New Result on Model Reference Robust Control: Tracking Performance Improvement1)

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Abstract This paper considers the tracking performance problem of a model reference robust control (MRRC) for plants with relative degree greater than one. A new algorithm is proposed based on the earlier research. It is shown that by applying a special transformation to the tracking system, the $\mathcal{L}_\infty$ bound of the tracking error can be achieved even when the high frequency gain is unknown, and both the tracking performance and the control effort can be improved significantly. Furthermore, the strictly positive real (SPR) condition, which is an essential assumption of the earlier design, can be removed.

Key words Robust control, model following, non-linear control, backstepping

1 Introduction

Model reference robust control was introduced by [1] as a new means of I/O based controller design for linear time invariant (LTI) plants with nonlinear input disturbance and has been extended to MIMO, non-minimum phase plants and the case in the presence of unmodeled dynamics[2~5]. For plants with relative degree greater than one ($n^* > 1$), the method can be considered as an extension of the backstepping design[6], where the final control signal is obtained through a series of fictitious control signals designed recursively. To overcome the influence of the nonlinear input disturbance, the update law is abandoned. Instead, the concept of bounding function is introduced in the control law design with the assumption that the parameters of the plant belong to a known compact set.

It is worth mentioning that in the area of model following with respect to SISO LTI plants, adaptive backstepping design algorithm has drawn a lot of attention recently due to its strong transient performance properties available in both $\mathcal{L}_2$ and $\mathcal{L}_\infty$[7~9]. The extension of the algorithm to plants with unmodeled dynamics and external disturbance shows that the tracking error is proportional to the size of the perturbations[10]. However, since in some missile control and guidance problems, a higher tracking precision is needed even with the existence of an external disturbance, this, of course, restricts the application of the algorithm.

We note that theoretically, a higher tracking precision can be achieved by using the MRRC. From a practical point of view, however, the tracking performance of the MRRC may be unacceptable. In fact, as the authors themselves pointed out, the control signal may change rapidly and have large amplitude by using their scheme if a higher tracking precision is needed. This situation is illustrated in the simulation results shown in the following figures, where we chose the same plant as that in example 2 of [1] and, to simplify the analysis, the model was chosen as

$$M(s) = 2/(s^2 + 6s + 5) = 2/(s + 1)(s + 5) \tag{1}$$

Let $L(s) = s + 5$; hence $M(s)L(s) = 2/(s + 1)$ is obviously an SPR function. In the simulation, the Matlab/Simulink toolbox was used. The input signal of the system, say, r was a square wave of amplitude 1 with frequency 1rad/sec. We chose the design parameters $\epsilon_1 = \epsilon_2 = 5$ and $\epsilon_1 = \epsilon_2 = 0.5$, respectively, where ϵ_1 and ϵ_2 were used in [1] to control the tracking precision, *i.e.*, the smaller the values of ϵ_1 and ϵ_2, the higher the tracking precision. Fig. 1 shows that the tracking error is, in general, large, while from Figures 3 and 4, it is clear that the amplitude of the control signal is high due to the smaller ϵ_1 and ϵ_2 though the tracking precision is better. It also should be pointed out that the simulation

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of example 2 of [1] was incorrect since it can be checked that $M(s)L(s)$ is not an SPR function when $L(s) = s + 10$. Therefore, it seems that the example would not be suitable to show the effectiveness of the algorithm.

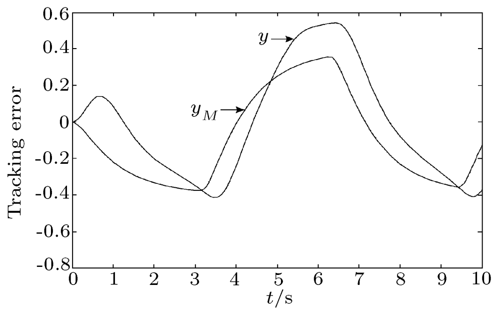

Fig. 1 Tracking error

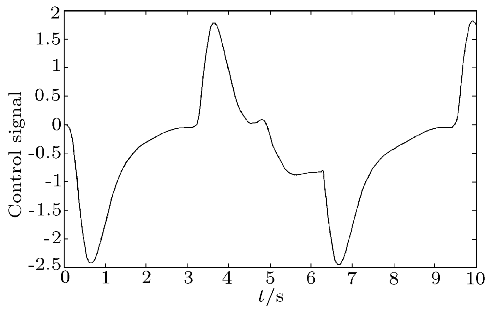

Fig. 2 Control signal

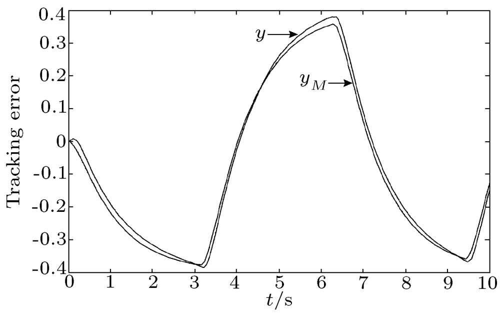

Fig. 3 Tracking error

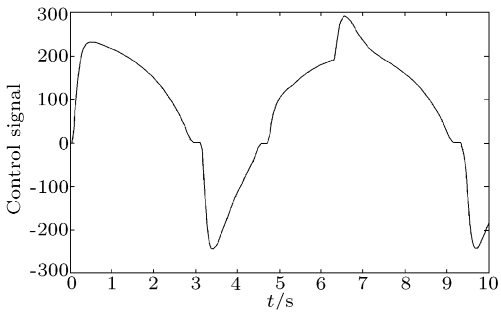

Fig. 4 Control signal

Given the motivation above, the objective of this paper is to improve the tracking performance of the MRRC for the case of $n^* > 1$ while maintaining a reasonable control effort. We will first introduce a special transformation to the tracking system and then a new Lyapunov function as well as a modified design algorithm will be given. We will show that by using the new algorithm, both the tracking performance and the control effort can be improved significantly. Furthermore, the SPR condition, which is an essential assumption of the earlier design, can be removed.

2 System and assumptions

The system to be controlled is

$$y = G_p(s)[u + d] = k_p(n_p(s)/d_p(s))[u + d] \tag{2}$$

where y and u are the system output and input, respectively, $G_p(s)$ is the plant transfer function with $d_p(s)$ and $n_p(s)$ being polynomials of degrees n and m, respectively, and d denotes a disturbance or uncertainty. In this paper, the reference model is given by

$$y_M = M(s)[r] = k_M \frac{1}{d_M(s)}[r],\ k_M > 0 \tag{3}$$

where $d_M(s)$ is a monic Hurwitz polynomial with $\deg(d_M(s)) = n - m := n^*$ and r is any piecewise continuous and uniformly bounded reference signal.

For the controlled system, the following assumptions are made:

(A_1) The parameters of G_p are unknown but belong to a known compact set; the order n, the relative degree n^*, and the sign of k_p are known and constant, and G_p is of minimum phase. Without loss of generality, it is assumed that $k_p > 0$. (A_2) The lumped disturbance and uncertainty term $d(y,t)$ is bounded by a known continuous function $\rho(y,t)$ such that for all $(y,t) \in R \times R^+$, $|d(y,t)| \leqslant \rho(y,t)$, $\forall t \geqslant 0$, where the bounding function $\rho(y,t)$ is assumed to be continuous, uniformly bounded with respect to t, and locally uniformly bounded with respect to the system output y. The uncertainty $d(y,t)$ is not necessarily continuous but, if it may be discontinuous, the existence of the solution of y is assumed.

Remark 1. Assumption (A_2) admits nonlinear functions and unmodeled dynamics associated with the system output y.

Define the control signal

$$u = \hat{\boldsymbol{\theta}}^{\mathrm{T}}\boldsymbol{\omega} + u_R \tag{4}$$

where u_R is the nonlinear control to be designed and the constant vector $\hat{\boldsymbol{\theta}} \in R^{2n}$ and the vector $\boldsymbol{\omega} \in R^{2n}$ will be defined below. Using the conventional technique of model reference adaptive control (MRAC), the tracking error $e := y - y_M$ can be expressed as

$$e = M(s)\kappa^*[\tilde{\boldsymbol{\theta}}^{\mathrm{T}}\boldsymbol{\omega} + d_f + u_R] + \varepsilon \tag{5}$$

where ε denotes a bounded, differentiable and exponentially decaying real function that represents non-zero initial conditions for all internal states of the tracking system,

$$\begin{aligned}
&\tilde{\boldsymbol{\theta}} := \hat{\boldsymbol{\theta}} - \boldsymbol{\theta}^*,\ \boldsymbol{\omega} := [\boldsymbol{\nu}_1^{\mathrm{T}}, y, \boldsymbol{\nu}_2^{\mathrm{T}}, r]^{\mathrm{T}}\\
&\hat{\boldsymbol{\theta}} := [\hat{\boldsymbol{\theta}}_1^{\mathrm{T}}, \hat{\theta}_0, \hat{\boldsymbol{\theta}}_2^{\mathrm{T}}, \hat{k}]^{\mathrm{T}},\ \boldsymbol{\theta}^* := [(\boldsymbol{\theta}_1^*)^{\mathrm{T}}, \theta_0^*, (\boldsymbol{\theta}_2^*)^{\mathrm{T}}, k^*]^{\mathrm{T}}\\
&\kappa^* := k_p/k_M = 1k^*,\ d_f := (1 - d_1(s))[d],\ d_1(s) := \hat{\boldsymbol{\theta}}_1^{\mathrm{T}}\mathrm{adj}(sI - \mathit{\Lambda})\boldsymbol{b}
\end{aligned} \tag{6}$$

where $\boldsymbol{\theta}^*$ is the matching vector for which the perfect tracking can be achieved[11], $\hat{\boldsymbol{\theta}}$ in the MRRC is an estimate of $\boldsymbol{\theta}^*$, and the input/output filters are defined, respectively, as

$$\dot{\boldsymbol{\nu}}_1 = \mathit{\Lambda}\boldsymbol{\nu}_1 + \boldsymbol{b}u,\ \boldsymbol{\nu}_1(0) = 0,\ \dot{\boldsymbol{\nu}}_2 = \mathit{\Lambda}\boldsymbol{\nu}_2 + \boldsymbol{b}y,\ \boldsymbol{\nu}_2(0) = 0;\ \mathit{\Lambda} \in R^{(n-1)\times(n-1)},\ \boldsymbol{b} \in R^{n-1} \tag{7}$$

where $\mathit{\Lambda}$ is a matrix such that $\deg(sI - \mathit{\Lambda})$ is a Hurwitz polynomial and $(\mathit{\Lambda}, \boldsymbol{b})$ is a controllable pair.

Write (5) in the following form for the case of $n^* > 1$:

$$e = M(s)L(s)\kappa^*[\tilde{\boldsymbol{\theta}}^{\mathrm{T}}\bar{\boldsymbol{\omega}} + d_L + L^{-1}(s)[u_R]] \tag{8}$$

where

$$L(s) := s^{n^*-1} + \alpha_1 s^{n^*-2} + \cdots + \alpha_{n^*-1} \tag{9}$$

and is assumed to be a Hurwitz polynomial such that $M(s)L(s)$ has relative degree one and, $\bar{\boldsymbol{\omega}}$ and d_L are defined, respectively, as

$$\bar{\boldsymbol{\omega}} := L^{-1}(s)[\boldsymbol{\omega}],\ d_L := L^{-1}(s)[d_f] \tag{10}$$

Hereafter, similar to [1], we do not consider ε since it does not affect the stability of the MRRC system and can only make the control law design a little bit complicated.

3 Improvement of the tracking performance

In the following, we show that by transforming (8) into a first-order differential equation, we can modify the control law of the previous research, so that the transient performance and the control effort can be improved significantly. To proceed, we introduce the following lemma.

Lemma 1.[12] Suppose $\eta = G(s)[v] = k_g(\beta(s)/\alpha(s))[v](k_g \neq 0)$ is a strictly proper system with $\alpha(s)$ and $\beta(s)$ being coprime and monic polynomials. If the relative degree of $G(s)$ is n^*, then there must exist a monic polynomial $\xi(s)$ with $\deg(\xi(s)) = n^*$ and a polynomial $\varphi(s)$ with $\deg(\varphi(s)) < \deg(\beta(s))$, such that

$$\alpha(s) = \xi(s)\beta(s) + \varphi(s) \tag{11}$$

$$\eta = k_g(1/\xi(s))[v] - (\varphi(s)/\xi(s)\beta(s))[\eta] \tag{12}$$

Proof. See [12].

By applying Lemma 1 to (8), it follows that

$$\dot{e} = (1/(s+\lambda))k_p[\tilde{\boldsymbol{\theta}}^{\mathrm{T}}\bar{\boldsymbol{\omega}} + d_L - \frac{\boldsymbol{\varphi}(s)}{k_p L(s)}[e] + L^{-1}(s)[u_R]] \tag{13}$$

where $\xi(s) = s+\lambda$ since $M(s)L(s)$ is of relative degree one, and the polynomial $\varphi(s)$ satisfies $\det(\varphi(s)) < \deg(L(s))$ and can be obtained *a priori*. Hence,

$$\dot{e} = -\lambda e + k_p(\tilde{\boldsymbol{\theta}}^{\mathrm{T}}\bar{\boldsymbol{\omega}} + d_L - \frac{\varphi(s)}{k_pL(s)}[e] + L^{-1}(s)[u_R]) \tag{14}$$

We are interested in the sign of λ in the above equation. In fact, we have the following lemma.

Lemma 2. Let $L(s)$ be given by (9) and d_M in (3) be defined as

$$d_M(s) = s^{n^*} + \beta_1 s^{n^*-1} + \cdots + \beta_{n^*} \tag{15}$$

Then

$$\lambda > 0,\ \text{if}\ \beta_1 > \alpha_1;\quad \lambda \leqslant 0,\ \text{if}\ \beta_1 \leqslant \alpha_1 \tag{16}$$

where α_1 can be found in (9). In particular, if $M(s)L(s)$ is an SPR function, then $\lambda > 0$.

Proof. From (9), (15) and taking (11) into consideration, $d_M(s)$ can be written as $d_M(s) = (s+\lambda)L(s) + \varphi(s)$. It is easy to check that $\lambda = \beta_1 - \alpha_1$ and therefore, (16) follows. Furthermore, it can be shown, after some straightforward calculations, that

$$\lim_{w\to\infty} w^2\mathrm{Re}[M(jw)L(jw)] = \beta_1 - \alpha_1 = \lambda \tag{17}$$

Hence, if $M(s)L(s)$ is an SPR function, from [13, Theorem 2.1], we have $\lambda > 0$. □

To design the control signal, let $z_1 := L^{-1}(s)[u_R]$. The controllable canonical form of z_1 is

$$\begin{aligned}&\dot{z}_1 = z_2,\ \dot{z}_i = z_{i+1},\ i = 2,\cdots,n^*-2\\ &\dot{z}_{n^*-1} = -\alpha_1 z_{n^*-1} - \alpha_2 z_{n^*-2} - \cdots - \alpha_{n^*-1}z_1 + u_R\end{aligned} \tag{18}$$

Further, we consider the following Lyapunov function

$$V = \frac{1}{2}e^2 + \frac{1}{2}k_p\sum_{i=1}^{n^*-1}(z_i - v_i)^2 \tag{19}$$

where v_i' are the control signals to be designed in a recursive manner. We introduce the modified version of Theorem 2 of [1] in the following theorem.

Theorem 1. Let the MRRC system given by (5) satisfy assumptions (A_1), (A_2). Let the signals $v_1,\cdots,v_{n^*-1}$, of (19) and the control signal $u_R = v_{n^*}$ be defined as follows.

$$\begin{aligned}
&v_1 := -\sigma e - \frac{\mu_1|\mu_1|^{\tau_1}}{|\mu|^{\tau_1+1} + \epsilon_1^{\tau_1+1}}g_1\\
&v_2 = u_R := -\gamma(z_1 - v_1) - e + \alpha_1 z_1 - \frac{\mu_2|\mu_2|^{\tau_2}}{|\mu_2|^{\tau_2+1} + \epsilon_2^{\tau_2+1}}g_2,\ \text{if}\ n^* = 2\\
&v_2 := -\gamma(z_1 - v_1) - e - \frac{\mu_2|\mu_2|^{\tau_2}}{|\mu_2|^{\tau_2+1} + \epsilon_2^{\tau_2+1}}g_2\\
&v_i := -\gamma(z_{i-1} - v_{i-1}) - (z_{i-2} - v_{i-2}) - \frac{\mu_i|\mu_i|^{\tau_i}}{|\mu_i|^{\tau_i+1} + \epsilon_i^{\tau_i+1}}g_i,\ i = 3,\cdots,n^*-1\\
&u_R = v_{n^*} = -\gamma(z_{n^*-1} - v_{n^*-1}) - (z_{n^*-2} - v_{n^*-2}) + (\alpha_1 z_{n^*-1} + \cdots + \alpha_{n^*-1}z_1) -\\
&\qquad \frac{\mu_{n^*}|\mu_{n^*}|^{\tau_{n^*}}}{|\mu_{n^*}|^{\tau_{n^*}+1} + \epsilon_{n^*}^{\tau_{n^*}+1}}g_{n^*}
\end{aligned} \tag{20}$$

where $\tau_1 \geqslant 0, \epsilon_1 > 0, \tau_j \geqslant 0, \epsilon_j > 0, \sigma > 0$ and γ are design parameters,

$$\gamma \geqslant \lambda + \underline{k}_p\sigma := \gamma_1 \tag{21}$$

with $\underline{k}_p$ is a lower bound of k_p, and the bounding functions g_1, g_j and the functions μ_1, μ_j are designed, respectively, as follows:

$$g_1 = \mathrm{BND}(|\tilde{\boldsymbol{\theta}}^{\mathrm{T}}\bar{\boldsymbol{\omega}} + d_L - \frac{\varphi(s)}{k_pL(s)}[e]|),\ \mu_1 = eg_1$$

$$g_i = \mathrm{BND}(|\dot{v}_{j-1}|),\ \mu_j = (z_{j-1} - v_{j-1})g_j,\ j = 2, \cdots, n^* \tag{22}$$

If the robust control is chosen as

$$u = \hat{\boldsymbol{\theta}}^{\mathrm{T}} \boldsymbol{\omega} + u_{_R},\ u_{_R} = v_{n^*} \tag{23}$$

then, all the signals of the closed loop system are uniformly bounded and the following $\mathcal{L}_\infty$ bound of the tracking error is guaranteed:

$$|e| \leqslant \sqrt{2\exp(-2\gamma_1 t)V(0) + k_p\epsilon/\gamma_1},\ \forall t \geqslant 0 \tag{24}$$

where $\epsilon := \sum_{i=1}^{n^*} \epsilon_i$.

Remark 2. The bounding function of any signal f, say, $\mathrm{BND}(|f|)$ is a known, continuous, nonnegative function that bounds the magnitude (or Euclidean norm) of f. Readers may refer to [1] for detail about the definition.

The bounding function $\mathrm{BND}(|\dot{v}_{j-1}|)$ in the recursive design is well-defined by using (14), the triangle inequality, the fact that the bounding functions, including $\rho(y,t)$ and $\mathrm{BND}(d_L(s))$, are not unique and can be chosen to be differentiable, and the relations listed below.

$$\begin{aligned} &\dot{z}_i = s^i L^{-1}(s)[u_{_R}],\ \bar{r}^{(i)} = s^i L^{-1}(s)[r],\ \bar{y}^{(i)} = s^i L^{-1}(s)[y] \\ &\dot{\bar{\boldsymbol{\nu}}}_1 = \Lambda\bar{\boldsymbol{\nu}}_1 + \boldsymbol{b}[\hat{\boldsymbol{\theta}}^{\mathrm{T}}\bar{\boldsymbol{\omega}} + z_1],\ \dot{\bar{\boldsymbol{\nu}}}_2 = \Lambda\bar{\boldsymbol{\nu}}_2 + \boldsymbol{b}[\bar{y}] \end{aligned} \tag{25}$$

Remark 3. From the assumption (A_1), both $\underline{k}_p$ and an upper bound of $\|\tilde{\boldsymbol{\theta}}\|$ can be obtained *a priori*.

Remark 4. The signals $v_1, \cdots, v_{n^*-1}$ are so-called fictitious control signals.

Remark 5. If ϵ_i's are chosen to be time-varying and differentiable, the above results still hold.

In comparison to Theorem 2 of [1], the main feature of the modification is that by using Lemma 1, we can transform (8) into a first-order differential equation as shown in (14), which paves the way for the introduction of the new Lyapunov function and the design parameters σ and γ. Since σ, γ as well as γ_1 are at designer′s disposal, the theorem states that the tracking error converges to a residual set that is a decreasing function of γ_1 and therefore, the tracking performance improvement can be achieved with larger σ, γ as well as γ_1. This is the main difference between our modification and the earlier research. On the other hand, we note that (24) relaxes the limitation of k_p in adaptive backstepping control[8], where the $\mathcal{L}_\infty$ bound is only available when k_p is known exactly.

Another interesting feature of the modification is that the SPR assumption of $M(s)L(s)$ can be removed. In fact, if $M(s)L(s)$ is not an SPR function, the method of [1] cannot be applied. Note that by Lemma 2, a non-SPR function implies that we may have $\lambda \leqslant 0$. However, by using the modification, we can choose σ such that γ_1 in (21) is greater than zero even with $\lambda \leqslant 0$. The proof of the theorem given below will show that $\gamma_1 > 0$ is essential to guarantee the stability of the MRRC system.

Proof of Theorem 1. The time derivative of the Lyapunov function yields

$$\begin{aligned} \dot{V} =& -\lambda e^2 + k_p[(\tilde{\boldsymbol{\theta}}^{\mathrm{T}}\bar{\boldsymbol{\omega}} + d_L - \frac{\varphi(s)}{k_p L(s)}[e])e + z_1 e] + k_p\sum_{i=1}^{n^*-1}(z_i - v_i)(\dot{z}_i - \dot{v}_i) = \\ & -\lambda e^2 + k_p e[(\tilde{\boldsymbol{\theta}}^{\mathrm{T}}\bar{\boldsymbol{\omega}} + d_L - \frac{\varphi(s)}{k_p L(s)}[e]) + v_1] + k_p(z_1 - v_1)(e + v_2 - \dot{v}_1) + \\ & k_p\sum_{i=2}^{n^*-2}(z_i - v_i)[(z_{i-1} - v_{i-1}) + v_{i+1} - \dot{v}_i] + k_p(z_{n^*-1} - v_{n^*-1})[(z_{n^*-2} - v_{n^*-2}) + \\ & \underbrace{(-\alpha_1 z_{n^*-1} - \cdots - \alpha_{n^*-1}z_1) + u_{_R}}_{\dot{z}_{n^*-1}} - \dot{v}_{n^*-1}] \end{aligned} \tag{26}$$

where (14) and the following relation have been used

$$(z_i - v_i)(\dot{z}_i - \dot{v}_i) = (z_i - v_i)(v_{i+1} - \dot{v}_i) + (z_i - v_i)(z_{i+1} - v_{i+1}) \tag{27}$$

Also, note that from (18), $\dot{z}_i = z_{i+1}$. Replacing (20) in (26) it follows that

$$\dot{V} \leqslant -\gamma_1 e^2 - \gamma k_p \sum_{i=1}^{n^*-1} (z_i - v_i)^2 + k_p \sum_{i=1}^{n^*} \epsilon_i \leqslant$$
$$-\gamma_1 e^2 - \gamma_1 k_p \sum_{i=1}^{n^*-1} (z_i - v_i)^2 + k_p \sum_{i=1}^{n^*} \epsilon_i = -2\gamma_1 V + k_p \epsilon,\ t \geqslant 0 \tag{28}$$

where we have used (21) and the inequalities

$$-\lambda e^2 + k_p e[(\tilde{\boldsymbol{\theta}}^{\mathrm{T}} \bar{\boldsymbol{\omega}} + d_L - \frac{\varphi(s)}{k - pL(s)}[e]) + v_1] = -(\lambda + k_p \sigma) e^2 + k_p[(\tilde{\boldsymbol{\theta}}^{\mathrm{T}} \bar{\boldsymbol{\omega}} + d_L - \frac{\varphi(s)}{k_p L(s)}[e])e -$$
$$\frac{|\mu_1|^{\tau_1+2}}{|\mu_1|^{\tau_1+1} + \epsilon_1^{\tau_1+1}}] \leqslant -(\lambda + k_p\sigma)e^2 + k_p(|\mu_1| - \frac{|\mu_1|^{\tau_1+2}}{|\mu_1|^{\tau_1+1} + \epsilon_1^{\tau_1+1}}) \leqslant -\gamma_1 e^2 + k_p \epsilon_1 \tag{29}$$

and

$$k_p(z_{i-1} - v_{i-1})(-\frac{\mu_i|\mu_i|^{\tau_i}}{|\mu_i|^{\tau_i+1} + \epsilon_i^{\tau_i+1}} g_i - \dot{v}_{i-1}) = k_p[-(z_{i-1} - v_{i-1})\dot{v}_{i-1} - \frac{\mu_i|\mu_i|^{\tau_i}}{|\mu_i|^{\tau_i+1} + \epsilon_i^{\tau_i+1}} \mu_i] \leqslant$$
$$k_p(|\mu_i| - \frac{|\mu_i|^{\tau_i+2}}{|\mu_i|^{\tau_i+1} + \epsilon_i^{\tau-i+1}}) \leqslant k_p \epsilon_i \tag{30}$$

respectively, where the proof of the inequalities

$$k_p(|\mu_j| - \frac{|\mu_j|^{\tau_j+2}}{|\mu_j|^{\tau_j+1} + \epsilon_j^{\tau_j+1}}) \leqslant k_p \epsilon_j,\ j = 1, 2, \cdots, n^* \tag{31}$$

in both (29) and (30) can be found in [1, p.2226].

The inequality (28) implies that

$$V \leqslant \exp(-2\gamma_1 t) V(0) + k_p \epsilon / 2\gamma_1,\ \forall t \geqslant 0 \tag{32}$$

which, in view of (19), gives $e^2/2 \leqslant \exp(-2\gamma_1 t)V(0) + k_p\epsilon/2\gamma_1, \forall t \geqslant 0$, and therefore, (24) follows. That is, a larger γ_1 can improve the transient and steady-state performance of the tracking error.

To prove all the signals of the close-loop system are uniformly bounded, we note that (32) and (19) imply that the tracking error e and $(z_i - v_i)$ belong to $\mathcal{L}_\infty$. The fact that $\bar{\boldsymbol{\omega}} \in \mathcal{L}_\infty$ is thus proved by noting that $y = e + y_M \in \mathcal{L}_\infty$ and therefore, $\bar{\boldsymbol{\nu}}_2 = L^{-1}(s)(sI - \Lambda)^{-1}\boldsymbol{b}[y]$ and $\bar{\boldsymbol{\nu}}_1 = L^{-1}(s)(sI - \Lambda)^{-1}\boldsymbol{b}[u] = L^{-1}(s)(sI - \Lambda)^{-1}\boldsymbol{b}G_p^{-1}(s)[y] \in \mathcal{L}_\infty$, where $\bar{\boldsymbol{\omega}}$ is given by (10), and $L^{-1}(s)(sI - \Lambda)^{-1}\boldsymbol{b}G_p^{-1}(s)$ is proper and stable because $G_p(s)$ is of minimum phase. Hence, by the same manner as in [1], it can be proved that $v_i, z_i, u_R \in \mathcal{L}_\infty$. Finally, we need to prove that $\boldsymbol{\omega} \in \mathcal{L}_\infty$. Since the reference signal r and the output y belong to $\mathcal{L}_\infty$, from (6), we only need to show that $\boldsymbol{\nu}_1 \in \mathcal{L}_\infty$. Taking into account (7) and (4), we have

$$|\dot{\boldsymbol{\nu}}_1| = |\Lambda \boldsymbol{\nu}_1 + \boldsymbol{b}u| < |\Lambda \boldsymbol{\nu}_1 + \boldsymbol{b}(\hat{\boldsymbol{\theta}}^{\mathrm{T}} \boldsymbol{\omega} + u_R)| \leqslant c + c\|(\boldsymbol{\nu}_1)_t\|_\infty + c\|\boldsymbol{\omega}\| \leqslant c + c\|(\boldsymbol{\nu}_1)_t\|_\infty \tag{33}$$

where the norm $\|x_t\|_\infty := \sup_{\tau \leqslant t} |x(\tau)|$, c generically denotes a positive constant and the fact that $\|\boldsymbol{\omega}\| \leqslant c + c\|(\boldsymbol{\nu}_1)_t\|_\infty$ is used. The inequality (33) implies that $\boldsymbol{\nu}_1$ is a regular signal [14, p.70]. Hence by using Corollary 3.6.3 of [14], $\bar{\boldsymbol{\omega}} \in \mathcal{L}_\infty$ implies that $\boldsymbol{\nu}_1 \in \mathcal{L}_\infty$. □

4 Simulation results

We chose the same plant, reference model and disturbance signal discussed in Section 1. The plant of relative degree two is

$$G_p(s) = 1/(s^2 + s + 1) \tag{34}$$

The reference model is given by (1). $L(s) = s + \alpha_1 = s + 5$. Since $M(s)L(s) = 2/(s+1)$, *i.e.*, there is a pole/zero cancellation, from (11) and (13) it is easy to check that $\varphi(s) = 0$ and $\lambda = 1$. The parameters of the input and output filters given by (7) are $\Lambda = -10$ and $b = 1$. The disturbance is

$d(y,t) = \cos t + 0.5\cos y + y^2 \sin t$, whose bounding function was chosen as $\rho(y,t) = 2 + y^2$. Note that the control law (20) holds only when $\varepsilon = 0$ as we mentioned at the end of the section 2. If $\varepsilon \neq 0$, similar to (5), we can rewrite (14) as follows.

$$\dot{e} = -\lambda e + k_p(\tilde{\boldsymbol{\theta}}^{\mathrm{T}}\bar{\boldsymbol{\omega}} + d_L - (\varphi(s)/(k_p L(s)))[e] + L^{-1}(s)[u_R]) + \bar{\varepsilon} \tag{35}$$

where $\bar{\varepsilon}$ has the same definition as ε and is not available for measurement also; hence, in view of (21) and (35), (22) has to be slightly modified as follows.

$$|\dot{v}_1| = \left|\frac{\partial v_1}{\partial e}\dot{e} + \frac{\partial v_1}{\partial g_1}\dot{g}_1\right| \leqslant g_2 + c\bar{\varepsilon}^2 \tag{36}$$

where the triangle inequality is used to separate $\bar{\varepsilon}$, c is any positive constant, and g_2 is the bounding function of $|\dot{v}_1|$ except $\bar{\varepsilon}$. Note that $c\bar{\varepsilon}^2$ decays exponentially and therefore does not affect the stability of the closed-loop system. To obtain $u_R = v_2$, let $\tau_1 = 1$, $\tau_2 = 0$, $\epsilon_1 = \epsilon_2 = 5$, $\sigma = 30$ and $\gamma = 31$. Then from (20) and taking into account (35) and (36), we can get v_1 and $u_R = v_2$, respectively, where we chose

$$g_1 = \mathrm{BND}(|\pm\tilde{\theta}^{\mathrm{T}}\bar{\boldsymbol{\omega}} + d_L|) = \|\tilde{\boldsymbol{\theta}}\|\sqrt{\bar{r}^2 + \bar{\boldsymbol{\nu}}_1^2 + \bar{\nu}_1^2 + \bar{y}^2 + 0.1} + (1/L(s))[\rho(y,t)] \tag{37}$$

We let $\hat{\boldsymbol{\theta}} = 0$, $\mathrm{BND}(\tilde{k}) = 4$ and $\mathrm{BND}(\tilde{\theta}_0) = \mathrm{BND}(\tilde{\boldsymbol{\theta}}_1) = \mathrm{BND}(\tilde{\boldsymbol{\theta}}_2) = 0.7$. Hence, $\|\tilde{\boldsymbol{\theta}}\| = \sqrt{17.47}$. We chose $k_p = 0.5$. Fig. 5 demonstrates the perfect transient performance when the initial condition is non-zero. In addition, to avoid peaking phenomenon of the control signal at the beginning of the simulation, the initialization technique of [7, p.751] was used and has improved the control effort significantly.

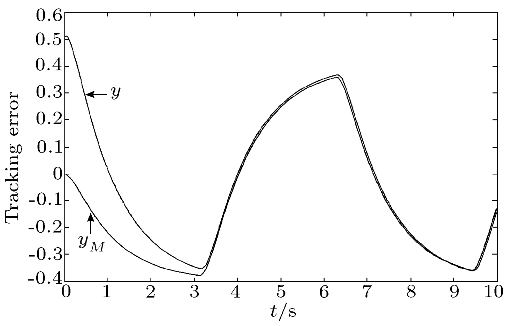

Fig. 5 Tracking error

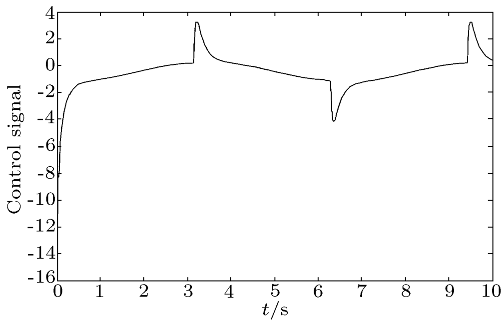

Fig. 6 Control signal

5 Conclusion

In this paper, a new model reference robust controller design algorithm has been proposed for plants with relative degree greater than one. The property of the algorithm is that both the transient performance and the control effort can be improved significantly. This is achieved by transforming the tracking system into a first-order system and by using a specially chosen Lyapunov function, which allows the introduction of some new design parameters. Furthermore, the strictly positive real (SPR) condition can be removed.

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