State Feedback Stabilzation for a Class of Time-delay Nonlinear Systems[1)]

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Abstract The problem of global stabilization by state feedback for a class of time-delay nonlinear system is considered. By constructing the appropriate Lyapunov-Krasovskii functionals (LKF) and using the backstepping design, a linear state feedback controller making the closed-loop system globally asymptotically stable is constructed.

Key words Nonlinear time-delay systems, state feedback, globally asymptotically stability, backstepping design method

1 Introduction

Nonlinear system with delay constitute basic mathematical models of real phenomena and time delays are often encountered in various engineering systems. Moreover, the existence of time delays is frequently a source of instability in some way. Hence, the problem of stability analysis of time-delay systems has been one of the main concerns of researchers wishing to inspect the properties of such systems.

In [1], the output feedback stabilization was considered for a class of time-delay nonlinear systems with delay only dependent on output. [2] studied the problem of almost disturbance decoupling for strict feedback nonlinear delayed system with adaptive L_2-gain disturbance attenuation. In [3], using backstepping method, the author considered the state feedback stabilization for a class of time-delay nonlinear systems, however, its main result is not correct in general[4]. So, it is very difficult and much valuable to proceed the stability analysis for nonlinear time-delay systems.

The backingstepping design method is a powerful tool to design stability controllers for nonlinear system[1,2,5]. In this note, based on the backstepping design method, the stability controllers for a class of nonlinear time-delay systems are constructed. Moreover, different from [1] and [2], our stability controllers are linear.

2 Preliminaries

In this paper, we consider the following single-input-single-output (SISO) delay systems

$$\begin{aligned}\dot{x}_i(t) &= x_{i+1}(t) + \phi_i(t, \boldsymbol{x}(t), \quad \boldsymbol{x}(t-d), u(t)), \quad i = 1, 2, \cdots, n-1 \\ \dot{x}_n(t) &= u(t) + \phi_n(t, \boldsymbol{x}(t), \quad \boldsymbol{x}(t-d), u(t))\end{aligned} \tag{1}$$

where $\boldsymbol{x} = [x_1, \cdots, x_n]^{\mathrm{T}} \in R^n$ is the state vector, $u \in R$ is the input, $d > 0$ is a known time delay of the system. In this paper, we always denote $x_i(t)$ and $\xi_i(t)$ by x_i and ξ_i. The mappings $\phi_i : R \times R^n \times R^n \times R \longrightarrow R, i = 1, 2, \cdots, n$, are continuous and satisfy the following linear growth condition.

Assumption 1. For $i = 1, 2, \cdots, n$, there exist constants $c \geqslant 0$ and $c_1 \geqslant 0$ such that

$$|\phi_i(t, \boldsymbol{x}, \boldsymbol{x}(t-d), u)| \leqslant c \sum_{k=1}^{i} |x_k| + c_1 \sum_{k=1}^{i} |x_k(t-d)| \tag{2}$$

Under the above assumption, we design the linear state feedback controller

$$u = \boldsymbol{K}\boldsymbol{x}(t)$$

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such that the closed-loop systems (1) and (3) are globally asymptotically stable (GAS) at the equilibrium $\boldsymbol{x} = 0$.

In the case of $c_1 = 0$ in Assumption 1, [5] and [6] have respectively studied the output feedback control and the state feedback control for system (1).

3 Main result

Theorem 1. Under Assumption 1, there exists a linear state feedback controller such that time-delay nonlinear system (1) is GAS.

Proof. We use backstepping design method to design the controller.

Step 1. Choose $\tilde{V}_1(x_1) = \frac{x_1^2}{2}$; then

$$\dot{\tilde{V}}_1 = x_1(x_2 + \phi_1(\cdot)) \leqslant x_1 x_2 + c x_1^2 + c_1 |x_1| \cdot |x_1(t-d)| \leqslant x_1(x_2 - x_2^*) + x_1 x_2^* + c x_1^2 + \frac{c_1}{4} x_1^2 + c_1 x_1^2(t-d)$$

Let

$$\xi_2 = x_2 - x_2^*,\ x_2^* = -b_1 x_1, \tag{4}$$

where $b_1 = n + c + \frac{c_1}{4} + nc_1 + \frac{1}{2}$, and we have $\dot{\tilde{V}}_1 \leqslant x_1\xi_2 - nx_1^2 - nc_1 x_1^2 - \frac{1}{2} x_1^2 + c_1 x_1^2(t-d)$.

Taking $V_1(x_1) = \tilde{V}_1 + \int_{t-d}^{t} nc_1 x_1^2(\tau)\mathrm{d}\tau$, we have $\dot{V}_1 \leqslant -nx_1^2 - (n-1)c_1 x_1^2(t-d) - \frac{1}{2} x_1^2 + x_1\xi_2$.

Inductive Step. Suppose at step m, there is a smooth LKF function $V_m(x_1, \xi_2, \cdots, \xi_m)$, and a set of controllers $x_1^*, \cdots, x_{m+1}^*$ defined by

$$\begin{aligned} &\xi_1 = x_1 - x_1^*, && x_1^* = 0 \\ &\xi_j = x_j - x_j^*, && x_j^* = -b_{j-1}\xi_{j-1}, \quad j = 2, 3, \cdots, m+1 \end{aligned} \tag{5}$$

such that

$$\dot{V}_m \leqslant -(n-(m-1)) \sum_{j=1}^{m} \xi_j^2 - (n-m)c_1 \sum_{j=1}^{m} \xi_j^2(t-d) - \frac{1}{2}\xi_m^2 + \xi_m \xi_{m+1} \tag{6}$$

We shall show that (6) also holds at step $m+1$.

Let $\tilde{V}_{m+1}(\xi_1, \cdots, \xi_{m+1}) = V_m(\xi_1, \cdots, \xi_m) + \frac{1}{2}\xi_{m+1}^2$; then

$$\dot{\tilde{V}}_{m+1} \leqslant -(n-(m-1)) \sum_{j=1}^{m} \xi_j^2 - (n-m)c_1 \sum_{j=1}^{m} \xi_j^2(t-d) + \frac{1}{2}\xi_{m+1}^2 + \xi_{m+1}\dot{\xi}_{m+1} \tag{7}$$

We use (2) and (5) to estimate $\xi_{m+1}\dot{\xi}_{m+1}$

$$\begin{aligned} \xi_{m+1}\dot{\xi}_{m+1} = \xi_{m+1}(\dot{x}_{m+1} + b_m\dot{\xi}_m) \leqslant \xi_{m+1}x_{m+2} + |\xi_{m+1}| \Bigg(\sum_{j=1}^{m+1} (r_{m+1,j}|\xi_j|) \Bigg) + \\ |\xi_{m+1}| \Bigg(\sum_{j=1}^{m} (l_{m+1,j} c_1 |\xi_j(t-d)|) + c_1 |\xi_{m+1}(t-d)| \Bigg) \leqslant \xi_{m+1}x_{m+2} + \\ \Bigg(\sum_{j=1}^{m} \bigg(\frac{r_{m+1,j}^2 + c_1 l_{m+1,j}^2}{4} \bigg) + r_{m+1,m+1} + \frac{c_1}{4} \Bigg) \xi_{m+1}^2 + \sum_{j=1}^{m} \xi_j^2 + \sum_{j=1}^{m+1} c_1 \xi_j^2(t-d) \end{aligned} \tag{8}$$

where $r_{m+1,j}$, $l_{m+1,j} (j = 1, 2, \cdots, m)$ and $r_{m+1,m+1}$ are nonnegative constants dependent on c, b_j $(j = 1, 2, \cdots, m)$.

Combing (7) and (8), we arrive at

$$\begin{aligned} \dot{\tilde{V}}_{m+1} \leqslant & -(n-m) \sum_{j=1}^{m} \xi_j^2 - (n-(m+1))c_1 \sum_{j=1}^{m} \xi_j^2(t-d) + \xi_{m+1}(x_{m+2} - x_{m+2}^*) + \xi_{m+1}x_{m+2}^* + \\ & \Bigg(\sum_{j=1}^{m} \bigg(\frac{r_{m+1,j}^2 + c_1 l_{m+1,j}^2}{4} \bigg) + r_{m+1,m+1} + \frac{c_1}{4} + \frac{1}{2} \Bigg) \xi_{m+1}^2 + c_1 \xi_{m+1}^2(t-d) \end{aligned} \tag{9}$$

Let

$$\xi_{m+2} = x_{m+2} - x^*_{m+2}, x^*_{m+2} = -b_{m+1}\xi_{m+1}$$

where

$$b_{m+1} = n - m + \sum_{j=1}^{m}\left(\frac{r^2_{m+1,j} + c_1\, l^2_{m+1,j}}{4}\right) + r_{m+1,m+1} + \frac{c_1}{4} + 1 + (n-m)c_1$$

then

$$\dot{\tilde{V}}_{m+1} \leqslant -(n-m)\sum_{j=1}^{m}\xi_j^2 - (n-(m+1))c_1\sum_{j=1}^{m}\xi_j^2(t-d) + \xi_{m+1}\xi_{m+2} - (n-m)\xi^2_{m+1} - \frac{1}{2}\,\xi^2_{m+1} - (n-m)c_1\,\xi^2_{m+1} + c_1\,\xi^2_{m+1}(t-d)$$

Takeing $V_{m+1}(\xi_1,\cdots,\xi_{m+1}) = \tilde{V}_{m+1} + \int_{t-d}^{t}(n-m)c_1\xi^2_{m+1}(\tau)\mathrm{d}\tau$, we get

$$\dot{V}_{m+1} \leqslant -(n-m)\sum_{j=1}^{m+1}\xi_j^2 - (n-(m+1))c_1\sum_{j=1}^{m+1}\xi_j^2(t-d) - \frac{1}{2}\,\xi^2_{m+1} + \xi_{m+1}\xi_{m+2}$$

Step n. Let $\tilde{V}_n(\xi_1,\cdots,\xi_n) = V_{n-1}(\xi_1,\cdots,\xi_{n-1}) + \frac{1}{2}\,\xi_n^2$. Similar to (9), we can get

$$\dot{\tilde{V}}_n \leqslant -\sum_{j=1}^{n-1}\xi_j^2 + \xi_n u + \left(\sum_{j=1}^{n-1}\left(\frac{r^2_{n,j} + c_1 l^2_{n,j}}{4}\right) + r_{n,n} + \frac{c_1}{4} + \frac{1}{2}\right)\xi_n^2 + c_1\xi_n^2(t-d)$$

where nonnegative constants $r_{n,j}, l_{n,j}(j = 1, 2, \cdots, n-1)$ and $r_{n,n}$ are dependent on $c, b_j(j = 1, 2, \cdots, n-1)$.

Taking

$$u = -b_n\xi_n = -b_n(x_n + b_{n-1}(x_{n-1} + \cdots + b_2(x_2 + b_1x_1)\cdots)) \tag{10}$$

where

$$b_n = 1 + \sum_{j=1}^{n-1}\left(\frac{r^2_{n,j} + c_1\, l^2_{n,j}}{4}\right) + r_{n,n} + \frac{c_1}{4} + \frac{1}{2} + c_1$$

we have

$$\dot{\tilde{V}}_n \leqslant -\sum_{j=1}^{n}\xi_j^2 - c_1\,\xi_n^2 + c_1\,\xi_n^2(t-d)$$

Taking $V_n = \tilde{V}_n + \int_{t-d}^{t} c_1\,\xi_n^2(\tau)\mathrm{d}\tau$, we have $\dot{V}_n \leqslant -\sum_{j=1}^{n}\xi_j^2$.

Up to now, we have constructed LKF $V_n = \frac{1}{2}\sum_{j=1}^{n}\xi_j^2 + c_1\int_{t-d}^{t}\sum_{j=1}^{n}((n+1-j)\xi_j^2(\tau))\mathrm{d}\tau$ for $\boldsymbol{\xi}$−system. Since $\dot{V}_n$ is negatively definite, $\boldsymbol{\xi}$−system under the control $u = -b_n\xi_n$ is GAS at the equilibrium $\boldsymbol{\xi} = 0$. Consequently, the closed-loop systems (1) and (10) are also GAS at the equilibrium $\boldsymbol{x} = 0$. □

4 Example

We consider the following system

$$\dot{x}_1 = x_2 + x_1(\sin x_2)^2 + d_1(t)x_1(t-d)$$
$$\dot{x}_2 = u + x_1 + x_2 + d_2(t)(x_1(t-d))^{\frac{1}{3}}(x_2(t-d))^{\frac{2}{3}}$$

where uncertain disturbance $d_i(t)$ satisfies $0 \leqslant d_i(t) \leqslant 1, i = 1, 2$.

It is easy to see that Assumption 1 is satisfied with $c = 1, c_1 = 1$. As proceeded in the proof of Theorem 1, we can choose $b_1 = \frac{23}{4}, b_2 = 569$, and the control $u = -569\xi_2 = -569\Big(x_2 + \frac{23}{4}x_1\Big)$.

Remark. Specifically, if $d_i(t) = 0(i = 1, 2)$, that is, $c_1 = 0$, we can get $b_1 = \frac{7}{2}, b_2 = 109$, consequently we have the control $u = -109\xi_2 = -109\Big(x_2 + \frac{7}{2}x_1\Big)$. It can be seen that the gain of the control for the system with delay is higher than the gain of the control for the system with no delay.

5 Conclusion

In this paper, we have studied the problem of global stabilization by state feedback, for a class of nonlinear delay system satisfying linear growth condition. We think there is much work to do for the more general nonlinear delay systems.

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