen K. Inverse control of systems with hysteresis and creep, *IEE Proceedings Control Theory Appl.*, 2001, **148**: 185∼192

33 Banks H T, Kurdila A, Webb G. Identification of hysteretic control influence operators representing smart actuators part 1: fomulation, *Tech. Rep.*, Center for Research in Scientific Computation, Dept. of Math, North Carolina State U., Raleigh, NC, Dept. of Aerospace Engr.; Dept. of Math, Texas A and M U., College Station, TX, 1996

34 Banks H T, Kurdila A, Webb G. Identification of hysteretic control influence operators representing smart actuators: Convergent approximation, *Tech. Rep.*, Center for Research in Scientific Computation, Dept. of Math, North Carolina State U., Raleigh, NC, Dept. of Aerospace Engr.; Dept. of Math, Texas A and M U., College Station, TX, 1997

35 Galinaitis W S. Two methods for modeling scalar hysteresis and their use in controlling actuators with hysteresis, *Ph.D Thesis*, Virginia Tech, Blacksburg, Virginia, USA, 1999

36 Ge P, Jouaneh M. Tracking control of a piezoceramic actuator, *IEEE Transactions on Control Systems Technology*, 1996, **4**(3): 209∼216

37 Mittal S, Meng C-H. Hysteresis compensation in electromagnetic actuators through Preisach model inverse, *IEEE/ASME Transactions Mechatron.*, 2000, **5**(4): 394∼409

38 Bernard Y, Mendes E, Bouillault F. Dynamic hysteresis modeling based on Preisach model, *IEEE Transactions on Magnetics*, 2002, **38**(2): 885∼888

39 Webb G, Kurdila A, Lagoudas D. Adaptive hysteresis model for model reference control with actuator hysteresis, *J. Guidance, Control and Dynamics*, 2000, **23**: 459∼465

40 Selmic R R, Lewis F L. Backlash compensation in nonlinear systems using dynamics inversion by neural networks, In: Proceedins 1999 IEEE International Conference Control App., Aug., 1163∼1168

41 Hwang C-L, Jan C, Chen Y-H. Piezomechanics using intelligent variable-structure control, *IEEE Transactions on Industrial Electronics*, 2001, **48**(1): 47∼59

42 Hwang C-L, Jan C. A reinforcement discrete neuro-adaptive control for unknown piezoelectric actuator systems with dominant hysteresis, *IEEE Transactions on Neural Networks*, 2003, **14**(1): 66∼78
43 Tianshun Lu, Tan Y H, Chun-Yi Su. Pole placement control of discrete time systems preceded with hysteresis via dynamic neural network compensator intelligent control, In: Proceedings of the 2002 IEEE International Symposium, 2002
44 Nakaguchi T, Isome S, Jin'no K. Tanaka M. Box Puzzling Problem Solver by Hysteresis Neural Networks, *IEICE Transactions Fundamentals*, 2001, **E84-A**(9): 2173∼2181
45 Meng-shiun Tsai, Jin-Shin Chen, Robust tracking control of a piezoactuator using a new approximate hysteresis model, *Tran. of ASME*, 2003, **125**: 96∼102
46 Pare T E. Robust stability and performance analysis of systems with hysteresis nonlinearities, In: Proceedings of American Control Conference, Philadolphia, Penn, 1998, 1904∼1908
47 Banning R, De Koning W L, Han J M A. Adriaens. State-space analysis and identification for a class of hysteretic systems, *Automatica*, 2001, **37**: 1883∼1892
48 Fertis D G, Lee C T. Inelastic response of variable stiffness members under cyclic loading, *J. Engineering Mechanics,* 1992, **118**(7): 14064∼1446
49 Su C-Y, Stepanenko Y, Svoboda J, Leung T P. Robust adaptive control of a class of nonlinear systems with unknown backlash-like hysteresis, *IEEE Transactions on Automatic Control*, 2000, **45**: 2427∼2432
50 Isidori A. *Nonlinear Control Systems: An Introduction*, 2nd ed. Berlin, Germany: Springer-Verlag, 1989

Qingqing Wang Received the M.Eng. degree in adaptive system control from Beijing University of Aeronautics and Astronautics, P.R.China, and the M.Sc. degree in applied math from University of Manitoba, Manitoba, Canada, in 1988 and 1991. She is currently a Ph.D candidate in the Department of Mechanical and Industrial Engineering, Concordia University, Montreal, QC, Canada.

Chun-Yi Su Received his bachelor degree in control engineering from Shaanxi Institute of Mechanical Engineering in 1982, his master and Ph.D. degrees in control engineering from South China University of Technology, P.R. China, in 1987 and 1990, respectively. His Ph.D. study was jointly directed at Hong Kong Polytechnic (now Hong Kong Polytechnic University), Hong Kong, P.R.China. After long stint at the University of Victoria (Canada), he joined the Concordia University (Canada) in 1998, where he is currently an Associate Professor of Mechanical Engineering and holds the Concordia Research Chair in Control. He has also held several short-time visiting positions in Japan, Singapore, China and New Zealand. Dr. Su's research covers control theory and its applications to various mechanical systems. His current main research interests are in control techniques for smart material based actuators, vehicle suspension and vibration, robotic and mechatronic systems, and fuzzy control for nonlinear systems. He is the author or co-author of over 100 publications, which have appeared in journals, as book chapters and in conference proceedings. In addition to his academic activities, he has worked extensively with industrial organizations on various projects. Dr. Su is a Senior Member of IEEE and a Member of ASME. He is an Associate Editor of IEEE Transactions on Control Systems Technology and Journal of Control Theory & Applications. He is on the editorial board of Mechatronics: An International Journal as well.

Yonghong Tan Obtained his Ph.D. degree in 1996 from Department of Control Engineering & Automation, Ghent University, Belgium. Presently, he is a professor at Lab. of Intelligent Systems and Control Engineering, Guilin University of Electronic Technology, Guilin, P.R.China. From 1996-1998, Dr. Tan was a post-doctoral fellow at School of Engineering Science, Simon Fraser University, Vancouver, Canada. In 2001, he was a visiting professor respectively at the Dept. of Chemical Engineering and Bio-resources, Colorado State University, USA and the Dept. of Mechanical and Industrial Engineering, Concordia University, Canada. His research interests include intelligent control of nonlinear system, modeling and control of the complex systems.