Stable Fault-tolerance Control for a Class of Networked Control Systems $^{1)}$

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Abstract In this paper, we use the matrix measure technique to study stable fault-tolerance control of networked control systems. State feedback networked control systems with the network-induced delay, parameter uncertainties, sensor failures and actuator failures are considered. State feedback gain K is designed for any invariant delay τ , and some theorems and sufficient conditions for stable fault-tolerance control are given. Example is presented to illustrate the effectiveness of these theorems.

Key words Networked control systems, network-induced delay, parameter uncertainties, sensor failures, actuator failures, stable fault-tolerance control, matrix measure

1 Introduction

NCS (networked control system) is a kind of feedback control system where the control loops are closed through a real-time network^[1~4]. The primary advantages of an NCS are reduced system wiring, reduced weight and power, lower cost, improved system reliability and performance, simpler installation and maintenance. It has been used extensively in modern complicated industry process, aircraft and space shuttle, nuclear power station, high-performance automobile, *etc.* However, because of the limited communication bandwidth and the loss of information sources, the network-induced delay is inevitable during information transmission and receiving. The network-induced delay can degrade the performance of control systems designed without considering it and can even destabilize the system^[1], which makes analysis and design of an NCS complex.

The problem of stability analysis for networked control system has attracted a considerable amount of interest in recent years. Zhang et al., discussed fundamental issues for system where network delay was introduced into the feedback^[1]. Stability results were derived for constant delay in the system and asynchronous dynamical system techniques were used to analyze the average stability of system with lost packets. Walsh et al. introduced Try-Once-Discard protocol and the notion of a maximal allowable transfer interval, denoted by $\tau^{[2]}$. Their goal was to find the value of τ for the globally exponential stability of NCS. Branicky showed that the stability of an NCS with network-induced delay could also be analyzed using a hybrid system stability analysis technique^[5]. In these papers, the problems of parameter uncertainties, sensor failures, actuator failures were not considered and the results were given in terms of the conventional methods of either Lyapunov functions or matrix eigenvalues^[1,2,5,6]. However, to the best of our knowledge, the network-induced delay of networked control systems often can be longer than one sampling period, and the system normally involves the problems of parameter uncertainties, sensor failures, actuator failures, which make the use of these conventional methods difficult and even ineffective. To overcome this dilemma, by using matrix norm and matrix measure, we will present how to simplify a complex and high order network-induced delay system. Based on that, for any invariant delay τ , when the system involves parameter uncertainties, sensor failures, actuator failures, we will present how to design state feedback gain K, which makes system attain stable fault-tolerance control.

The paper is organized as follows. In Section 2, we present the structure and the mathematical model of NCS. In Section 3, some definitions and theorems are given for stable fault-tolerance control. Example is presented in Section 4. The conclusion and our future work are given in the last section.

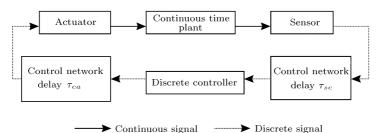
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2 Description of the problem

The NCS model considering network-induced delay is shown in Fig. 1^[1]. Network-induced delay in NCS occurs when sensors, actuators, and controllers exchange data across the network. There are two sources of delays from the network: sensor-to-controller τ_{sc} and controller-to-actuator τ_{ca} . Any computational delay caused by controller can be absorbed into either τ_{sc} or τ_{ca} without loss of generality. τ_{sc} and τ_{ca} can be lumped together as $\tau = \tau_{sc} + \tau_{ca}$, which can be constant, time-variant, or random.



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Fig. 1 The model of networked control system

To simplify the system analysis, this paper considers the following conditions:

1) The system is time-invariant linear system. The network-induced delay is any constant which can be achieved by using an appropriate network protocol^[1] and other technique^[7,8]. The network-induced delay can be longer than one sampling period. Assume the network-induced delay is $\tau = (d-1)h + \tau'$, where $0 \leq \tau' \leq h$, h is one sampling period, and d is a positive integer.

2) We choose the mode for controller-event-driven^[9], and guarantee no data loss and its sequence^[10].

We consider the issues of state feedback, and the closed-loop system equation can be written as^[1]

$$\begin{cases} \dot{\boldsymbol{x}}(t) = A\boldsymbol{x}(t) + B\boldsymbol{u}(t), & t \in [kh + \tau, (k+1)h + \tau] \\ \boldsymbol{u}(t^+) = K\boldsymbol{x}(t-\tau), & t \in \{kh + \tau, k = 0, 1, 2, \cdots\} \end{cases}$$
(1)

where $u(t) \in \mathbb{R}^m$ is the control input vector, $x(t) \in \mathbb{R}^n$ is the state vector, $A \in \mathbb{R}^{n \times n}$ and $B \in \mathbb{R}^{n \times m}$ are known constant matrices, $K \in \mathbb{R}^{m \times n}$ is state feedback gain matrix, $u(t^+)$ is piecewise continuous and only changes value at $kh + \tau$.

3 Design of stable fault-tolerance control

The matrix measure can be used to study the stability of linear systems. The key idea is that the simple system can be obtained using the matrix measure technique. In this paper, we will use this technique to study complex networked control systems.

Definition 1. Let $|\mathbf{x}|$. denote a vector norm of \mathbf{x} on C^n , where $\cdot = 1, 2, \infty, \cdots$, and ||A(t)||. is the matrix norm of A(t) induced by this vector norm. The symbol $\mu[A(t)]$ denotes the matrix measure derived from the matrix norm ||A(t)|| and defined as^[11,12]

$$\mu[A(t)] = \lim_{\theta \to 0^+} \frac{\|I + \theta A(t)\| - 1}{\theta}$$

$$\tag{2}$$

where I is the identity matrix with the same dimension as A(t).

For convenience, we list only two properties of $\mu[A(t)]$ in the following lemma.

Lemma 1^[12]. μ .[A(t)] is defined for any induced norm and has the following properties a) For any $\alpha_j \ge 0$ ($1 \le j \le k$) and any matrix $A_j(t)$ ($1 \le j \le k$), we have

$$\mu\left[\sum_{k=0}^{k}\alpha_{k}A_{k}(t)\right] \leq \sum_{k=0}^{k}\alpha_{k}\mu\left[A_{k}(t)\right]$$

$$\mu \cdot \left\lfloor \sum_{j=1} \alpha_j A_j(t) \right\rfloor \leqslant \sum_{j=1} \alpha_j \mu \cdot [A_j(t)]$$

b) For any norm and any A(t), we have $\mu[A(t)] \leq ||A(t)||$.

Theorem 1. Let $\mu[A(t)]$ be given by (2), for $t_0 \leq t$; then we have

$$\Big|\exp\Big\{\int_{t_0}^t A(s)\mathrm{d}s\Big\}\Big\|.\leqslant \exp\Big\{\int_{t_0}^t \mu.[A(s)]\mathrm{d}s\Big]$$

Proof. Consider the following linear system

$$\dot{\boldsymbol{x}}(t) = A(t)\boldsymbol{x}(t)$$

The solution of (3) is

$$\boldsymbol{x}(t) = \boldsymbol{x}(t_0) \exp\left\{\int_{t_0}^t A(s) \mathrm{d}s\right\}$$

Using the known inequality (Coppel's Inequality^[11,13])

$$\|\boldsymbol{x}(t)\|_{\cdot} \leq \|\boldsymbol{x}(t_0)\|_{\cdot} \exp\left\{\int_{t_0}^t \mu_{\cdot}[A(s)] \mathrm{d}s\right\}$$

For any $\boldsymbol{x}(t_0)$, we have $\|\boldsymbol{x}(t_0) \exp\{\int_{t_0}^t A(s) ds\}\| \le \|\boldsymbol{x}(t_0)\| . \exp\{\int_{t_0}^t \mu . [A(s)] ds\}$ Considering the definition of norm, we have

$$\max_{\|\boldsymbol{x}(t_0)\|_{\cdot}=1} \left\| \boldsymbol{x}(t_0) \exp\left\{ \int_{t_0}^t A(s) \mathrm{d}s \right\} \right\|_{\cdot} = \left\| \exp\left\{ \int_{t_0}^t A(s) \mathrm{d}s \right\} \right\|_{\cdot}$$

Hence,

$$\Big|\exp\Big\{\int_{t_0}^t A(s)\mathrm{d}s\Big\}\Big\|_{\cdot} \leqslant \exp\Big\{\int_{t_0}^t \mu_{\cdot}[A(s)]\mathrm{d}s\Big\}$$

The theorem is proved.

Definition 2. Diagonal matrix $F = diag(f_1, f_2, \dots, f_m)$ inserted between state feedback matrix K and state vector x(t) denotes sensor failure, where

$$f_i = \begin{cases} 1, & \text{ith sensor work} \\ 0, & \text{ith sensor failure} \end{cases}, \ i = 1, 2, \cdots, n$$

Definition 3. Diagonal matrix $L = diag(l_1, l_2, \dots, l_n)$ inserted between control input vector u(t)and B control matrix denotes actuator failure, where

$$l_i = \begin{cases} 1, & \text{ith sensor work} \\ 0, & \text{ith sensor failure} \end{cases}, \ i = 1, 2, \cdots, m$$

Definition 4. We define the matrices $\Delta A(t)$ and $\Delta B(t)$ representing time-varying parameter uncertainties in the system model with appropriate dimensions and assume

$$\|\Delta A(t)\|_{\cdot} \leqslant a, \quad \|\Delta B(t)\|_{\cdot} \leqslant b$$

Theorem 2. Considering the characteristic equation

$$f(\lambda) = \lambda^{d+1} - a\lambda^d - b\lambda - c = 0 \tag{4}$$

where a, b and c are nonnegative real numbers, $d = 1, 2, 3, \cdots$, all the characteristic roots of (4) satisfy $|\lambda| < 1$ if and only if 1 - a - b - c > 0.

Proof. Let us rewrite (4) in the form

$$\lambda^{d+1} = a\lambda^d + b\lambda + c \tag{5}$$

Taking norms on both sides of (5), we obtain

$$\lambda|^{d+1} = |a\lambda^d + b\lambda + c| \leqslant a|\lambda|^d + b|\lambda| + c$$

or

$$|\lambda|^{d+1} - a|\lambda|^d - b|\lambda| - c \leqslant 0 \tag{6}$$

No. 2

(3)

It shows that if λ is the characteristic root of (4), then (6) must be satisfied.

Assume that $g(|\lambda|) = |\lambda|^{d+1} - a|\lambda|^d - b|\lambda| - c$. Then differentiating $g(|\lambda|)$ with respect to $|\lambda|$ we have

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$$g'(|\lambda|) = (d+1)|\lambda|^d - ad|\lambda|^{d-1} - b = d|\lambda|^{d-1}(|\lambda| - a) + |\lambda|^d - b$$
(7)

Necessity would be obvious from Jury criterion, *i.e.*, when all the characteristic roots of (4) satisfy $|\lambda| < 1$, the inequality f(1) > 0 must be satisfied, *i.e.*, 1 - a - b - c > 0.

For sufficiency, let us prove it by contradiction. Assume if 1 - a - b - c > 0, the characteristic equation (4) has the root that satisfies $|\lambda| = \alpha \ge 1$.

Since 1 - a - b - c > 0, if $|\lambda| \ge 1$, then $|\lambda| > a$, $|\lambda|^d > b$. Hence from (7), we have

$$g'(|\lambda|) > 0$$

It shows that $g(|\lambda|)$ is an increasing function when $|\lambda| \ge 1$. Furthermore, g(1) > 0 for $|\lambda| = 1$. Hence if $|\lambda| = \alpha \ge 1$, the following can be obtained

 $g(\alpha) > 0$

This contradicts equation (6). It implies that if 1 - a - b - c > 0, (4) has not the characteristic root that satisfies $|\lambda| = \alpha \ge 1$. This completes the proof.

Theorem 3. Consider system (1), for any constant fixed matrix A, and any invariant networkinduced delay τ . The parameter uncertain matrices are $\Delta A(t)$ and $\Delta B(t)$. If there exists a matrix norm such that $\|\Delta A(t)\| \leq a$, $\|\Delta B(t)\| \leq b$, $\mu(A) + a < 0$, and the system involves parameter uncertainties, sensor failures, actuator failures, then we can design state feedback gain K, such that when $(\|B\| + b)\|K\| < -(\mu(A) + a)$, the discrete system of system (1) can attain stable fault-tolerance control.

Proof. Considering the system with parameter uncertainties, sensor failures, and actuator failures and sampling the closed-loop system (1), we obtain the following discrete system model^[1,14]:

$$\boldsymbol{x}(k+1) = \boldsymbol{\Phi}(k+1,k)\boldsymbol{x}(k) + \int_{kh+\tau'}^{kh+h} \boldsymbol{\Phi}(k+1,t)(B+\Delta B(t))LKFdt\boldsymbol{x}(k-d+1) + \int_{kh}^{kh+\tau'} \boldsymbol{\Phi}(k+1,t)(B+\Delta B(t))LKFdt\boldsymbol{x}(k-d)$$
(8)

where $\Phi(k+1,t)$ is the transition matrix of $(A + \Delta A(t))$. From Theorem 1 and Lemma 1, we obtain

$$\begin{split} \|\varPhi(k+1,t)\|_{\cdot} &= \Big\|\exp\Big\{\int_{t}^{kh+h}[A+\Delta A(s)]\mathrm{d}s\Big\}\Big\|_{\cdot} \leqslant \exp\Big\{\int_{t}^{kh+h}\mu_{\cdot}[A+\Delta A(s)]\mathrm{d}s\Big\} \leqslant \exp\Big\{\int_{t}^{kh+h}[\mu_{\cdot}(A)+\mu_{\cdot}(\Delta A(s))]\mathrm{d}s\Big\} \leqslant \exp\Big\{\int_{t}^{kh+h}[\mu_{\cdot}(A)+a]\mathrm{d}s\Big\} = \exp\{(\mu_{\cdot}(A)+a)(kh+h-t)\} \end{split}$$

Similarly, we have $\|\Phi(k+1,k)\| \le \exp\{(\mu \cdot (A) + a)h\}$. Thus, evaluating the norm $\|*\|$. of both sides of (8) and simplifying it yield

$$\|\boldsymbol{x}(k+1)\|_{\cdot} \leqslant \exp\{(\mu.(A)+a)h\}\|\boldsymbol{x}(k)\|_{\cdot} + \int_{0}^{h-\tau'} \exp\{(\mu.(A)+a)t\}(\|B\|_{\cdot}+b)\|LKF\|_{\cdot}dt\|\boldsymbol{x}(k-d)\|_{\cdot}dt\|\boldsymbol{x}(k-d)\|_{\cdot}dt\|\boldsymbol{x}(k-d)\|_{\cdot}dt\|\boldsymbol{x}(k-d)\|_{\cdot}dt\|\boldsymbol{x}(k-d)\|_{\cdot}dt\|\boldsymbol{x}(k-d)\|_{\cdot}dt\|\boldsymbol{x}(k-d)\|_{\cdot}dt\|\boldsymbol{x}(k-d)\|_{\cdot}dt\|\boldsymbol{x}(k-d)\|_{\cdot}dt\|\boldsymbol{x}(k-d)\|_{\cdot}dt\|\boldsymbol{x}(k-d)\|_{\cdot}dt\|\boldsymbol{x}(k-d)\|_{\cdot}dt\|\boldsymbol{x}(k-d)\|_{\cdot}dt\|\boldsymbol{x}(k-d)\|_{\cdot}dt\|\boldsymbol{x}(k-d)\|_{\cdot}dt\|\boldsymbol{x}(k-d)\|_{\cdot}dt\|\boldsymbol{x}(k-d)\|_{\cdot}dt\|\boldsymbol{x}(k-d)\|_{\cdot}dt\|\boldsymbol{x}(k-d)\|_{\cdot}dt\|\boldsymbol{x}(k-d)\|_{\cdot}dt\|\boldsymbol{x}(k-d)\|_{\cdot}dt\|\boldsymbol{x}(k-d)\|_{\cdot}dt\|\boldsymbol{x}(k-d)\|_{\cdot}dt\|\boldsymbol{x}(k-d)\|_{\cdot}dt\|\boldsymbol{x}(k-d)\|_{\cdot}dt\|\boldsymbol{x}(k-d)\|_{\cdot}dt\|\boldsymbol{x}(k-d)\|_{\cdot}dt\|\boldsymbol{x}(k-d)\|_{\cdot}dt\|\boldsymbol{x}(k-d)\|_{\cdot}dt\|\boldsymbol{x}(k-d)\|_{\cdot}dt\|\boldsymbol{x}(k-d)\|_{\cdot}dt\|\boldsymbol{x}(k-d)\|_{\cdot}dt\|\boldsymbol{x}(k-d)\|_{\cdot}dt\|\boldsymbol{x}(k-d)\|_{\cdot}dt\|\boldsymbol{x}(k-d)\|_{\cdot}dt\|\boldsymbol{x}(k-d)\|_{\cdot}dt\|\boldsymbol{x}(k-d)\|_{\cdot}dt\|\boldsymbol{x}(k-d)\|_{\cdot}dt\|\boldsymbol{x}(k-d)\|_{\cdot}dt\|\boldsymbol{x}(k-d)\|_{\cdot}dt\|\boldsymbol{x}(k-d)\|_{\cdot}dt\|\boldsymbol{x}(k-d)\|_{\cdot}dt\|\boldsymbol{x}(k-d)\|_{\cdot}dt\|\boldsymbol{x}(k-d)\|_{\cdot}dt\|\boldsymbol{x}(k-d)\|_{\cdot}dt\|\boldsymbol{x}(k-d)\|_{\cdot}dt\|\boldsymbol{x}(k-d)\|_{\cdot}dt\|\boldsymbol{x}(k-d)\|_{\cdot}dt\|\boldsymbol{x}(k-d)\|_{\cdot}dt\|\boldsymbol{x}(k-d)\|_{\cdot}dt\|\boldsymbol{x}(k-d)\|_{\cdot}dt\|\boldsymbol{x}(k-d)\|_{\cdot}dt\|\boldsymbol{x}(k-d)\|_{\cdot}dt\|\boldsymbol{x}(k-d)\|_{\cdot}dt\|\boldsymbol{x}(k-d)\|_{\cdot}dt\|\boldsymbol{x}(k-d)\|_{\cdot}dt\|\boldsymbol{x}(k-d)\|_{\cdot}dt\|\boldsymbol{x}(k-d)\|_{\cdot}dt\|\boldsymbol{x}(k-d)\|_{\cdot}dt\|\boldsymbol{x}(k-d)\|_{\cdot}dt\|\boldsymbol{x}(k-d)\|_{\cdot}dt\|\boldsymbol{x}(k-d)\|_{\cdot}dt\|\boldsymbol{x}(k-d)\|_{\cdot}dt\|\boldsymbol{x}(k-d)\|_{\cdot}dt\|\boldsymbol{x}(k-d)\|_{\cdot}dt\|\boldsymbol{x}(k-d)\|_{\cdot}dt\|\boldsymbol{x}(k-d)\|_{\cdot}dt\|\boldsymbol{x}(k-d)\|_{\cdot}dt\|\boldsymbol{x}(k-d)\|_{\cdot}dt\|\boldsymbol{x}(k-d)\|_{\cdot}dt\|\boldsymbol{x}(k-d)\|_{\cdot}dt\|\boldsymbol{x}(k-d)\|_{\cdot}dt\|\boldsymbol{x}(k-d)\|_{\cdot}dt\|\boldsymbol{x}(k-d)\|_{\cdot}dt\|\boldsymbol{x}(k-d)\|_{\cdot}dt\|\boldsymbol{x}(k-d)\|_{\cdot}dt\|\boldsymbol{x}(k-d)\|_{\cdot}dt\|\boldsymbol{x}(k-d)\|_{\cdot}dt\|\boldsymbol{x}(k-d)\|_{\cdot}dt\|\boldsymbol{x}(k-d)\|_{\cdot}dt\|\boldsymbol{x}(k-d)\|_{\cdot}dt\|\boldsymbol{x}(k-d)\|_{\cdot}dt\|\boldsymbol{x}(k-d)\|_{\cdot}dt\|\boldsymbol{x}(k-d)\|_{\cdot}dt\|\boldsymbol{x}(k-d)\|_{\cdot}dt\|\boldsymbol{x}(k-d)\|_{\cdot}dt\|\boldsymbol{x}(k-d)\|_{\cdot}dt\|\boldsymbol{x}(k-d)\|_{\cdot}dt\|\boldsymbol{x}(k-d)\|_{\cdot}dt\|\boldsymbol{x}(k-d)\|_{\cdot}dt\|\boldsymbol{x}(k-d)\|_{\cdot}dt\|\boldsymbol{x}(k-d)\|_{\cdot}dt\|\boldsymbol{x}(k-d)\|_{\cdot}dt\|\boldsymbol{x}(k-d)\|_{\cdot}dt\|\boldsymbol{x}(k-d)\|_{\cdot}dt\|\boldsymbol{x}(k-d)\|_{\cdot}dt\|\boldsymbol{x}(k-d)\|_{\cdot}dt\|\boldsymbol{x}(k-d)\|_{\cdot}dt\|\boldsymbol{x}(k-d)\|_{\cdot}dt\|\boldsymbol{x}(k-d)\|_{\cdot}dt\|\boldsymbol{x}(k-d)\|_{\cdot}dt\|\boldsymbol{x}(k-d)\|_{\cdot}dt\|\boldsymbol{x}(k-d)\|_{\cdot}dt\|\boldsymbol{x}(k-d)\|_{\cdot}dt\|\boldsymbol{x}(k-d)\|_{\cdot}dt\|\boldsymbol{x}(k-d)\|_{\cdot}dt\|\boldsymbol{x}(k-d)\|_{\cdot}dt\|\boldsymbol{x}(k-d)\|_{\cdot}dt\|\boldsymbol{x}(k-d)\|_{\cdot}dt\|\boldsymbol{x}(k-d)\|_{\cdot}dt\|\boldsymbol{x}(k-d)\|_{\cdot}dt\|\boldsymbol{x}(k-d)\|_{\cdot}dt\|\boldsymbol{x}(k-d)\|_{\cdot}dt\|\boldsymbol{x}(k-d)\|_{\cdot}dt\|\boldsymbol{x}(k-d)\|$$

Assume that

$$\boldsymbol{z}(k+1) = \exp\{(\mu.(A) + a)h\}\boldsymbol{z}(k) + \int_{0}^{h-\tau'} \exp\{(\mu.(A) + a)t\}(\|B\|.+b)\|LKF\|.dt\boldsymbol{z}(k-d+1) + \exp\{(\mu.(A) + a)(h-\tau')\}\int_{0}^{\tau'} \exp\{(\mu.(A) + a)t\}(\|B\|.+b)\|LKF\|.dt\boldsymbol{z}(k-d)$$
(9)
$$\boldsymbol{z}(k) = \|\boldsymbol{x}(k)\|., \text{ for } -d \leq k \leq 0$$

It is obvious that the following is satisfied

$$\|\boldsymbol{x}(k+1)\|_{\cdot} \leq \boldsymbol{z}(k+1)$$

Hence, asymptotic stability of z(k+1) implies that of x(k+1). Now we design state feedback gain K, which makes z(k+1) asymptotically stable. The characteristic equation of equation (9) is

$$\lambda^{d+1} - \exp\{(\mu.(A) + a)h\}\lambda^{d} - \frac{(||B||.+b)||LKF||.}{\mu.(A) + a}(\exp\{(\mu.(A) + a)(h - \tau')\} - 1)\lambda - \frac{(||B||.+b)||LKF||.}{\mu.(A) + a}(\exp\{(\mu.(A) + a)h\} - \exp\{(\mu.(A) + a)(h - \tau')\}) = 0$$
(10)

Note that for $\mu(A) + a < 0$, the coefficients of equation (10) satisfy the conditions of Theorem 2, therefore, system (9) is asymptotically stable if and only if

$$1 - \exp\{(\mu.(A) + a)h\}\frac{(||B||. + b)||LKF||.}{\mu.(A) + a}(\exp\{(\mu.(A) + a)(h - \tau)'\} - 1) - \frac{(||B||. + b)||LKF||.}{\mu.(A) + a}(\exp\{(\mu.(A) + a)h\} - \exp\{(\mu.(A) + a)(h - \tau')\}) > 0$$

i.e.,

$$(||B||.+b)||LKF||. < -(\mu.(A)+a)$$
(11)

Note that

$$(||B||.+b)||LKF||. \leq (||B||.+b)||L||.||K||.||F||. \leq (||B||.+b)||K||$$

Thus, as long as the condition $(||B||.+b)||K||. < -(\mu.(A)+a)$ is satisfied, equation (11) must be satisfied. It implies that for any invariant delay τ , $(||B||.+b)||K||. < -(\mu.(A) + a)$ is a sufficient condition for the stability of system (8). Namely, the discrete system of system (1) can attain stable fault-tolerance control. The theorem is proved.

Corollary 1. For $\exp\{(\mu . (A) + a)\} < \alpha < 1$, if

$$(||B||.+b)||K||. < \frac{-\alpha^d(\alpha - \exp\{(\mu.(A) + a)h\})(\mu.(A) + a)}{\alpha - \exp\{(\mu.(A) + a)h\} + \exp\{(\mu.(A) + a)(h - \tau')\} - \alpha \exp\{(\mu.(A) + a)(h - \tau')\}}$$

then the characteristic roots of (10) satisfy $|\lambda| < \alpha < 1$.

Proof. (10) can be rewritten in the form

$$\alpha^{d+1} \left(\frac{\lambda}{\alpha}\right)^{d+1} - \alpha^{d} \exp\{(\mu.(A) + a)h\} \left(\frac{\lambda}{\alpha}\right)^{d} - \alpha \frac{(\|B\|. + b)\|LKF\|.}{\mu.(A) + a} (\exp\{(\mu.(A) + \alpha)(h - \tau')\} - 1) \left(\frac{\lambda}{\alpha}\right) - \frac{(\|B\|. + b)\|LKF\|.}{\mu.(A) + a} (\exp\{(\mu.(A) + a)h\} - \exp\{(\mu.(A) + a)(h - \tau')\}) = 0$$

Next, the proof is similar to the proof in Theorem 3. If

$$\alpha^{d+1} - \alpha^d \exp\{(\mu.(A) + a)h\} - \alpha \frac{(||B||. + b)||LKF||.}{\mu.(A) + a} (\exp\{(\mu.(A) + a)(h - \tau')\} - 1) - \frac{(||B||. + b)||LKF||.}{\mu.(A) + a} (\exp\{(\mu.(A) + a)h\} - \exp\{(\mu.(A) + a)(h - \tau')\}) > 0$$

then,

$$(\|B\|.+b)\|LKF\|. < \frac{-\alpha^d(\alpha - \exp\{(\mu.(A) + a)h\})(\mu.(A) + a)}{\alpha - \exp\{(\mu.(A) + a)h\} + \exp\{(\mu.(A) + a)(h - \tau')\} - \alpha \exp\{(\mu.(A) + a)(h - \tau')\}}$$

Note that $(\|B\|.+b)\|LKF\|. \leq (\|B\|.+b)\|K\|$, hence, if

$$(\|B\|.+b)\|K\|. < \frac{-\alpha^d(\alpha - \exp\{(\mu.(A) + a)h\})(\mu.(A) + a)}{\alpha - \exp\{(\mu.(A) + a)h\} + \exp\{(\mu.(A) + a)(h - \tau')\} - \alpha \exp\{(\mu.(A) + a)(h - \tau')\}}$$
(12)

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Then the characteristic roots of (10) satisfy $\frac{|\lambda|}{\alpha} < 1$, *i.e.*, $|\lambda| < \alpha$. Noting (12), $\alpha > \exp\{(\mu.(A) + a)\}$ should be satisfied. The proof is completed. When $|\lambda| < \alpha < 1$, that is, the discrete system (8) can attain stable fault-tolerance control more quickly.

4 Simulation example

We give an application of Theorem 3. Consider the system

$$A = \begin{pmatrix} -3 & 0 \\ 0 & -3 \end{pmatrix}, B = \begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix}, \Delta A(t) = \begin{pmatrix} 0.1 \sin t & 0 \\ 0 & 0.1 \cos t \end{pmatrix}, \Delta B(t) = \begin{pmatrix} -0.1 \sin t & 0 \\ 0 & -0.1 \cos t \end{pmatrix}$$

Since $\|\Delta A(t)\|_1 \leq a = 0.1$, $\|\Delta B(t)\|_1 \leq b = 0.1$, $\mu_1[A(t)] = \max_j \{\operatorname{Re}[a_{ij}(t)] + \sum_{\substack{i=1 \ i \neq j}}^n |a_{ij}(t)|\}^{[12]}$
 $\mu_1(A) + a = -2.9 \leq 0$ we can consider the inequality $(\|B\|_1 + b)\|K\|_1 \leq 2.9$ or $\|k\|_1 \leq 0.9355$. Assume

 $\mu_1(A) + a = -2.9 < 0$, we can consider the inequality $(||B||_1 + b)||K||_1 < 2.9$ or $||k||_1 < 0.9355$. Assume $K = \begin{pmatrix} 0.5 & 0.4 \\ 0.2 & 0.3 \end{pmatrix}$. The initial condition of this system is chosen at $\mathbf{x}(0) = (1 - 1)^{\mathrm{T}}$ and $\mathbf{x}(k) = 0$ for $-d \leq k < 0$. The values of τ and h are chosen arbitrarily. Suppose $\tau = 1.2$ s, h = 1s, we obtain d = 2, $\tau' = 0.2$ s. Here only four cases for different L and F are shown in Fig. 2. From the simulations results we see that the system with parameter uncertainties, sensor failures, actuator failures can attain stable fault-tolerance control. For different τ , L and F, other cases simulated (not shown in here) also give same conclusion.

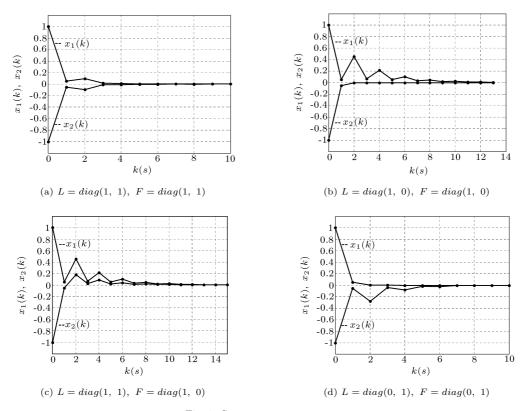


Fig. 2 State response curves

5 Conclusions and perspective

In this paper, we use the matrix measure technique to study stable fault-tolerance control of networked control system. Some Theorems are derived for stable fault-tolerance control of networked control system with the network-induced delay, parameter uncertainties, sensor failures and actuator failures. The example is presented to illustrate the effectiveness of these Theorems.

To simplify the system analysis, a great deal of previous results are only limited to the case that delay is less than a sample period or is only limited to a small interval. Expect for the premise that delay is constant, there is no limitation in this paper, which makes the Theorem 3 very useful for NCS.

We will model an NCS that the delay is random or time-variant, and study the stability, the fault detection and the fault-tolerance control.

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