Switching Feedback Stabilization for Discrete Linear Multiple-input Systems[1)]

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Abstract New idea of stabilization for discrete linear multiple-input system is proposed based on switching technique and single-input control. The system discussed here denotes coupled single-input objects to be controlled. The central processing unit chooses an object at each discrete instant according to periodic switching strategy and controls it by local state feedback. Stabilization of a multiple-input system is turned into stabilization of single-input systems under periodic switching strategy, which is easy to be realized in practice. On the other hand, only one central processing unit can realize all local controllers, which decreases the cost and increases the usage of the resources.

Key words Multiple-input, switching, single-input, CPU

1 Introduction

Multiple-input systems exist in industrial installation, electrical networks, aerospace engineering, chemical processes, social economic system, biological systems, *etc.* Many methods for analyses and design of linear multiple-input systems have been put forward for time domain or frequency domain[1~3].

Multiple-input discrete linear system discussed here consists of m ($\geqslant$2) single-input discrete subsystems with coupling, each subsystem denotes a controlled object[4]. In order to stabilize the multiple-input system, we present a method by which the central processing unit (CPU) chooses only one subsystem at each discrete instant according to the periodic switching strategy (PSS) and controls it by local controller. PSS means the choosing of subsystems is periodic, and the period is m. LC means it is constructed only by the corresponding substate, not the whole state vector.

This idea is illuminated by the research of switching control[5~7] and possesses the following characters. Firstly, at each discrete instant it is single-input subsystem for which we design the local controller. Secondly, PSS has a simple structure and can be run conveniently. Thirdly, There is only one local controller running at each discrete instant. One CPU can realize all controllers due to this character, so we can save lots of hardware and make full use of various related resources.

2 Descriptions of System

Consider m ($\geqslant$2) discrete linear single-input subsystems

$$\boldsymbol{x}_i(t+1) = A_{ii}\boldsymbol{x}(t) + \boldsymbol{b}_{ii}u_i(t), \quad t \in N,\ i \in \underline{m} \tag{1}$$

where $N = \{0, 1, 2, \cdots\}, \underline{m} = \{1, 2, \cdots, m\}$. Such systems are denoted as $(A_{ii}, \boldsymbol{b}_{ii})$ for simplicity. All $(A_{ii}, \boldsymbol{b}_{ii})$ and their linear coupling constitute a multiple-input system

$$\boldsymbol{x}(t+1) = A\boldsymbol{x}(t) + B\boldsymbol{u}(t), \quad t \in N \tag{2}$$

where $A = (A_{ij})_{m\times m}, B = (\boldsymbol{b}_{ij})_{m\times m}, A_{ij} \in R^{n_i\times n_j}, \boldsymbol{b}_{ij} \in R^{n_i}, \boldsymbol{x}^{\mathrm{T}}(t) = (\boldsymbol{x}_1^{\mathrm{T}}(t), \boldsymbol{x}_2^{\mathrm{T}}(t), \ldots, \boldsymbol{x}_m^{\mathrm{T}}(t)), \boldsymbol{x}_i(t) \in R^{n_i}, \boldsymbol{u}^{\mathrm{T}}(t) = [u_1(t), u_2(t), \cdots u_m(t)], u_i \in R, i \in \underline{m}$. R denotes the space of real number, R^n denotes the space of real vectors with dimensions n, $R^{n\times m}$ denotes the space of real matrix with dimensions $n\times m$. System (2) can be described as

$$\boldsymbol{x}_i(t+1) = A_{ii}\boldsymbol{x}(t) + \boldsymbol{b}_{ii}u_i(t) + \sum_{j\neq 1}[A_{ij}\boldsymbol{x}_j(t) + \boldsymbol{b}_{ij}u_j(t)], \quad t \in N,\ i,j \in \underline{m} \tag{3}$$

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3 Main Results

In our plan, CPU controls each single-input subsystem $(A_{ii}, \boldsymbol{b}_{ii})$ periodically by the corresponding local controller. For arbitrary $l \in N, i \in \underline{m}, t(l,i) \triangleq lm + i - 1$, where the symbol $\triangleq$ denotes be defined as. The control strategy is

$$u_i(t) = \begin{cases} k_i \boldsymbol{x}_i(t), & t = t(l,t) \\ 0, & t \neq t(l,i) \end{cases}, \quad l \in N, i \in \underline{m} \tag{4}$$

Lemma 1[4]**.** The necessary and sufficient condition for assigning poles arbitrarily for a discrete linear system by state feedback is that the system is reachable.

We need the first hypothesis based on Lemma 1.

$\mathbf{H}_1$**.** Single-input discrete systems $(A_{ii}^m, A_{ii}^{m-1}\boldsymbol{b})$ are all reachable, $i \in \underline{m}$

$$H_i \triangleq A_{ii}^{m-i}(A_{ii} + \mathbf{b}_{ii}k_i)A_{ii}^{i-1}, \quad i = \in \underline{m} \tag{5}$$

Under hypothesis H1, for arbitrary $r_i \in [0,1)$, there exists gain k_i so that

$$p(H_i) = p(A_{ii}^m + A_{ii}^{m-1}\boldsymbol{b}_{ii}k_i) = r_i, \quad i \in \underline{m} \tag{6}$$

where $p(\bullet)$ denotes spectrum radius of matrices.

Hence, for arbitrary positive definite matrix $Q_i \in R^{n_i \times n_i}$, the discrete Lyapunov equation

$$H_i^{\mathrm{T}} P_i H_i - P_i + Q_i = 0 \tag{7}$$

has a unique positive definite solution $P_i \in R^{n_i \times n_i}, i \in \underline{m}$

$$A_{ij}^{(s)} \triangleq \begin{cases} A_{ij} + \boldsymbol{b}_{is}k_s, & j = s \\ A_{ij}, & j \neq s \end{cases}, \quad i,j,s \in \underline{m},\ A^{(s)} \triangleq [A_{ij}^{(s)}]_{m \times m} \tag{8}$$

Then the closed-loop system of control strategy (4) and system (3) is

$$x(t+1) = A^{(i)}x(t), \quad t = t(l,i), \quad l \in N,\ i \in \underline{m} \tag{9}$$

$$\bar{A} \triangleq (\bar{A}_{ij})_{m \times m} \triangleq \prod_{s=1}^{m} A^{(m-s+1)},\ \bar{A}_{ij} \in R^{n_i \times n_j},\ H_{ij} \triangleq \begin{cases} \bar{A}_{ij}, & j \neq i \\ \bar{A}_{ii} - H_i, & j = i \end{cases}, \xi_i(l) \triangleq \boldsymbol{x}_i(lm), l \in N \tag{10}$$

The discrete system

$$\xi(l+1) = H_i \xi_i(l) + \sum_{j=1}^{m} H_{ij}\xi_j(l), l \in N, i \in \underline{m} \tag{11}$$

is called the induced system of closed-loop system (9). It is easy to proof the following lemma:

Lemma 2. The closed-loop system (9) is asymptotically stable if and only if its induced system (11) is asymptotically stable

$$W_{ii} \triangleq -Q_i + 2H_i^{\mathrm{T}} P_i H_{ii} + \sum_{k=1}^{m} H_{ki}^{\mathrm{T}} P_k H_{ki} \tag{12}$$

$$W_{ij} \triangleq 2H_i^{\mathrm{T}} P_i H_{ij} + \sum_{k=1}^{m} H_{ki}^{\mathrm{T}} P_k H_{kj}, \quad j \neq i \tag{13}$$

$$W \triangleq (W_{ij})_{m \times m}, \quad \bar{W} \triangleq -(W^{\mathrm{T}} + W)/2 = (\bar{W}_{ij})_{m \times m} \tag{14}$$

$\lambda_{\min}(\bullet)$ denotes the minimum eigenvalue, $\|\bullet\|$ denotes consistent norm of vectors or matrices

$$\tilde{W} \triangleq (\tilde{w}_{ij})_{m \times m},\ \tilde{w} = \begin{cases} \lambda_{\min}(\bar{W}_{ii}), & j = i \\ -\parallel \bar{W}_{ij} \parallel, & j \neq i \end{cases} \tag{15}$$

We suppose system (3) satisfies the following hypothesis based on (12)∼(15).

H_2. The symmetric matrix $\tilde{W}$ is positively definite.

We will discuss the stability of the induced system (11)

$$z_i(lm) \triangleq \| \boldsymbol{x}(lm) \|, \ i \in \underline{m}, \quad \boldsymbol{z}(lm) \triangleq (z_1(lm), z_2(lm), \cdots, z_m(lm))^{\mathrm{T}} \tag{16}$$

Theorem 1. The induced system (11) is asymptotically stable under H_2.

Proof. Defining positively definite function

$$V(lm) = \sum_{i=1}^{m} \boldsymbol{x}_i^{\mathrm{T}}(lm) P_i \boldsymbol{x}_i(lm) \tag{17}$$

It can be proved that

$$\triangle V(lm) = V(lm+m) - V(lm) \leqslant -\boldsymbol{z}^{\mathrm{T}}(lm)\tilde{W}\boldsymbol{z}(lm) \tag{18}$$

H_2 shows that functionis $\triangle V(lm)$ negative definitely, so the proof is accomplished. □

Lemma 2 and Theorem 1 imply the following conclusion:

Theorem 2. Controllers (4) can stabilize multiple-input system (3) if H_1 and H_2 hold. The gains $k_i (i \in \underline{m})$ are determined by (6).

4 Simulation

The example has two single-input subsystems with orders 2×2, so $m = 2$.

$$A_{11} = \begin{pmatrix} 2.5778 & 0.4948 \\ -2.2236 & 0.9972 \end{pmatrix}, \ \boldsymbol{b}_{11} = \begin{pmatrix} 0.6004 \\ -0.9429 \end{pmatrix}, \ A_{22} = \begin{pmatrix} -1.0543 & 0.8563 \\ -0.0042 & -1.1624 \end{pmatrix}, \ \boldsymbol{b}_{22} = \begin{pmatrix} -0.3089 \\ 0.1848 \end{pmatrix}$$

The coupling is described as

$$A_{12} = \begin{pmatrix} 0.0269 & 0.0120 \\ 0.0089 & -0.0678 \end{pmatrix}, \ \boldsymbol{b}_{12} = \begin{pmatrix} 0.0166 \\ -0.0199 \end{pmatrix}, \ A_{21} = \begin{pmatrix} 0.0550 & -0.0344 \\ 0.0401 & -0.0725 \end{pmatrix}, \ \boldsymbol{b}_{21} = \begin{pmatrix} 0.0752 \\ -0.0511 \end{pmatrix}$$

Both $(A_{11}, \boldsymbol{b}_{11})$ and $(A_{22}, \boldsymbol{b}_{22})$ are controllable, therefore, H_1 is satisfied. eig($\bullet$) denotes the set of the eigenvalues of a matrix. Direct calculation shows

$$\mathrm{eig}(A_{11}^2) = \{2.7103 \pm 2.4892i\}, \quad \mathrm{eig}(A_{22}^2) = \{1.2278 \pm 0.0568i\}$$

Subsystems $(A_{11}, \boldsymbol{b}_{11}), (A_{22}, \boldsymbol{b}_{22})$ and the system are all unstable. Let $r_1 = r_2 = 0.25 \in [0, 1)$; then the corresponding gains are

$$\boldsymbol{k}_1 = (-2.2351 \quad 1.1624), \quad \boldsymbol{k}_2 = (-2.5380 \quad 3.4094)$$

The calculation shows

$$H_1 = \begin{pmatrix} 3.1281 & 3.0359 \\ -2.8641 & -2.7598 \end{pmatrix}, \quad H_2 = \begin{pmatrix} 0.2860 & -0.0029 \\ 0.5011 & 0.2135 \end{pmatrix}$$

Let $Q_1 = Q_2 \triangleq I_2$, where I_2 denotes identity matrix with order 2. Further calculation shows

$$\tilde{W} = \begin{pmatrix} 0.9397 & -0.6177 \\ -0.6177 & 0.9001 \end{pmatrix}, \quad \lambda_{\min}(\tilde{W}) = 0.3018 > 0$$

so $\tilde{W}$ is positively definite and H_2 is also satisfied.

The following Fig. 1 describes the asymptotical stability of the closed-loop system. The abscissas denote discrete instants. The polygonal line above represents the norm of substate $x_1(t)$ and polygonal line below represents the negative norm of substate $x_2(t)$ for a better contrastive effect.

5 Conclusion

This paper discusses discrete linear multiple-input system and presents a method for system stabilization, in which local state feedback controllers run according to periodic switching strategy.

The problem lies on the control of subsystems and the definiteness of a matrix. The former is easy for all subsystems are single-input, and the later is also easy because of the low dimension and symmetry of object matrix.

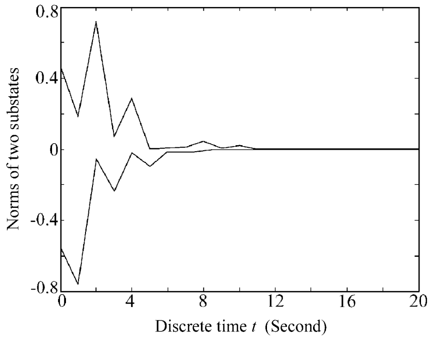

Fig. 1 The asymptotical stability of the closed-loop syetem

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