On Robust H_2 Estimation

Lihua Xie
(*School of Electrical and Electronic Engineering, BLK S2, Nanyang Technological University, Nanyang Avenue,* 639798 *Singapore*)
(E-mail: elhxie@ntu.edu.sg)

Abstract The problem of state estimation for uncertain systems has attracted a recurring interest in the past decade. In this paper, we shall give an overview on some of the recent development in the area by focusing on the robust H_2 (Kalman) filtering of uncertain discrete-time systems. The robust H_2 estimation is concerned with the design of a fixed estimator for a family of plants under consideration such that the estimation error covariance is of a minimal upper bound. The uncertainty under consideration includes norm-bounded uncertainty and polytopic uncertainty. In the finite horizon case, we shall discuss a parameterized difference Riccati equation approach for systems with norm-bounded uncertainty and pinpoint the difference of state estimation between systems without uncertainty and those with uncertainty. In the infinite horizon case, we shall deal with both the norm-bounded and polytopic uncertainties using a linear matrix inequality (LMI) approach. In particular, we shall demonstrate how the conservatism of design can be improved using a slack variable technique. We also propose an iterative algorithm to refine a designed estimator. An example will be given to compare estimators designed using various techniques.

Key words Discrete-time systems, uncertain systems, robust H_2 estimation, Riccati equations, linear matrix inequalities

1 Introduction

One of the fundamental problems in control systems and signal processing is the estimation of the state variables of a dynamic system through available noisy measurements. In the past three decades, this problem has attracted the interests of many researchers and one of the popular methods is based on the minimization of the variance of the estimation error, *i.e.*, the celebrated Kalman filtering approach (see, e.g. [1]). The filtering algorithm requires the knowledge of a perfect dynamic model for the signal generating system and that the noise sources should be white processes with known statistics. Thus, the standard Kalman filter may not be robust against modeling uncertainty and disturbances. This has motivated many studies on design of a robust filter which is able to yield a suboptimal solution with respect to the nominal system and in addition offer a guaranteed estimation performance for any possible uncertainties under consideration.

In the past decades, the robust estimation problem has been dealt with under various performance measures. Of particular interest, the study on robust estimation of systems with norm-bounded parameter uncertainty under the H_∞ performance was initiated in [2,3] which was followed by the study of guaranteed cost filtering in [4]. The guaranteed cost filtering is concerned with the design of a filter to ensure an upper bound on the estimation error variances for all admissible parameter uncertainties under investigation and was first addressed in [5]. The problem is also known as *robust Kalman filtering or robust H_2 estimation* and has been studied in [4,6∼13] for uncertain continuous-time systems and [14∼ 22] for uncertain discrete-time systems.

There are essentially two approaches to the robust H_2 estimation, namely the Riccati equation approach and the LMI approach. The Riccati equation approach was applied in the early stage of the development; see, e.g. [4,6,7,14]. Under this approach, the robust filter design is related to the solutions of two parameterized Riccati equations. The advantage of this approach is that the effect of parameter uncertainty on the structure and gain of the optimal filter is clearly demonstrated, which provides useful insights on the problem. In particular, in the finite horizon, the approach reveals that unlike the classical Kalman filtering an optimal filter at time k does not necessarily lead to an optimal filter at $k+1$. This implies that the recursive computation of the optimal covariance matrix in the classical Kalman filter is no longer valid for the robust Kalman filtering. The difficulty of the Riccati equation approach also lies

Received February 12, 2004; in revised form June 25, 2004

in the fact that it requires a search of scaling parameter(s) such that the Riccati equations have solutions and the estimation error variance is minimized.

In recent years, a lot of interests have been focused on the LMI approach; see e.g. [9,11,12,18,23]. The LMI approach has an advantage in numerical computation and optimization. In fact, it has been demonstrated that the design of robust filter for systems under either norm-bounded or polytopic parameter uncertainty can be characterized as a convex optimization involving the LMIs. In the early development, a fixed Lyapunov function was applied for all the members of the plant family; see e.g. [9,17]. And it is generally conservative. Much of the recent interest in the area has been towards developing techniques which could lead to a less conservative design. In [18,22,23], a parameter-dependent Lyapunov function technique was introduced based on a slack variable technique. The work was inspired by a new result in [24] on robust stability of uncertain discrete-time systems. The slack variable technique has been demonstrated to be able to offer much improved results. Very recently, a further reduction of design conservatism has been proposed in [20] where extra slack variables are introduced.

In this paper, the problem of finite and infinite horizon robust H_2 estimation for uncertain discrete-time systems is revisited. Both the finite horizon and infinite horizon cases are considered. We shall review the development of the area in the past decade and propose an iterative algorithm to refine a designed infinite horizon filter. An example will be given to compare estimators designed using various techniques.

The rest of the paper is organized as follows. The system under investigation and the statement of the robust H_2 estimation will be given in Section 2. Section 3 studies techniques for systems with norm-bounded uncertainty using both the Riccati equation and LMI approaches. Systems with polytopic uncertainty will be investigated in Section 4. An example will be given to compare various design techniques of robust H_2 estimation in Section 5. Finally, we shall draw some conclusions in Section 6.

2 Problem formulation

Consider the following asymptotically stable system:

$$\boldsymbol{x}_{k+1} = A_\delta \boldsymbol{x}_k + B\boldsymbol{w}_k \tag{1}$$

$$\boldsymbol{y}_k = C_\delta \boldsymbol{x}_k + D\boldsymbol{w}_k \tag{2}$$

$$\boldsymbol{z}_k = L\boldsymbol{x}_k \tag{3}$$

where $\boldsymbol{x}_k \in \mathcal{R}^n$ is the system state vector, $\boldsymbol{y}_k \in \mathcal{R}^r$ is the measurement, $\boldsymbol{z}_k \in \mathcal{R}^p$ is the signal to be estimated and $\boldsymbol{w}_k \in \mathcal{R}^m$ is the noise input with zero-mean and unit variance. The initial state $\boldsymbol{x}_0$ of the system is a random vector of zero-mean and variance $\bar{S}_0 > 0$ and is independent of $\boldsymbol{w}_k$ for any $k \geqslant 0$.

Note that for the case when the process noise and measurement noise are different (usually so in practice), say $B_1\boldsymbol{w}_{1k}$ and $D_1\boldsymbol{w}_{2k}$, we can simply put $B = [B_1 \;\; 0]$, $D = [0 \;\; D_1]$ and let $\boldsymbol{w}_k = [\boldsymbol{w}_{1k}^{\mathrm{T}} \;\; \boldsymbol{w}_{2k}^{\mathrm{T}}]^{\mathrm{T}}$ in the system model (1)∼(2).

For the sake of clarity of presentation, we assume that system parameter uncertainty only appears in A_δ and C_δ whereas B, D and L are known matrices. It should be noted that it is rather straightforward to extend the solutions of this paper to the case where uncertainty is also present in matrices B, D and L.

Two types of uncertainty are commonly considered in literature, namely, the norm-bounded uncertainty[2,14,15,19] and the polytopic uncertainty[11,13,17,18,20,23].

a) Norm-bounded uncertainty

In this characterization, we allow the uncertainty to be time-varying and matrices A_δ and C_δ have the form:

$$\Omega_N = \left\{ (A_\delta, C_\delta) \mid \begin{bmatrix} A_\delta \\ C_\delta \end{bmatrix} = \begin{bmatrix} A \\ C \end{bmatrix} + \begin{bmatrix} H_1 \\ H_2 \end{bmatrix} F_k E \right\} \tag{4}$$

where $F_k = diag\{F_{1k}, F_{2k}, \cdots, F_{\ell k}\}$ with $F_{ik} \in \mathcal{R}^{\alpha_i \times \beta_i}$ is an unknown real time-varying matrix satisfying

$$F_{ik}^{\mathrm{T}} F_{ik} \leqslant I, \ \forall k \geqslant 0, \ i = 1, 2, \cdots, \ell \tag{5}$$

and H_1, H_2 and E are known real constant matrices of appropriate dimensions that specify how the elements of the nominal matrices A and C are affected by the uncertainty in F_k.

b) Polytopic uncertainty

In this case, matrices A_δ and C_δ belong to the following uncertainty polytope:

$$\Omega_P = \left\{(A_\delta, C_\delta) \mid (A_\delta, C_\delta) = \sum_{i=1}^{N_v} \alpha_i (A^{(i)}, C^{(i)}),\ \alpha_i \geqslant 0,\ \sum_{i=1}^{N_v} \alpha_i = 1\right\} \tag{6}$$

where $(A^{(i)}, C^{(i)})$ is the i-th vertex of the polytope.

Remark 1. Note that by the Schur complement (5) is equivalent to

$$\begin{bmatrix} -I & F_{ik}^{\mathrm{T}} \\ F_{ik} & -I \end{bmatrix} \leqslant 0$$

Therefore, the uncertainty set Ω_N also belongs to a convex set. However, Ω_N is in general not a polyhedral set except for some special cases such as when the uncertainty matrix F_k is diagonal. On the other hand, since Ω_N is a convex set, it is always possible to approximate Ω_N using a polyhedral set. But a good approximation may require a polytope with a large number of vertices which will be numerically costly in designing a robust H_2 estimator as seen later. The norm-bounded uncertainty characterization may be restrictive in some applications but it is less computationally demanding to design a robust H_2 estimator with such a characterization than that of the polytopic uncertainty.

In this paper, we are concerned with the design of an estimator for the uncertain system (1)$\sim$(3) to achieve an optimal guaranteed estimation performance regardless of the uncertainty characterized in either (4) or (6). Both the finite horizon and infinite horizon cases are considered.

In the finite horizon case, the system matrices in (1)$\sim$(3) are allowed to be time-varying, *i.e.*, A, B, C, D, L, H_1, H_2 and E are replaced by A_k, B_k, C_k, D_k, L_k, H_{1k}, H_{2k} and E_k, respectively. However, for simplicity, we drop the subscript k whenever no confusion is caused.

We consider the following problems.

a) Finite horizon robust H_2 estimation

Our objective is to design a robust one-step ahead predictor of the form

$$(\mathcal{F}_P): \hat{\boldsymbol{x}}_{k+1} = \hat{A}_k \hat{\boldsymbol{x}}_k + \hat{B}_k \boldsymbol{y}_k, \quad \hat{\boldsymbol{x}}_0 = 0 \tag{7}$$

$$\hat{\boldsymbol{z}}_k = L\boldsymbol{x}_k \tag{8}$$

where $\hat{A}_k$ and $\hat{B}_k$ are time-varying matrices to be determined in order that the variance of the estimation error $\boldsymbol{z}_k - \hat{\boldsymbol{z}}_k$ is guaranteed to be smaller than a certain bound for all uncertainty matrices F_k satisfying (5), *i.e.*, the estimation error dynamics satisfies

$$\mathcal{E}[(\boldsymbol{x}_k - \hat{\boldsymbol{x}}_k)(\boldsymbol{x}_k - \hat{\boldsymbol{x}}_k)^{\mathrm{T}}] \leqslant \Sigma_k \tag{9}$$

with Σ_k being an optimized upper bound of the prediction error covariance over the class of robust quadratic predictors to be defined later, where $\mathcal{E}$ denotes the mathematical expectation.

b) Infinite horizon robust H_2 estimation

For the infinite horizon case, the system matrices A, B, C, D, L, H_1, H_2 and E are confined to be constant matrices and our objective is to design a time-invariant filter

$$(\mathcal{F}_F): \hat{\boldsymbol{x}}_{k+1} = \hat{A}\hat{\boldsymbol{x}}_k + \hat{B}\boldsymbol{y}_k, \quad \hat{\boldsymbol{x}}_0 = 0 \tag{10}$$

$$\hat{\boldsymbol{z}}_k = \hat{C}\boldsymbol{x}_k + \hat{D}\boldsymbol{y}_k \tag{11}$$

such that for any uncertainty from Ω_N or Ω_P the filtering error dynamics $\boldsymbol{e}_k = \boldsymbol{x}_k - \hat{\boldsymbol{x}}_k$ is asymptotically stable and

$$\mathcal{E}[(\boldsymbol{x}_k - \hat{\boldsymbol{x}}_k)(\boldsymbol{x}_k - \hat{\boldsymbol{x}}_k)^{\mathrm{T}}] \leqslant \Sigma \tag{12}$$

for a constant matrix $\Sigma = \Sigma^{\mathrm{T}} \geqslant 0$. Furthermore, $trace(\Sigma)$ is to be minimized.

Remark 2. In the case of norm-bounded uncertainty, the robust estimation problem in the H_∞ sense was first investigated in [2] via a Riccati equation approach. This work was later extended to solve the robust H_2 estimation in [14] and further improved by [15,19]. The difficulty with this approach is the requirement of solving parameterized Riccati equations which is not easy in general. In particular, when the uncertainty is of a block-diagonal structure, to reduce the conservatism of design, a scaling matrix is usually introduced and the selection of such a scaling matrix for an optimal filtering performance turns out to be difficult. In [9] and [25], an LMI approach was adopted for the infinite horizon robust H_2 estimation. The nice feature of this approach is that the search of the scaling matrix and filter parameters are convex and can be easily carried out by using the LMI Toolbox[26].

For the polytoic uncertainty, the LMI approach has attracted significant interest in the past few years; see, e.g. [12,13,17,18,20,23]. In [17], a convex optimization solution was obtained by applying a fixed Lyapunov function for all admissible uncertainties, which is conservative. A slack-variable technique was introduced in [13,18,23] in order to alleviate the design conservatism. Recently, a further improved result has been achieved by introducing extra slack variables in optimization in [20].

3 Robust H_2 filtering of systems with norm-bounded uncertainty

In this section, we shall present solutions to the robust H_2 estimation of systems with norm-bounded uncertainty. For the simplicity of presentation, we assume that $BD^{\mathrm{T}} = 0$ which implies that the process noise and measurement noise are independent of each other.

3.1 Finite horizon case

In terms of system (1)~(3) with uncertainty Ω_N and predictor (7)~(8), the state-space equations for the estimation error $\boldsymbol{e}_k$ are as follows:

$$\boldsymbol{\xi}_{k+1} = (\bar{A} + \bar{H}F_k\bar{E})\boldsymbol{\xi}_k + G\boldsymbol{w}_k, \quad \boldsymbol{\xi}(0) = \boldsymbol{\xi}_0 \tag{13}$$

$$\boldsymbol{\eta}_k = \bar{L}\boldsymbol{\xi}_k \tag{14}$$

where $\boldsymbol{e}_k = \boldsymbol{x}_k - \hat{\boldsymbol{x}}_k$, $\boldsymbol{\xi}_k = \begin{bmatrix}\boldsymbol{e}_k^{\mathrm{T}} & \hat{\boldsymbol{x}}_k^{\mathrm{T}}\end{bmatrix}^{\mathrm{T}}$ with $\boldsymbol{\xi}_0 = \begin{bmatrix}\boldsymbol{x}_0^{\mathrm{T}} & \boldsymbol{0}\end{bmatrix}^{\mathrm{T}}$ and

$$\bar{A} = \begin{bmatrix} A - \hat{B}C & A - \hat{A} - \hat{B}C \\ \hat{B}C & \hat{A} + \hat{B}C \end{bmatrix}, \quad G = \begin{bmatrix} B - \hat{B}D \\ \hat{B}D \end{bmatrix} \tag{15}$$

$$\bar{H} = \begin{bmatrix} H_1 - \hat{B}H_2 \\ \hat{B}H_2 \end{bmatrix}, \quad \bar{E} = [E \quad E], \quad \bar{L} = [L \quad 0] \tag{16}$$

Definition 1[15,19]. The estimator (7)~(8) is said to be a robust quadratic one-step ahead predictor if for some $\varepsilon_k > 0$, there exists a bounded $\bar{\Sigma}_k = \bar{\Sigma}_k^{\mathrm{T}} \geqslant 0$ that satisfies the following difference Riccati equation (DRE):

$$\bar{\Sigma}_{k+1} = \bar{A}\bar{\Sigma}_k\bar{A}^{\mathrm{T}} + \bar{A}\bar{\Sigma}_k\bar{E}^{\mathrm{T}}(\varepsilon_k^{-1}I - \bar{E}\bar{\Sigma}_k\bar{E}^{\mathrm{T}})^{-1}\bar{E}\bar{\Sigma}_k\bar{A}^{\mathrm{T}} + \varepsilon_k^{-1}\bar{H}\bar{H}^{\mathrm{T}} + GG^{\mathrm{T}} \tag{17}$$

and such that $I - \varepsilon_k\bar{E}\bar{\Sigma}_k\bar{E}^{\mathrm{T}} > 0$, where $\bar{\Sigma}_0 = \begin{bmatrix} \bar{S}_0 & 0 \\ 0 & 0 \end{bmatrix}$.

It is easy to show that for a given robust one-step ahead predictor, an upper bound for the prediction error covariance can be given by

$$\mathcal{E}(\boldsymbol{\eta}_k\boldsymbol{\eta}_k^{\mathrm{T}}) \leqslant L\Sigma_k L^{\mathrm{T}} \tag{18}$$

where Σ_k is the (1,1)-block of $\bar{\Sigma}_k$.

Before presenting a solution to the optimal robust H_2 one-step ahead prediction, we introduce the following two DREs.

$$P_{k+1} = AP_kA^{\mathrm{T}} + AP_kE^{\mathrm{T}}\left(\frac{I}{\varepsilon_k} - EP_kE^{\mathrm{T}}\right)^{-1}EP_kA^{\mathrm{T}} + \frac{1}{\varepsilon_k}H_1H_1^{\mathrm{T}} + BB^{\mathrm{T}}, \quad P_0 = \bar{S}_0 \tag{19}$$

$$\Sigma_{k+1} = A\Sigma_kA^{\mathrm{T}} - (A\Sigma_k\tilde{C}_k^{\mathrm{T}} + \tilde{B}_k\tilde{D}_k^{\mathrm{T}})(\tilde{C}_k\Sigma_k\tilde{C}_k^{\mathrm{T}} + \tilde{R}_k)^{-1}(A\Sigma_k\hat{C}_k^{\mathrm{T}} + \tilde{B}_k\tilde{D}_k^{\mathrm{T}})^{\mathrm{T}} + \tilde{B}_k\tilde{B}_k^{\mathrm{T}},\ \Sigma_0 = \bar{S}_0 \tag{20}$$

where

$$\tilde{C}_k = \sqrt{\varepsilon_k}\begin{bmatrix} C \\ E \end{bmatrix}, \quad \tilde{B}_k = \left[B \quad \frac{1}{\sqrt{\varepsilon_k}}H_1 \quad 0\right]$$

$$\tilde{D}_k = \begin{bmatrix} \bar{D}_k \\ 0 \end{bmatrix}, \quad \tilde{R}_k = \begin{bmatrix} \bar{D}_k\bar{D}_k^{\mathrm{T}} & 0 \\ 0 & -I \end{bmatrix}, \quad \bar{D}_k = [0 \quad H_2 \quad \sqrt{\varepsilon_k}D]$$

We make the following assumption.

Assumption 1. $rank[A \quad H_1 \quad B] = n$.

Theorem 2[19]. Consider the uncertain system (1)∼(3) with a single block of uncertainty Ω_N ($\ell = 1$) and satisfying Assumption 1. There exists a robust quadratic one-step ahead predictor for the system that minimizes the bound on the error variance in (18) if and only if for some $\varepsilon_k > 0$, there exists a solution $P_k = P_k^{\mathrm{T}} > 0$ over $[0, N]$ to the DRE (19) such that $P_k^{-1} - \varepsilon_k E^{\mathrm{T}} E > 0$.

Under this condition, a robust quadratic one-step ahead predictor with an optimized upper bound of error covariance is given by

$$\hat{\boldsymbol{x}}_{k+1} = (A + A_{e,k})\hat{\boldsymbol{x}}_k + K_k\Big[\boldsymbol{y}_k - (C + C_{e,k})\hat{\boldsymbol{x}}_k\Big], \quad \hat{\boldsymbol{x}}_0 = \mathbf{0} \tag{21}$$

$$\hat{\boldsymbol{z}}_k = L\hat{\boldsymbol{x}}_k \tag{22}$$

where

$$A_{e,k} = \varepsilon_k A \Sigma_k E^{\mathrm{T}}\Big(I - \varepsilon_k E \Sigma_k E^{\mathrm{T}}\Big)^{-1} E \tag{23}$$

$$C_{e,k} = \varepsilon_k C \Sigma_k E^{\mathrm{T}}\Big(I - \varepsilon_k E \Sigma_k E^{\mathrm{T}}\Big)^{-1} E \tag{24}$$

$$K_k = \left(AQ_kC^{\mathrm{T}} + \frac{1}{\varepsilon_k}H_1H_2^{\mathrm{T}}\right)\left(R_{\varepsilon_k} + CQ_kC^{\mathrm{T}}\right)^{-1} \tag{25}$$

$$R_{\varepsilon_k} = DD^{\mathrm{T}} + \frac{1}{\varepsilon_k}H_2H_2^{\mathrm{T}} \tag{26}$$

and $\Sigma_k = \Sigma_k^{\mathrm{T}} > 0$ is a solution of DRE (20) over $[0, N]$ and satisfies $Q_k^{-1} = \Sigma_k^{-1} - \varepsilon_k E^{\mathrm{T}} E > 0$.

Moreover, the optimized error covariance bound is Σ_k.

Remark 3. Note from Theorem 2 that the predictor parameters (23)∼(25) are not related to the solution of DRE (19). However, the existence of a solution to (19) is needed to ensure that the derived predictor is a robust quadratic one.

The matrices $A_{e,k}$ and $C_{e,k}$ of the predictor can be viewed respectively as correction matrices on A and C in consideration of the norm-bounded uncertainty in A_δ and C_δ. We observe from (20) and (23)∼(25) that when the parameter uncertainty in system (1)∼(2) disappears, the robust predictor of Theorem 2 reduces to the standard finite-horizon Kalman predictor for the nominal system of (1)∼(2). In this situation, (19) becomes redundant.

Remark 4. From Theorem 2, it is clear that the optimal gauranteed one-step ahead prediction performance at time k depends on system data and the scaling parameters $\varepsilon_0, \ \varepsilon_1, \ \cdots, \varepsilon_{k-1}$. This projects a significant difference between the optimal estimation of systems without uncertainty and that of systems with uncertainty. For the former case, it is well known that the optimal estimation at time $k-1$ will lead to an optimal estimation at k. This enables a recursive calculation of the optimal estimation covariance based on one DRE. This desirable property, however, is no longer valid for the latter[27]. It means that to obtain an optimal robust estimator at time k, the scaling parameters $\varepsilon_0, \ \varepsilon_1, \ \cdots, \varepsilon_{k-2}$ have to be recomputed and so do the corresponding solutions of (19) and (20). The optimization over the scaling parameters may be numerically costly with the increasing value of k.

To reduce the computational complexity, instead of optimizing the cost over all the k scaling parameters, we may optimize $trace(\Sigma_k)$ over a fixed length of scaling parameters $\varepsilon_{k-1}, \varepsilon_{k-2}, \cdots, \varepsilon_{k-d}$, where d is called a window in [27] in which a semidefinite programming approach is adopted. This can be done as follows. Given the computed suboptimal P_{k-d} and Σ_{k-d}, for any $k - d \leqslant i \leqslant k - 1$ it follows from [19] that given $P_i > 0$ and $\Sigma_i > 0$, the solution $\Sigma_{i+1} > 0$ of (20) exists if $0 < \varepsilon_i < \bar{\varepsilon}_i = \|EP_iE^{\mathrm{T}}\|^{-1}$. Hence, we can grid the interval $(0, \bar{\varepsilon}_i)$ and for each ε_k from the interval calculate the corresponding P_{i+1} and Σ_{i+1}. The optimization of $trace(\Sigma_k)$ over $\varepsilon_{k-1}, \varepsilon_{k-2}, \cdots, \varepsilon_{k-d}$ will then involve the calculation of P_i and Σ_i of a tree structure. The optimal solution will be the one with minimal $trace(\Sigma_k)$. The performance of the robust predictor and its computational cost will depend on the size of window, d. The larger the d the better the performance but the computational cost is also higher. We note that in [28] the case of $d = 1$ was applied for the design of a suboptimal robust LQR controller.

3.2 Infinite horizon case

In this section we shall address the robust H_2 estimation of systems with norm-bounded uncertainty in the infinite horizon case via an LMI approach. An algebraic Riccati equation approach for the single block uncertainty case can be found in [19].

Theorem 3[25]. Suppose that the system (1)$\sim$(3) is quadratically stable (see [29] for the definition). Then, there always exists a robust H_2 filter of the form (10)$\sim$(11). Further, the optimal robust H_2 filter can be obtained by solving the following optimization

$$\min_{\Gamma>0,R>0,S>0,\mathcal{A},\mathcal{B},\mathcal{C},\mathcal{D}} trace(\Sigma) \tag{27}$$

subject to

$$\begin{bmatrix} -\Sigma & * & * & * \\ RB+\mathcal{B}D & -R & * & * \\ SB & -S & -S & 0 \\ L-\mathcal{D}D & 0 & 0 & -I \end{bmatrix} < 0 \tag{28}$$

$$\begin{bmatrix} -R+E^{\mathrm{T}}\Gamma E & * & * & * & * & * \\ -S+E^{\mathrm{T}}\Gamma E & -S+E^{\mathrm{T}}\Gamma E & * & * & * & * \\ RA+\mathcal{B}C & \mathcal{A} & -R & * & * & * \\ SA & SA & -S & -S & * & * \\ L-\mathcal{D}C & L-\mathcal{C} & 0 & 0 & -I & * \\ 0 & 0 & H_1^{\mathrm{T}}R+H_2^{\mathrm{T}}\mathcal{B}^{\mathrm{T}} & H_1^{\mathrm{T}}S & -H_2^{\mathrm{T}}\mathcal{D}^{\mathrm{T}} & -\Gamma \end{bmatrix} < 0 \tag{29}$$

where '$*$' denotes entry that can be deduced from the symmetry property of the matrix, $\Gamma = diag\{\varepsilon_1 I_{\alpha_1}, \varepsilon_2 I_{\alpha_2}, \cdots, \varepsilon_\ell I_{\alpha_\ell}\}$ is a scaling matrix and $R,\ S,\ \mathcal{A},\ \mathcal{B},\ \mathcal{C},\ \mathcal{D}$ are matrices to be determined.

Indeed, given optimal solutions ($\Gamma > 0, R > 0,\ S > 0,\ \mathcal{A},\ \mathcal{B},\ \mathcal{C},\ \mathcal{D}$), an optimal robust H_2 filter can be computed through

$$MN^{\mathrm{T}} = I - RS^{-1} \tag{30}$$

$$\hat{D} = \mathcal{D} \tag{31}$$

$$\hat{C} = (\mathcal{C} - \mathcal{D}C)S^{-1}N^{-\mathrm{T}} \tag{32}$$

$$\hat{B} = M^{-1}\mathcal{B} \tag{33}$$

$$\hat{A} = M^{-1}(\mathcal{A} - RA - M\hat{B}C)S^{-1}N^{-\mathrm{T}} \tag{34}$$

Remark 5. It is observed that LMIs (28) and (29) are linear in $\Gamma,\ R,\ S,\ \mathcal{A},\ \mathcal{B},\ \mathcal{C}$ and $\mathcal{D}$. Therefore, the optimization in (27) is convex. As compared to the results in [14,16] where one-step ahead prediction was considered, the result in Theorem 3 is numerically much more attractive as the former requires searching for scaling parameters for which no effective algorithms are available. Furthermore, Theorem 3 also allows to handle both the one-step ahead predictor ($\hat{D} = 0$) and the filtering cases.

4 Robust H_2 filtering of systems with polytopic uncertainty

In this section, we shall study the robust H_2 filtering for systems with polytopic uncertainty in the infinite horizon case. In the past few years, the LMI approach has been prevailing in dealing with this problem. Here, we shall capture some of these developments. The finite horizon case can be found in [18] where a DLMI (difference LMI) technique was applied. We shall not address this case here.

First, denote $\boldsymbol{\xi} = [\boldsymbol{x}^{\mathrm{T}}\ \ \hat{\boldsymbol{x}}^{\mathrm{T}}]^{\mathrm{T}}$. It follows from (1)$\sim$(3) and (10)$\sim$(11) that

$$\boldsymbol{\xi}_{k+1} = \bar{A}_\delta \boldsymbol{\xi}_k + \bar{B}\boldsymbol{w}_k \tag{35}$$

$$\boldsymbol{z}_k - \hat{\boldsymbol{z}}_k = \bar{C}_\delta \boldsymbol{\xi}_k + \bar{D}\boldsymbol{w}_k \tag{36}$$

where

$$\bar{A}_\delta = \begin{bmatrix} A_\delta & 0 \\ \hat{B}C_\delta & \hat{A} \end{bmatrix},\quad \bar{B} = \begin{bmatrix} B \\ \hat{B}D \end{bmatrix},\quad \bar{C}_\delta = [L - DC_\delta \quad -\hat{C}],\quad \bar{D} = -\hat{D}D \tag{37}$$

Recall that for any uncertainty from Ω_P, an upper bound for the H_2 norm square of the system (35)$\sim$(36) can be computed by the following minimization[18]:

$$\min_P\ trace(\bar{C}_\delta P \bar{C}_\delta^{\mathrm{T}} + \bar{D}\bar{D}^{\mathrm{T}}) \tag{38}$$

subject to

$$\bar{A}_\delta P \bar{A}_\delta^{\mathrm{T}} - P + \bar{B}\bar{B}^{\mathrm{T}} < 0 \tag{39}$$

A robust H_2 filtering based on a fixed Lyapunov function matrix P was given in [17], which is generally conservative.

Since the uncertainty in A_δ and C_δ comes from the polytope Ω_P, it is clear[30] that (38)~(39) hold if

$$\begin{bmatrix} \Sigma & \bar{C}^{(i)}G & \bar{D} \\ * & G+G'-P_i & 0 \\ \bar{D}^{\mathrm{T}} & 0 & I \end{bmatrix} > 0 \tag{40}$$

and

$$\begin{bmatrix} P_i & \bar{A}^{(i)}G & \bar{B} \\ * & G+G'-P_i & 0 \\ * & * & I \end{bmatrix} > 0, \quad i=1,2,\cdots,N_v \tag{41}$$

where $\bar{A}^{(i)}$ and $\bar{C}^{(i)}$ denote respectively matrices $\bar{A}_\delta$ and $\bar{C}_\delta$ at the i-th vertex of the polytope Ω_P.

The above result exploits the vertex-dependent parameter P_i. This can be seen as follows. Suppose that (41) is satisfied at all the vertices of Ω_P. Then for any uncertainty $\delta \in \Omega_P$, *i.e.* $(A_\delta, C_\delta) = \sum_{i=1}^{N_v} \alpha_i (A^{(i)}, C^{(i)})$,

$$\begin{bmatrix} \Sigma & \bar{C}_\delta G & \bar{D} \\ * & G+G'-\sum_{i=1}^{N_v}\alpha_i P_i & 0 \\ * & * & I \end{bmatrix} > 0, \quad \begin{bmatrix} \sum_{i=1}^{N_v}\alpha_i P_i & \bar{A}_\delta G & \bar{B} \\ * & G+G'-\sum_{i=1}^{N_v}\alpha_i P_i & 0 \\ * & * & I \end{bmatrix} > 0 \tag{42}$$

which clearly shows that $\sum_{i=1}^{N_v}\alpha_i P_i$ is a parameter-dependent Lyapunov matrix. A robust H_2 one-step ahead predictor design based on the parameter-dependent Lyapunov function is recalled below.

Theorem 4[18,23]. Consider the system (1)~(3) with polytopic uncertainty Ω_P. The optimal robust H_2 one-step ahead predictor can be given by (10)~(11) with

$$\hat{A} = MS^{-1}, \quad \hat{B} = M^{-1}Z, \quad \hat{C} = S_C$$

where matrices M, S, Z and S_C are solutions to the following convex optimization:

$$\min_{M,R,X,Z,S,S_C,H_j^{(i)},\ j=1,2,3} \quad trace(\Sigma) \tag{43}$$

subject to

$$\begin{bmatrix} \Sigma & L-S_C & L \\ * & H_1^{(i)} & H_2^{(i)} \\ * & * & H_3^{(i)} \end{bmatrix} > 0 \tag{44}$$

and

$$\begin{bmatrix} -R^{\mathrm{T}}-R+H_1^{(i)} & -R^{\mathrm{T}}-X-M^{\mathrm{T}}+H_2^{(i)} & R^{\mathrm{T}}A^{(i)} & R^{\mathrm{T}}A^{(i)} & R^{\mathrm{T}}B \\ * & -X-X^{\mathrm{T}}+H_3^{(i)} & X^{\mathrm{T}}A^{(i)}+ZC^{(i)}+S & X^{\mathrm{T}}A^{(i)}+ZC^{(i)} & X^{\mathrm{T}}B+ZD \\ * & * & -H_1^{(i)} & -H_2^{(i)} & 0 \\ * & * & * & -H_3^{(i)} & 0 \\ * & * & * & * & -I \end{bmatrix} < 0 \tag{45}$$

for $i = 1,\ 2,\ \cdots,\ N_v$.

Remark 6. Observe that (44)~(45) are linear in unknown parameters $M,\ R,\ X,\ Z,\ S, S_C,\ H_j^{(i)},\ j = 1,2,3$. Hence, the optimization in (43) can be solved by convex optimization using the LMI Tool[26]. It was demonstrated in [18] that Theorem 4 gives a much less conservative design than that of [17]. Certainly, it is worth noting that even with the deployment of the parameter-dependent Lyapunov function the result is still sufficient but not necessary and thus there exists some degree of conservatism. Therefore, efforts remain to be made to improve the design.

Note that (39) is equivalent to

$$\begin{bmatrix} \bar{A}_\delta & \bar{B} \end{bmatrix} diag\{P, I\} \begin{bmatrix} \bar{A}_\delta^{\mathrm{T}} \\ \bar{B}^{\mathrm{T}} \end{bmatrix} - P < 0 \tag{46}$$

or

$$\begin{bmatrix} \bar{A}_\delta & \bar{B} \end{bmatrix}^{\mathrm{T}} Q \begin{bmatrix} \bar{A}_\delta & \bar{B} \end{bmatrix} - diag\{Q, I\} < 0 \tag{47}$$

where $Q = P^{-1}$.

The following lemma can be found in [31].

Lemma 1[31]. There exists a matrix $Q = Q^{\mathrm{T}} > 0$ to (47) if there exists a solution $(F, G, Q^{(i)},\ i = 1, 2, \cdots, N_v)$ with $Q^{(i)} = Q^{(i)\mathrm{T}}$ such that

$$\begin{bmatrix} -diag\{Q^{(i)}, I\} + \begin{bmatrix} \bar{A}^{(i)\mathrm{T}} \\ \bar{B}^{\mathrm{T}} \end{bmatrix} F + F^{\mathrm{T}}[\bar{A}^{(i)} \quad \bar{B}] & -F^{\mathrm{T}} + \begin{bmatrix} \bar{A}^{(i)\mathrm{T}} \\ \bar{B}^{\mathrm{T}} \end{bmatrix} G \\ -F + G^{\mathrm{T}}[\bar{A}^{(i)} \quad \bar{B}] & Q^{(i)} - (G + G^{\mathrm{T}}) \end{bmatrix} < 0 \tag{48}$$

Remark 7. As compared to (41), (48) contains an extra slack variable F which will help reduce the conservatism in evaluating the H_2 performance of a given robust filter. The result was also explored to give an improved design of robust H_2 filter in [20] which is recalled below. When F is set to be zero, (48) will be reduced to (41).

Theorem 5[20]. Consider the system (1)$\sim$(3) over the polytope Ω_P. A filter of the form (10)$\sim$(11) that gives a suboptimal guaranteed filtering error covariance bound can be derived from the following optimization

$$\min_{(R, W, S_A, S_B, S_C, S_D, M, H_1^{(i)}, H_2^{(i)}, H_3^{(i)}, i=1,2,\cdots N, \lambda_1, \lambda_2)} trace(\Sigma)$$

subject to

$$\begin{bmatrix} \lambda_1(A^{(i)\mathrm{T}}R + R^{\mathrm{T}}A^{(i)}) - H_1^{(i)} & * & * & * & * \\ \lambda_1 W^{\mathrm{T}}A^{(i)} + \lambda_2(S_B C^{(i)} + S_A) - H_2^{(i)\mathrm{T}} & -\lambda_2(S_A + S_A^{\mathrm{T}}) - H_3^{(i)} & * & * & * \\ \lambda_1 B^{\mathrm{T}}R & \lambda_1 B^{\mathrm{T}}W + \lambda_2 D^{\mathrm{T}}S_B^{\mathrm{T}} & -I & * & * \\ R^{\mathrm{T}}A^{(i)} - \lambda_1 R & -\lambda_1 W - \lambda_2 M^{\mathrm{T}} & R^{\mathrm{T}}B & H_1^{(i)} - (R + R^{\mathrm{T}}) & * \\ W^{\mathrm{T}}A^{(i)} + S_B C^{(i)} + S_A & -S_A + \lambda_2 M^{\mathrm{T}} & W^{\mathrm{T}}B + S_B D & H_2^{(i)\mathrm{T}} - (W^{\mathrm{T}} + M) & H_3^{(i)} + (M + M^{\mathrm{T}}) \end{bmatrix} < 0 \tag{49}$$

and

$$\begin{bmatrix} \Sigma & * & * & * \\ L^{\mathrm{T}} - C^{(i)\mathrm{T}}S_D^{\mathrm{T}} - S_C^{\mathrm{T}} & H_1^{(i)} & * & * \\ S_C^{\mathrm{T}} & H_2^{(i)\mathrm{T}} & H_3^{(i)} & * \\ -D^{\mathrm{T}}S_D^{\mathrm{T}} & 0 & 0 & I \end{bmatrix} > 0 \tag{50}$$

for $i = 1, 2, \cdots, N_v$. The suboptimal filter is given by

$$\hat{A} = M^{-1}S_A, \quad \hat{B} = M^{-1}S_B, \quad \hat{C} = S_C, \quad \hat{D} = S_D \tag{51}$$

Remark 8. It should be mentioned that when $\lambda_1 = \lambda_2 = 0$ and $\hat{D} = 0$, Theorem 5 recovers the result of Theorem 4 where a one-step ahead predictor is adopted. It is then clear that the result in Theorem 5 is more general and guaranteed to be less conservative than those in Theorem 4 due to the extra degrees of freedom in optimization provided by the free parameters λ_1 and λ_2.

Remark 9. Observe that for given λ_1 and λ_2, (49) and (50) are linear in R, W, S_A, S_B, S_C, S_D, M, $H_1^{(i)}$, $H_2^{(i)}$ and $H_3^{(i)}$, and hence can be solved by employing the LMI Tool[26]. The problem is then how to find the optimal values of λ_1 and λ_2 in order to minimize the filtering error variance bound. One way to address the tuning issue is to first solve the feasibility problem of the LMIs (49)$\sim$(50) with $i = 1, 2, \cdots, N_v$ using Matlab's LMI toolbox[26] and obtain a set of initial scaling parameters. Then, applying a numerical optimization algorithm, such as the program **fminsearch** in the optimization toolbox of Matlab[32] a locally convergent solution to the problem is obtained. The search is demonstrated in the examples of Section 5. Based on our experience, this optimization procedure can be efficient for the optimization in Theorem 5 as it only involves the search of two parameters λ_1 and λ_2.

Note that Theorem 5 has been derived with a specialized matrix F of the form

$$F = [\varLambda G \quad 0]$$

where $\varLambda = diag\{\lambda_1 I, \lambda_2 I\}$ in order to linearize the matrix inequality[20]. This, however, is restrictive. In the following we will propose an iterative algorithm which can be applied to refine the filter designed using Theorem 5.

To this end, we denote

$$\overline{AB}^{(i)} = \begin{bmatrix} A^{(i)} & 0 & B \\ 0 & 0 & 0 \end{bmatrix}, \quad \widehat{AB} = [\hat{A} \quad \hat{B}], \quad \overline{CD} = \begin{bmatrix} 0 & I & 0 \\ C^{(i)} & 0 & D \end{bmatrix}$$

Then, (48) can be rewritten as

$$\begin{bmatrix} -diag\{Q^{(i)}, I\} + \overline{AB}^{(i)\mathrm{T}} F + F^{\mathrm{T}} \overline{AB}^{(i)} + \overline{CD}^{(i)\mathrm{T}} \widehat{AB}^{\mathrm{T}} [0 \quad I] F + F^{\mathrm{T}} \begin{bmatrix} 0 \\ I \end{bmatrix} \widehat{AB}\overline{CD}^{(i)} & * \\ -F + G^{\mathrm{T}} \overline{AB} + G^{\mathrm{T}} \begin{bmatrix} 0 \\ I \end{bmatrix} \widehat{AB}\overline{CD}^{(i)} & Q^{(i)} - (G + G^{\mathrm{T}}) \end{bmatrix} < 0 \tag{52}$$

The following iterative procedure can be applied.

Step 1. Given the filter parameters $(\hat{A}, \hat{B}, \hat{C}, \hat{D})$, F, G and $Q^{(i)}$ may be found by minimizing $trace(\Sigma)$ subject to (48) and (40). The initial $(\hat{A}, \hat{B}, \hat{C}, \hat{D})$ can be the suboptimal filter designed by Theorem 5.

Step 2. With F, G and $Q^{(i)}$ obtained in Step 1, an improved filter can be obtained by minimizing $trace(\Sigma)$ subject to (52) and (40).

Repeat the above steps until $trace(\Sigma_{k-1} - \Sigma_k) < \mu$, where μ is a prescribed tolerance and Σ_k is the matrix Σ of (40) at the k-th iteration.

It should be emphasized that the above iteration always converges.

5 Illustrative examples

Consider the example in [14]:

$$\boldsymbol{x}_{k+1} = \begin{bmatrix} 0 & -0.5 \\ 1 & 1+\delta \end{bmatrix} \boldsymbol{x}_k + \begin{bmatrix} -6 \\ 1 \end{bmatrix} w_k \tag{53}$$

$$y_k = [\,-100 \quad 10\,]\, \boldsymbol{x}_k + v_k \tag{54}$$

$$z_k = [1 \quad 0] \boldsymbol{x}_k \tag{55}$$

where w_k and v_k are uncorrelated zero-mean white noise signals with unit variances, respectively. δ is the uncertain parameter satisfying $|\delta| \leqslant \delta_0$, where δ_0 is known to be a positive real number. Obviously, the uncertainty in this system can be represented by norm-bounded uncertainty.

Consider $\delta_0 = 0.3$. The above system matrices can be put into the form (4) with single block uncertainty and

$$H_1 = \begin{bmatrix} 0 \\ 10 \end{bmatrix}, \quad H_2 = 0, \quad E = [0 \quad 0.03]$$

Use the results in Theorems 2 and 3, the optimal robust one-step ahead predictor (setting $\hat{D} = 0$) based on the Riccati equation approach and the LMI approach can be obtained. Theorem 3 also allows us to compute the optimal filter (with $\hat{D} \neq 0$). A performance comparison of the optimal one-step ahead predictors and the optimal filter is given in Table 1.

Table 1 Performance of robust predictor and filter for the norm-bounded uncertainty characterization

	$\delta = -0.3$	$\delta = 0$	$\delta = 0.3$	bound
Robust filter of [14]	64.0	61.4	64.4	98.7
One-step ahead predictor by Theorem 2	52.8	51.1	54.4	69.2
One-step ahead predictor by Theorem 3	49.3	52.2	65.9	66.9
Optimal filter by Theorem 3	0.85	0.77	0.90	1.23

As expected, the optimal robust filter gives a much better performance than the optimal robust one-step ahead predictor.

Next, consider the discrete-time system in the form of (1)~(3) with [17]

$$A = \begin{bmatrix} 0.9 & 0.1 + 0.06\alpha \\ 0.01 + 0.05\beta & 0.9 \end{bmatrix}, \ B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$
$$C = [1 \quad 0], \ D = [0 \quad 0 \quad 1.414], \ L = [1 \quad 1]$$

where $|\alpha| \leqslant 1$ and $|\beta| \leqslant 1$. This is a two-block structured uncertainty which can be described by a four-vertex ploytope. The value of the H_2 guaranteed cost based upon the method in [12] (see also Theorem 3) is 44.0039. To make a fair comparison, we also adopt a one-step ahead predictor by letting $\hat{D} = 0$. Using Theorem 5 and the **fminsearch** in Remark 9, the optimal scaling parameters are obtained as $\lambda_1 = -0.9842$ and $\lambda_2 = -0.9747$ with the initial value $[\lambda_1, \lambda_2] = [-0.5, -0.5]$. The optimal H_2 guaranteed cost is 19.4682 with the corresponding predictor

$$\hat{A} = \begin{bmatrix} 0.0705 & 0.0263 \\ 1.2779 & 0.5492 \end{bmatrix}, \ \hat{B} = \begin{bmatrix} 0.9114 \\ -0.9972 \end{bmatrix}, \ \hat{C} = [1.2885 \quad 0.2382]$$

Further, starting from the above predictor and using the iterative algorithm in Section 4, a much less conservative minimum bound of 15.9759 can be obtained and the predictor is given by

$$\hat{A} = \begin{bmatrix} 0.0710 & 0.0262 \\ 1.2764 & 0.5496 \end{bmatrix}, \ \hat{B} = \begin{bmatrix} 0.9110 \\ -0.9960 \end{bmatrix}, \ \hat{C} = [1.2852 \quad 0.2433]$$

The above iteration only takes two steps to converge. The actual performance of the resultant predictor is given in Fig. 1 which verifies that the cost for any admissible uncertainty is below the derived upper bound.

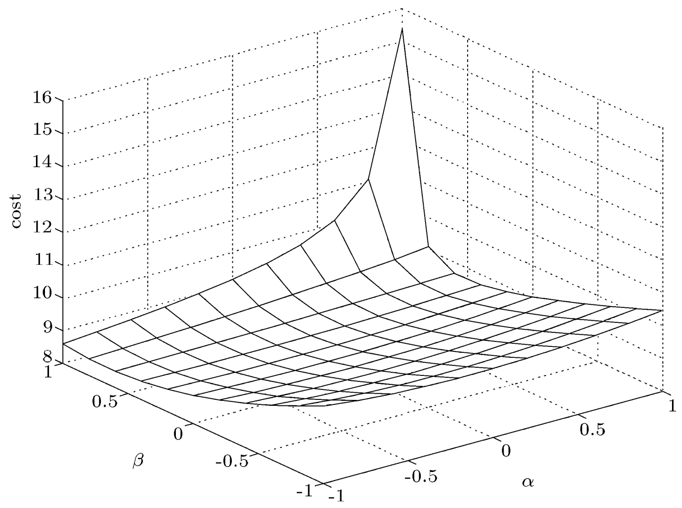

Fig. 1 Actual cost of the filter for various uncertain parameters α and β

6 Conclusion

This paper has revisited the robust H_2 estimation of discrete-time linear uncertain systems with both the norm-bounded uncertainty and the polytopic uncertainty. We have provided an overview on the development in this field in the past decades. In particular, the Riccati equation approach has been presented for the finite horizon case and the problem of optimizing the covariance matrix of estimation error which is non-recursive has been addressed. The LMI approach has been applied for the design of robust estimator in the infinite horizon case. We have demonstrated how slack variables can be incorporated to give a less conservative design. At this stage, efforts are being made on new techniques which are able to greatly reduce the design conservatism by using non-traditional methods such as applying non-quadratic Lyapunov functions.

References

1 Anderson B D O, Moore J B. *Optimal Filtering.* Englewood Cliffs, N.J.: Prenetice-Hall, 1979

2 Xie L, De Souza C E, Fu M. H_∞ estimation for discrete-time linear uncertain systems, *International J. Robust Nonlinear Control*, 1991, **1**: 111∼123

3 Fu M, De Souza C E, Xie L. H_∞ estimation for uncertain systems, *International J. Robust and Nonlinear Control*, 1992, **2**: 87∼105

4 Petersen I R, McFarlane D C. Robust state estimation for uncertian systems, In: Proceedings 30^{th} IEEE Conference on Decision and Control, Brighton, U.K., 1991, 2630∼2631

5 Jain B N. Guaranteed error estimation in uncertain systems, *IEEE Transsctions on Automatic Control*, 1975, **20**: 230∼232

6 Xie L, Soh Y C. Robust Kalman filtering for uncertian systems, *Systems & Control Letters*, 1994, **22**: 123∼129

7 Petersen I R, McFarlane D C. Optimal guaranteed cost control and filtering for uncertain linear systems, *IEEE Transactions on Automatic Control*, 1994, **39**: 1971∼1977

8 Shaked U, De Souza C E. Robust minimum variance filtering, *IEEE Transactions on Signal Processing*, 1995, **43**: 2474∼2483

9 Li H, Fu M. A linear matrix inequality approach to robust H_∞ filtering, *IEEE Transactions on Signal Processing*, 1997, **45**: 2338∼2350

10 Bolzern P, Colaneri P, De Nicolao G. Finite escape and convergence properties of guaranteed-cost robust filters, *Automatica*, 1997, **33**(1): 31∼47

11 Geromel J C, De Oliviera M. H_2 and H_∞ robust filtering for convex bounded uncertain systems, In: Proceedings of the 37th IEEE CDC, Tampa, FL, 1998, 146∼161

12 Geromel J C. Optimal Linear Filtering under Parameter Uncertainty, *IEEE Transactions on Signal Processing*, 1999, **47**: 168∼175

13 Tuan H D, Apkarian P, Nguyen T Q. Robust and reduced-order filtering: new LMI-based characterizations and methods, *IEEE Transactions on Signal Processing*, 2001, **49**(2): 2975∼2984

14 Xie L, Soh Y C. De Souza C E. Robust Kalman filtering for uncertain discrete-time systems, *IEEE Transactions on Automatic Control*, 1994, **39**: 1310∼1314

15 Petersen I R, McFarlane D C. Optimal guaranteed cost filtering for uncertain discrete-time linear systems, *International J. Robust Nonlinear Control*, 1996, **6**: 267∼280

16 Theodor Y, Shaked U. Robust discrete-time minimum-variance filtering, *IEEE Transactions on Signal Processing*, 1996, **44**: 181∼189

17 Geromel J C, Bernussou J, Garcia G, De Oliveira M C. H_2 and H_∞ robust filtering for discrete-time linear systems, *SIAM J. Control Optimization*, 2000, **38**(5): 1353∼1368

18 Shaked U, Xie L, Soh Y C. New approaches to robust minimum variance filter design, *IEEE Transactions on Signal Processing*, 2001, **49**: 2620∼2629

19 Zhu X, Soh Y C, Xie L. Design and analysis of discrete-time robust Kalman filters, *Automatica*, 2002, **38**(6): 1069∼1077

20 Xie L, Lu L, Zhang D, Zhang H. Improved robust H_2 and H_∞ filtering for uncertain discrete-time systems, *Automatica*, 2004, **40**: 873∼880

21 Sayed A H. A framework for state-space estimation with uncertain models, *IEEE Transactions on Automatic Control*, 2001, **46**(7): 998∼1013

22 Barbosa K A, De Souza C E, Trofino A. Robust H_2 filtering for discrete-time uncertain linear systems using parameter-dependent Lyapunov functions, In: Proceedings 2002 American Control Conference, Anchorage, AK, 2002, 3224∼3229

23 Geromel J C, De Oliveira M, Bernussou J. Robust filtering of discrete-time linear systems with parameter dependent Lyapunov functions, *SIAM J. Control Optimization*, 2002, **41**: 700∼711

24 De Oliveira M C, Bernussou J, Geromel J C. A new discrete-time robust stability condition, *Systems & Control Letters*, 1999, **37**(4): 261∼265

25 Xie L, Soh Y C, Du C. Robust H_2 estimation and control, In: Proceedings 7$^{\text{th}}$ International Conference on Control, Automation, Robotics and Vision, Singapore, 2002

26 Gahinet P, Nemirovski A, Laub A J, Chilali M. LMI Control Toolbox. Natick, MA: The Math Works, Inc., 1995

27 Fu M, De Souza C E, Luo Z. Finite horizon robust Kalman filter design, *IEEE Transactions on Signal Processing*, 2001, **49**(9): 2103$\sim$2112

28 Xie L, Soh Y C. Guaranteed cost control of uncertain discrete-time systems, Control Theory and Advanced Technology, Japan, Part 2, 1995, **10**(4): 1235$\sim$1251

29 De Souza C E, Fu M, Xie L. H_∞ analysis and synthesis of discrete-time systems with time-varying uncertainty, *IEEE Transactions on Automatic Control*, USA, 1993, **38**: 459$\sim$462

30 De Oliviera M C, Geromel J C, Bernussou J. An LMI optimization approach to multiobjective controller design for discrete-time systems, In: Proceedings 38$^{\text{th}}$ IEEE Conference Decision and Control, Phoenix, AZ, 1999, 3611$\sim$3616

31 Peaucelle D, Arzelier D, Bachelier O, Bernussou J. A new robust $\mathcal{D}$-stability condition for real convex polytopic uncertainty, *Systems & Control Letters*, 2000, **40**: 21$\sim$30

32 Coleman T, Branch M, Grace A. Optimization Toolbox for Use with Matlab, The MathWorks Inc. Natick, Mass, 1999

33 De Souza C E, Trofino A. An LMI approach to the design of robust H_2 filters, In: Recent Advances on Linear Matrix Inequality Methods in Control, L. El Ghaoui and S. Niculescu (Eds.), SIAM, 1999

34 Xie L. H_∞ output feedback control of systems with parameter uncertainty, *International J. Control*, 1996, **63**: 741$\sim$750

Lihua Xie Received the bachelor and master degrees in electrical engineering from Nanjing University of Science and Technology, P.R.China in 1983 and 1986, respectively, and the Ph.D. degree in electrical engineering from the University of Newcastle, Australia, in 1992. From April 1986 to January 1989, he held teaching appointments in the Department of Automatic Control, Nanjing University of Science and Technology, P.R.China. Since 1992 he has been with the School of Electrical and Electronic Engineering, Nanyang Technological University, Singapore, where he is currently a professor. He is an adjunct professor of Harbin Institute of Technology and Dalian University of Technology. He also held visiting appointments with the University of Melbourne and the Hong Kong Polytechnic University. He was the General Co-Chair of the 2002 International Conference on Control and Automation, Xiamen, and the Program Chair of the Fourth International Conference on Control and Automation, Montreal, June 2003 and the Program Chair of the Eighth International Conference on Control, Automation, Robotics and Vision, Kunming, December 2004. He is currently an associate editor of IEEE Transactions on Automatic Control, International Journal of Control, Automation and Systems and Journal of Control Theory and Applications. He served as an associate editor of the Conference Editorial Board, IEEE Control Systems Society from 2000 to 2004. He has authored and co-authored one book and over 110 articles in refereed international journals. His current research interests include robust control, networked control systems, control of disk drive systems and smart structures, and estimation theory with applications in autonomous vehicles.