New Results on Global Synchronization of Chua′s Circuit[1)]

LIAO Xiao-Xin LUO Hai-Geng ZHANG Gang
JIAN Ji-Gui ZONG Xiao-Jun XU Bing-Ji
(*Department of Control Science and Engineering, Huazhong University of Science and Technology, Wuhan* 430070)
(E-mail: xiaoxinliao@163.net)

Abstract Some new results on global exponential synchronization and global synchronization for Chua′s circuit are derived by means of Lyapunov functions and some other mathematic methods, which improve the existing results in the literatures. A strict and complete proof of the result is also given as a complement to the relevant literature where the proof was incomplete. The paper offers some new approaches for studying chaos synchronization for Chua′s circuit.

Key words Chaos, global exponential synchronization, global synchronization

1 Introduction

Since Pecora and Carroll in U.S. Navy Laboratory firstly realized synchronization of chaotic system by electronic circuit in 1990[1], the theories and applications of chaos control and synchronization have attracted considerable interests[2∼23]. A great deal of papers concerning chaos have emerged. Because chaotic communication possesses many advantages, such as high security, large dynamical storage capacity, low power consumption, low observability, low cost, *etc*, it is a new promising research subject in communication and control engineering.

The strange attractor of Chua′s circuit is different from the attractors of quadratic systems such as Lorenz system, Chen system, Rössler system, Lü system and unified system[22,23]. It is the simplest piecewise-linear chaotic attractor which is firstly realized by electronic circuit in laboratory and it displays abundant dynamical behaviors. Naturally, more and more researchers have devoted to this system, such as [19] and [7,10,11,12,20,21].

This paper obtains some new results on global exponential synchronization and global synchronization for Chua′s circuit, using Lyapunov functions and some other mathematic methods. It also improves and generalizes the results of [12,20,21] as corollaries. Furthermore, a strict and complete proof to the result of [16] where the proof was incomplete is given. We aim to offer some new approaches for studying chaos synchronization for Chua′s circuit.

2 Equations for Chua′s circuit and the existed results

Consider the differential equation for Chua′s circuit as follows.

$$\begin{cases} \dot{x} = p(-x + y - f(x)) \\ \dot{y} = x - y + z \\ \dot{z} = -qy \end{cases} \tag{1}$$

where $f(x) = bx + \frac{1}{2}(a-b)(|x+E| - |x-E|)$, and $p > 0$, $q > 0$, $a < b < 0$, $0 < E < +\infty$ are constants. When $p = 10.0$, $q = 14.87$, $a = -1.27$, $b = -0.68$, $E = 1$, it is the chaotic attractor of Chua′s circuit in [16,p82].

Applying two mutually coupled Chua′s circuits to chaos synchronization investigation, we obtain the following differential equations for the transmitter and receiver circuits, respectively.

$$\text{drive}: \begin{cases} \dot{x}_d = p(y_d - x_d - f(x_d)) + \delta_x(x_r - x_d) \\ \dot{y}_d = x_d - y_d + z_d + \delta_y(y_r - y_d) \\ \dot{z} = -qy_d + \delta_z(z_r - z_d) \end{cases} \tag{2}$$

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$$\text{response}: \begin{cases} \dot{x}_r = p(y_r - x_r - f(x_r)) + \delta_x(x_d - x_r) \\ \dot{y}_r = x_r - y_r + z_r + \delta_y(y_d - y_r) \\ \dot{z}_r = -qy_r + \delta_z(z_d - z_r) \end{cases} \tag{3}$$

where $\delta_x, \delta_y, \delta_z$ are feedback constants.

[10,11] obtained the following results.

Theorem A. If there exist constants $\delta_1 < \delta_2$, such that for $\delta_x, \delta_y, \delta_z \in (\delta_1, \delta_2)$, matrices

$$\begin{pmatrix} -p - p\lambda - 2\delta_x & p & 0 \\ 1 & -1 - 2\delta_y & 1 \\ 0 & -1 & -2\delta_z \end{pmatrix}$$

are all Hurwitz matrices when $\lambda = a, \lambda = b$ and if the initial conditions locate in the same attraction domain, then for all $\delta_x, \delta_y, \delta_z \in (\delta_1, \delta_2)$, the submanifold $x_d = x_r, y_d = y_r, z_d = z_r$ is stable in the phase space $\{x_d, y_d, z_d, x_r, y_r, z_r\} \in R^6$.

[12] oppugned the above result. We still think even if the above theorem can be strictly demonstrated by other methods, it is difficult to confirm infinite matrices are Hurwitz matrices. Secondly, this result is inconvenient in practice since it can not give any message about the attraction domain.

[16] improved the above theorem. If one lets $\delta_x = \delta_y = \delta_z = 0$ in transmitter system (2) and only considers the case $\delta_x^2 + \delta_y^2 + \delta_z^2 \neq 0$ in receiver system (3), a fairly common result can be derived.

Theorem B. If $pa + \delta_x \geqslant 0, \delta_y \geqslant 0, \delta_z \geqslant 0$, then systems (2) and (3) globally synchronize.

But we cannot confirm that the conclusion is true from the proof when $\delta_y = 0, \delta_z = 0, pa + \delta_x = 0$. In fact, we can not demonstrate it even if we use LaSalle invariant principle. Hence, [20,21] derived the following result under stronger conditions.

Theorem C. Suppose $\delta_y = \delta_z = 0$. If $pa + \delta_x > 0$ then the submanifold $\{x_d, y_d, z_d, x_r, y_r, z_r | x_d = x_r, y_d = y_r, z_d = z_r\}$ of systems (2) and (3) in R^6 globally synchronizes, *i.e.* the trivial solution of the following error dynamical system

$$\begin{cases} \dot{e}_x = pe_y - pe_x - p[f(x_d) - f(x_r)] - \delta_x e_x \\ \dot{e}_y = e_x - e_y + e_z \\ \dot{e}_z = -qe_y \end{cases} \tag{4}$$

is asymptotically stable (where $e_x = x_d - x_r, e_y = y_d - y_r, e_z = z_d - z_r$).

This paper proves that systems (2) and (3) are globally exponentially synchronized under the same hypothesis as [20,21]. Moreover, a new theorem on global exponential synchronization with different conditions in [20,21] is derived. Lastly, a strict and complete proof to the result of [16] where the proof was incomplete is given. Some new approaches for studying chaos synchronization of Chua′s circuit are offered.

3 Results of global exponential synchronization

We firstly give the definition of global exponential synchronization of systems (2) and (3).

Definition 1. It is said that two chaotic circuits (2) and (3) are globally exponentially synchronized if for any given initial values $x_d(0), y_d(0), z_d(0) \in R^3$, and corresponding $x_r(0), y_r(0), z_r(0) \in R^3$ the trivial solution of system (4) is globally exponentially stable, *i.e.*,

$$e_x^2(t) + e_y^2(t) + e_z^2(t) \leqslant ke^{-\alpha t}$$

where $\alpha > 0$ is a constant, k is a constant depending on initial values $e_x(0), e_y(0), e_z(0)$.

Theorem 1. If $\delta_x + pa > 0, \delta_y = 0, \delta_z = 0$, then the trivial solution of system (4) is globally exponentially stable, which implies systems (2) and (3) are globally exponentially synchronized.

Proof. Because the matrix $\text{diag}(q, pq, p)$ is positive definite, we can select $0 < \varepsilon << 1$, such that

$$W := \begin{pmatrix} q & 0 & 0 \\ 0 & pq & -\frac{\varepsilon}{2} \\ 0 & -\frac{\varepsilon}{2} & p \end{pmatrix}$$

is also positive definite. Construct a positive definite and radially unbounded Lyapunov function, *i.e.*,

$$V(e) := \begin{pmatrix} e_x \\ e_y \\ e_z \end{pmatrix} \begin{pmatrix} q & 0 & 0 \\ 0 & pq & -\frac{\varepsilon}{2} \\ 0 & -\frac{\varepsilon}{2} & p \end{pmatrix} \begin{pmatrix} e_x \\ e_y \\ e_z \end{pmatrix} \tag{5}$$

Computing the time derivative of $V(e)$ along the solution of system (4), we have

$$\begin{aligned} \left.\frac{\mathrm{d}V(e)}{\mathrm{d}t}\right|_{(4)} =& 2qe_x\dot{e}_x + 2pqe_y\dot{e}_y + 2pe_z\dot{e}_z - \varepsilon\dot{e}_y e_z - \varepsilon e_y\dot{e}_z \leqslant \\ & -2(pa+\delta_x)qe_x^2 - 2pqe_x^2 + 4pqe_x^2 - 2pqe_y^2 - \varepsilon e_x e_z + \varepsilon e_x e_z - \varepsilon e_z^2 + \varepsilon q e_y^2 = \\ & \begin{pmatrix} e_x \\ e_y \\ e_z \end{pmatrix} \begin{pmatrix} -2(pa+\delta_x)q - 2pq & 2pq & -\frac{\varepsilon}{2} \\ 2pq & -2pq+\varepsilon q & \frac{\varepsilon}{2} \\ -\frac{\varepsilon}{2} & \frac{\varepsilon}{2} & -\varepsilon \end{pmatrix} \begin{pmatrix} e_x \\ e_y \\ e_z \end{pmatrix} := e^{\mathrm{T}}\Omega e \\ & \Delta_1 := -2(pa+\delta_x)q - 2pq < 0 \\ & \Delta_2 := [-2(pa+\delta_x)q - 2pa](-2pq+\varepsilon q) - 4p^2q^2 = \\ & 4pq^2(pa+\delta_x) - 2\varepsilon q^2(pa+\delta_x) - 2\varepsilon pq^2 > 0, \quad \text{when } 0 < \varepsilon << 1 \\ & \Delta_3 := \det\Omega = -4\varepsilon pq^2(pa+\delta_x) + o(\varepsilon^2) < 0, \quad \text{when } 0 < \varepsilon << 1 \end{aligned} \tag{6}$$

Thereby, Ω is negative definite.

In conclusion, we can choose $0 < \varepsilon << 1$, such that W is positive definite and Ω is negative definite.

Consequently, we have

$$\left.\frac{\mathrm{d}V}{\mathrm{d}t}\right|_{(4)} \leqslant \lambda_{\max}(\Omega)\|e\|^2 \leqslant \frac{\lambda_{\max}(\Omega)}{\lambda_{\max}(W)}V \tag{7}$$

$$\lambda_{\min}(W)\|e\|^2 \leqslant V(e(t)) \leqslant V(e(0))\exp\left(\frac{\lambda_{\max}(\Omega)}{\lambda_{\max}(W)}t\right)$$

Further more, we have estimation formula

$$\|e\|^2 \leqslant \frac{V(e(t))}{\lambda_{\min}(W)} \leqslant \frac{V(e(0))}{\lambda_{\min}(W)}\exp\left(\frac{\lambda_{\max}(\Omega)}{\lambda_{\max}(W)}t\right) \tag{8}$$

therefore, the result is true. This completes the proof of Theorem 1. □

Remark 1. Theorem 1 improves the corresponding results in [20,21], and generalizes synchronization to global exponential synchronization.

We give another new result of global exponential synchronization in succession. The conditions are different from those in [7,20,21].

Consider the following errors dynamical system

$$\begin{cases} \dot{e}_x = pe_y - pe_x - p[f(x_d) - f(x_r)] - \delta_x e_x \\ \dot{e}_y = e_x - e_y + e_z - \delta_y e_y \\ \dot{e}_z = -qe_y \end{cases} \tag{9}$$

Theorem 2. If $\delta_x^2 + \delta_y^2 + \delta_z^2 = 0$ in system (2), and δ_x, δ_y in system (3) satisfy

$$-pq < (pa+\delta_x)q < 0, \quad \delta_y > \frac{-pq(pa+\delta_x)}{pa+\delta_x+p}$$

then the zero solution of system (9) is globally exponentially stable. Thereby, systems (2) and (3) are globally exponentially synchronized.

Proof. We still use the Lyapunov function (5). Computing the time derivative of $V(e)$ along the solution of system (9), we have

$$\left.\frac{\mathrm{d}V}{\mathrm{d}t}\right|_{(9)} = \begin{pmatrix} e_x \\ e_y \\ e_z \end{pmatrix} \begin{pmatrix} -2(pa+\delta_x)q - 2pq & 2pq & -\frac{\varepsilon}{2} \\ 2pq & -2pq+\varepsilon q - 2\delta_y & \frac{\varepsilon}{2} \\ -\frac{\varepsilon}{2} & \frac{\varepsilon}{2} & -\varepsilon \end{pmatrix} \begin{pmatrix} e_x \\ e_y \\ e_z \end{pmatrix} := e^{\mathrm{T}}\tilde{\Omega}e$$

because

$$\hat{\Delta}_1 := -2(pa+\delta_x)q - 2pq < 0$$
$$\hat{\Delta}_2 := [-2(pa+\delta_x)q - 2pq][-2(pq+\delta_y)+\varepsilon q] - 4p^2q^2 =$$
$$4pq^2(pa+\delta_x) + 4q\delta_y(pa+\delta_x+p) + o(\varepsilon) > 0, \quad \text{when } \delta_y > \frac{-pq(pa+\delta_x)}{pa+\delta_x+p}, 0<\varepsilon<<1$$
$$\hat{\Delta}_3 := \det\tilde{\Omega} = -4\varepsilon[pq^2(pa+\delta_x) + \delta_y(pa+\delta_x+p)q] + o(\varepsilon^2) < 0,$$
$$\text{when } \delta_y > \frac{-pq(pa+\delta_x)}{pa+\delta_x+p}, 0<\varepsilon<<1$$

$\tilde{\Omega}$ is negative definite. Accordingly, we have

$$\left.\frac{\mathrm{d}V}{\mathrm{d}t}\right|_{(9)} \leqslant \lambda_{\max}(\tilde{\Omega})\|e\|^2 \leqslant \frac{\lambda_{\max}(\tilde{\Omega})}{\lambda_{\max}(W)}V$$
$$\lambda_{\min}(W)\|e\|^2 \leqslant V(e(t)) \leqslant V(e(0))\exp\left(\frac{\lambda_{\max}(\tilde{\Omega})}{\lambda_{\max}(W)}t\right)$$

Further more, we have estimation formula

$$\|e\|^2 \leqslant \frac{V(e(t))}{\lambda_{\min}(W)} \leqslant \frac{V(e(0))}{\lambda_{\min}(W)}\exp\left(\frac{\lambda_{\max}(\tilde{\Omega})}{\lambda_{\max}(W)}t\right) \tag{10}$$

therefore, the result of Theorem 2 is true. This completes the proof of Theorem 2. □

Remark 2. Theorem 2 is a new result which is different from the results in [10,11,16,20,21].

4 Result of global synchronization

Definition 2. If the trivial solution of system (4) is globally asymptotically stable, then systems (2) and (3) are said to be globally synchronized.

To consummate the result that could not be strictly proved in [16], we use the following equality. According to the property of $f(x)$ and median value theorem, we have the following evident equalities.

$$f(x_d) - f(x_r) := \begin{cases} a(x_d - x_r), & \text{when } |x_d| \leqslant E \text{ and } |x_r| \leqslant E, \text{ or } x_d \leqslant -E \text{ and } x_d \leqslant -E \\ b(x_d - x_r), & \text{when } x_d \geqslant E \text{ and } x_r \geqslant E \\ k(x_d, x_r)(x_d - x_r), & x_d, x_r \text{ are not in the same interval of } (-\infty,-E),(-E,E) \text{ or } (E,+\infty) \end{cases}$$

where $k(x_d, x_r) \in (a, b)$ is a constant depending on x_d, x_r.

Theorem 3. If $pa+\delta_x \geqslant 0$ ($\delta_y = \delta_z = 0$), then the trivial solution of system (4) is at least globally asymptotically stable, thereby chaotic circuits (2) and (3) are at least globally synchronized.

Proof. First step. Suppose $pa+\delta_x > 0$, and construct a positive definite and radially unbounded Lyapunov function

$$V := \frac{1}{2}[qe_x^2 + pqe_y^2 + pe_z^2] \tag{11}$$

Taking the time derivative of V along the solution of system (4), we get

$$\left.\frac{\mathrm{d}V}{\mathrm{d}t}\right|_{(4)} = qe_x\dot{e}_x + pqe_y\dot{e}_y + pe_z\dot{e}_z \leqslant$$
$$qe_x[-p - k(x_d,x_r)p - \delta_x]e_x + pqe_ye_x + pqe_ye_x + pqe_y(e_x - e_y + e_z) - pqe_ye_z =$$
$$-[k(x_d,x_r)p + \delta_x]qe_x^2 - pq(e_x-e_y)^2 \leqslant$$
$$-(pa+\delta_x)qe_x^2 - pq(e_x-e_y)^2 \tag{12}$$

From (12), it is obvious that $\left.\frac{\mathrm{d}V}{\mathrm{d}t}\right|_{(4)}$ is negative definite about e_x, e_y, but is negative semi-definite about e_x, e_y, e_z. Therefore, we know $e_x \to 0, e_y \to 0$ by stability theory of partial components[24], but can not ensure $e_z \to 0$. It is evident that the trivial solution of system (4) is stable for all variables e_x, e_y, e_z

according to (12). However, we can easily prove $e_z \to 0$ $(t \to \infty)$ by LaSalle invariant principle and (12).

Second step. Assume $pa + \delta_x = 0$. When at least one of x_d, and x_r does not belong to interval $[-E, E]$, similar to the method of the first step, we still choose Lyapunov function (11), and have

$$V = \frac{1}{2}[qe_x^2 + pqe_y^2 + pe_z^2]$$
$$\frac{\mathrm{d}}{\mathrm{d}t}\Big|_{(4)} \leqslant -(pk(x_d, x_r) + \delta_x)qe_x^2 - pq(e_x - e_y)^2 < 0, \quad \text{when } e_x^2 + e_y^2 \neq 0$$

This implies $e_x(t) \to 0$, $e_y(t) \to 0$ $(t - \infty)$, under the hypothesis that at least one of x_d and x_r does not belong to interval $[-E, E]$.

Now, we prove $e_z(t) \to 0$ $(t \to \infty)$, under the hypothesis that at least one of x_d and x_r does not belong to interval $[-E, E]$.

We take $-p[f(x_d) - f(x_r)] - \delta_x e_x$ as a nonlinear perturbation of the first equation in error dynamical system (4).

Consider the linear part of system (4)

$$\begin{cases} \dot{e}_x = pe_y - pe_x \\ \dot{e}_y = e_x - e_y + e_z \\ \dot{e}_z = -qe_y \end{cases} \tag{13}$$

The system matrix of linear system (13) is

$$B := \begin{pmatrix} -p & p & 0 \\ 1 & -1 & 1 \\ 0 & -q & 0 \end{pmatrix}$$

and its characteristic equation is

$$\det(\lambda E - B) = \lambda^3 + (p+1)\lambda^2 + q\lambda + pq = 0 \tag{14}$$

According to the Hurwitz criterion in [25], the sufficient and necessary condition that the zeros of polynomial (14) have only negative real parts is

$$q > 0, \quad 0 < pq < (p+1)q = pq + q \tag{15}$$

Since $p > 0, q > 0$, (15) is obviously satisfied, thereby, matrix B is a Hurwitz matrix.

So, there exist constants $M \geqslant 1$ and $\alpha > 0$, and we have estimation formula

$$\|e^{B(t-\tau)}\| \leqslant Me^{-\alpha(t-\tau)}, \quad t \geqslant \tau$$

Applying the method of variation of parameter with initial conditions $(e_x(0), e_y(0), e_z(0) \in R^3)$ the common solution of (4) can be expressed as

$$\begin{pmatrix} e_x(t) \\ e_y(t) \\ e_z(t) \end{pmatrix} = e^{Bt}\begin{pmatrix} e_x(0) \\ e_y(0) \\ e_z(0) \end{pmatrix} + \int_0^t e^{B(t-\tau)}\begin{pmatrix} -pke_x(\tau) - \delta_x e_x(\tau) \\ 0 \\ 0 \end{pmatrix}\mathrm{d}\tau \tag{16}$$

Hence, we have

$$\left\|\begin{matrix} e_x(t) \\ e_y(t) \\ e_z(t) \end{matrix}\right\| \leqslant Me^{-\alpha t}\left\|\begin{matrix} e_x(0) \\ e_y(0) \\ e_z(0) \end{matrix}\right\| + M\int_0^t e^{-\alpha(t-\tau)}|pa + \delta_x| \cdot |e_x(\tau)|\mathrm{d}\tau =$$
$$Me^{-\alpha t}\left\|\begin{matrix} e_x(0) \\ e_y(0) \\ e_z(0) \end{matrix}\right\| + Me^{-\alpha t}\int_0^t e^{\alpha\tau}|pa + \delta_x| \cdot |e_x(\tau)|\mathrm{d}\tau \tag{17}$$

The first term of (17) tends to be zero when t tends to infinity, *i.e.*, $Me^{-\alpha t}\left\|\begin{matrix} e_x(0) \\ e_y(0) \\ e_z(0) \end{matrix}\right\| \to 0(t \to +\infty)$.

By means of L′Hospital rule and $e_x(t) \to 0$ $(t \to \infty)$, we have

$$\lim_{t\to\infty} \frac{M\int_0^t e^{\alpha\tau}\|pa+\delta_x\|\cdot\|e_x(\tau)\|\mathrm{d}\tau}{e^{\alpha\tau}} = \lim_{t\to\infty}\frac{M\|pa+\delta_x\|\cdot|e_x(t)|}{\alpha} = 0$$

Therefore, $e_x(t), e_y(t), e_z(t) \to 0$ $(t \to \infty)$, hence the trivial solution of system (4) is globally asymptotically stable, i.e., systems (2) and (3) globally synchronize.

If $x_d, x_z \in [-E, E]$, then system (4) is reduced to linear system:

$$\begin{cases} \dot{e}_x = pe_y - pe_x - (pa+\delta_x)e_x = pe_y - pe_x \\ \dot{e}_y = e_x - e_y + e_z \\ \dot{e}_z = -qe_y \end{cases}$$

that is, system (13). We have proved the coefficient matrix of system (13) is Hurwitz matrix. Therefore, $e_x(t), e_y(t), e_z(t) \to 0$ $(t \to \infty)$. □

In conclusion, the trivial solution of system (4) is globally asymptotically stable, hence systems (2) and (3) globally synchronize. This completes the proof of the theorem.

Remark 3. Theorem3 gives a strict and perfect proof to the result in [16] where the proof was incomplete, and improve the results of [20,21].

5 Conclusion

For Chua′s chaotic circuit, we can always realize chaos synchronization *via* the simplest linear feedback control. In this paper, we have proposed the concept of global exponential synchronization, generalized the exist result, and given strict and perfect proof in theory. Because the conditions and the results are expressed with mathematic analytic formulae, we do not give numerical simulation example.

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Liao Xiao-Xin Professor in the Department of Control Science and Engineering at Huazhong University of Science and Technology, P. R. China. His research interests include neural network and nonlinear control.

Luo Hai-Geng, Zhang Gang, Jian Ji-Gui, Zong Xiao-Jun Ph. D. candidates of the Department of Control Science and Engineering, Huazhong University of Science and Technology, P. R. China.

Xu Bing-Ji Associate professor in the Department of Control Science and Engineering at Huazhong University of Science and Technology, P. R. China. His research interests include nonlinear systems and stability.