

Reliable Controller Design with Mixed H_2/H_∞ Performance for Descriptor Systems¹⁾

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Abstract The design problem of a reliable observer-based controller with mixed H_2/H_∞ performance for descriptor systems is discussed using linear matrix inequality (LMI) approach. First, the design problems of H_2 and H_∞ controllers are discussed, respectively. Secondly, when there are partial actuator faults for the descriptor system, controllers with H_2 and H_∞ performance are designed, respectively. Finally, a numerical example is given to illustrate the design procedures and their effectiveness.

Key words Descriptor systems, mixed H_2/H_∞ performance, reliable controller, linear matrix inequality

1 Introduction

H_2 optimal control for linear systems possesses many excellent properties except robustness. H_∞ control can overcome robust problems of system, yet it sacrifices other properties^[1]. Therefore, Berstein *et al.*^[2] put forward the design idea of mixed H_2/H_∞ control. The approach has received extensive attention and made much progress^[3] since it can better resolve the problem of robust performance and performance index.

In recent years, there have been great interests in design reliable controllers for linear systems. Using algebraic Riccati equation, Veillette^[4] has brought forward the design method to reliable controllers with H_∞ performance. Veillette^[5] has also devised a class of reliable linear quadratic (LQ) controllers. In some cases, many other performances are sacrificed for the sake of system stability when reliable controllers are designed. In order to optimize capability index and ensure the stability of the system, it is necessary to use the reliable controller design which mixes H_2/H_∞ capability index^[6].

The development of descriptor systems is later than regular systems. Descriptor systems have applications in many fields, such as power systems, aviation, aerospace, petrochemical industry, and so on^[7~9]. The reliable controller design of descriptor systems is more complex than regular systems due to the impulse of descriptor systems^[10]. In the past, many scholars studied these problems about integrity control^[11] and reliable H_∞ control of descriptor systems^[10]. They usually paid the price of losing other capabilities of the systems. The paper discusses the reliable controller design problem which mixes H_2/H_∞ capability index for the descriptor systems. The purpose of design is to search for a descriptor controller so that if part actuator turn to fault, the descriptor systems are admissible (regular, stable, impulse-free) and H_∞ norm of the transfer function is less than the given constant scalar, and optimize the H_2 performance index of the systems.

2 Systems description

Consider descriptor systems described by

$$E\dot{x} = Ax + B_1w + B_2u, \quad z_0 = C_0x + D_{02}u, \quad z_1 = C_1x + D_{12}u, \quad y = C_2x + D_2w \quad (1)$$

where $x \in R^n$ is the state, $w \in R^q$ is the exogenous disturbance, $u \in R^m$ is the control input, $z_0 \in R^p$, $z_1 \in R^l$ are the controlled outputs and $y \in R^r$ is the measured output. $E, A, B_1, B_2, C_0, C_1, C_2, D_{02}, D_{12}$ and D_2 are known constant matrices of appropriate dimensions. E is singular matrix. Assume that (1) is regular and satisfies

(A₁) (E, A, C_2) is detectable, impulse controllable and observable.

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$$(A_2) D_{i2}^T [C_i \quad D_{i2}] = [0 \quad I], \quad i = 0, 1.$$

Design descriptor controller:

$$E\xi = A\xi + B_1 w + B_2 u - L(y - D_2 w - C_2 \xi) \quad (2a)$$

$$u = K\xi \quad (2b)$$

where L is the observer gain matrix and K is the feedback gain matrix. (2a) is a descriptor observer, together with (2b) form the descriptor controller (2). Then the resulting closed-loop descriptor systems can be written as

$$\bar{E}x_e = \bar{A}x_e + \bar{B}w, \quad z_0 = \bar{C}_0 x_e, \quad z_1 = \bar{C}_1 x_e \quad (3)$$

where

$$\begin{aligned} x_e &= \begin{bmatrix} I & I \\ 0 & I \end{bmatrix}^{-1} \begin{bmatrix} x \\ \xi \end{bmatrix}, \quad \bar{A} = \begin{bmatrix} A + LC_2 & 0 \\ -LC_2 & A + B_2 K \end{bmatrix}, \quad \bar{E} = \text{diag}[E \quad E] \\ \bar{B} &= [0 \quad B_1^T]^T, \quad \bar{C}_0 = [C_0 \quad C_0 + D_{02}K], \quad \bar{C}_1 = [C_1 \quad C_1 + D_{12}K] \end{aligned}$$

Let

$$A_0 = A + LC_2, \quad A_k = A + B_2 K, \quad C_{0k} = C_0 + D_{02}K, \quad C_{1k} = C_1 + D_{12}K \quad (4)$$

Then the transfer functions from z_0 to w and from z_1 to w are respectively:

$$T_{z_0 w}(s) = C_{0k}(sE - A_k)^{-1}B_1, \quad T_{z_1 w}(s) = C_{1k}(sE - A_k)^{-1}B_1$$

Remark 1. The poles of the closed-loop descriptor systems (3) are composed of the poles of descriptor systems (1) and descriptor controller (2) when the assumption (A₁) is satisfied. Chen *et al.*^[12] use the controller similar to (2) to discuss a class of descriptor systems H_∞ control. When descriptor systems (1) are impulse, the controller (2) is equivalent to a regular controller^[9].

When descriptor systems (1) are regular, it follows from [9] that there exist invertible matrices P and Q such that

$$PEQ = \begin{bmatrix} I_r & 0 \\ 0 & N \end{bmatrix}, \quad PAQ = \begin{bmatrix} A_1 & 0 \\ 0 & I_{n-r} \end{bmatrix} \quad (5)$$

where $A_1 \in R^{r \times r}$, $N \in R^{(n-r) \times (n-r)}$ is nilpotent matrix. Let $\bar{B}_1 = QPB_1$.

3 H_2 and H_∞ control for descriptor systems

3.1 H_2 control for descriptor systems

H_2 performance is analyzed for descriptor systems (3), and the following theorems are obtained.

Theorem 1. Assume that (1) is regular and satisfies (A₁), and that $T_{z_1 w}$ is strictly proper. If there exist matrices X and Y satisfying

$$\begin{bmatrix} A_k^T X + X^T A_k & C_{1k}^T \\ C_{1k} & -I \end{bmatrix} < 0 \quad (6)$$

$$\begin{bmatrix} E^T X & X^T E \bar{B}_1 \\ \bar{B}_1^T E^T X & Y \end{bmatrix} \geq 0 \quad (7)$$

then the closed-loop descriptor systems (3) are admissible and $\|T_{z_1 w}(s)\|_2 \leq \sqrt{\text{tr}(Y)}$.

Design a controller (2) to make the close-loop descriptor systems (3) admissible and $\|T_{z_1 w}(s)\|_2$ bound with LMI method. Then we have

Theorem 2. Assume that descriptor systems (1) satisfy (A₁) and (A₂). If there exist invertible matrices \bar{X}_1 and W such that

$$\begin{bmatrix} \bar{X}_1^T A^T + A \bar{X}_1 - B_2 B_2^T & \bar{X}_1^T C_1^T \\ C_1 \bar{X}_1 & -I \end{bmatrix} < 0 \quad (8)$$

$$\begin{bmatrix} \bar{X}_1^T E^T & E \bar{B}_1 \\ \bar{B}_1^T E^T & W \end{bmatrix} \geq 0 \quad (9)$$

and $T_{z_1 w}(s)$ is strictly proper, then the closed-loop descriptor systems (3) are admissible and $\|T_{z_1 w}(s)\|_2 \leq \sqrt{\text{tr}(w)}$. In this case, the matrix $K = -B_2^T \bar{X}_1^{-1}$ and L makes (E, A_0) admissible.

3.2 H_∞ control for descriptor systems

Analyzing the H_∞ performance of descriptor systems (3), we can obtain the following results.

Theorem 3. Assume that descriptor systems (1) satisfy (A₁). Then the following states are equivalent:

- i) The closed-loop descriptor systems (3) are admissible and $\|T_{z_0w}(s)\|_\infty < r$.
- ii) There exists an invertible matrix P satisfying,

$$\begin{bmatrix} A_k^T P + P^T A_k & P^T B_1 & C_{0k}^T \\ B_1^T P & -r^2 I & 0 \\ C_{0k} & 0 & -I \end{bmatrix} < 0 \tag{10a}$$

$$E^T P = P^T E \geq 0 \tag{10b}$$

In the following, we design a controller (2) to make the close-loop descriptor systems (3) admissible and $\|T_{z_0w}(s)\|_\infty < r$ with LMI method.

Theorem 4. Assume that regular descriptor systems (1) satisfy (A₁) and (A₂). Then the following states are equivalent:

- i) The closed-loop descriptor systems (3) are admissible and $\|T_{z_0w}(s)\| < r$.
- ii) There exists an invertible matrix \bar{X}_2 satisfying

$$\begin{bmatrix} \bar{X}_2^T A^T + A \bar{X}_2 - B_2 B_2^T & B_1 & \bar{X}_2^T C_0^T \\ B_1^T & -r^2 I & 0 \\ C_0 \bar{X}_2 & 0 & -I \end{bmatrix} \tag{11a}$$

$$\bar{X}_2^T E^T = E \bar{X}_2 \geq 0 \tag{11b}$$

In this case, the matrix $K = -B_2^T \bar{X}_2^{-1}$ and L makes (E, A_0) admissible.

4 Reliable controller design with mixed H_2/H_∞ performance

Actuators of descriptor systems (1) can be classified into two sets. The set of actuators that are susceptible to faults is denoted as $\Omega \subset \{1, 2, \dots, m\}$. These controllers of systems (1) are redundant, but they can improve the system performance. The complementary set of actuators is denoted $\bar{\Omega} = \{1, 2, \dots, m\} - \Omega$, and is assumed never to fail. Let $B_2 = [B_{2\Omega} \ B_{2\bar{\Omega}}]$, where $B_{2\Omega}$ and $B_{2\bar{\Omega}}$ are the control matrices associated with the sets Ω and $\bar{\Omega}$, respectively. Let $\varphi \subseteq \Omega$ correspond to a particular subset of the actuators that actually experience a fault for systems (1), and assume that the actuator faults are modeled as u_φ whose elements correspond to the set of faulty actuators φ . Similarly, $B_2 = [B_{2\varphi} \ B_{2\bar{\varphi}}]$, where $\bar{\varphi} = \bar{\Omega} \cup (\Omega - \varphi)$. We denote

$$u = \begin{bmatrix} u_\varphi \\ u_{\bar{\varphi}} \end{bmatrix}, K = \begin{bmatrix} K_\varphi \\ K_{\bar{\varphi}} \end{bmatrix}, D_{02} = [D_{02\varphi} \ D_{02\bar{\varphi}}], D_{12} = [D_{12\varphi} \ D_{12\bar{\varphi}}]$$

The new disturbance input can be denoted as $\bar{w} = \begin{bmatrix} w \\ u_\varphi \end{bmatrix}$. So the closed-loop descriptor systems (3) become the following form

$$\begin{aligned} \bar{E} x_e &= \bar{A} x_e + \bar{B} \bar{w} \\ z_0 &= \bar{C}_0 x_e \\ z_1 &= \bar{C}_1 x_e \end{aligned} \tag{12}$$

where

$$\begin{aligned} \bar{A} &= \begin{bmatrix} A_0 & 0 \\ -LC_2 & A + B_{2\bar{\varphi}} K_{\bar{\varphi}} \end{bmatrix}, \bar{B} = \begin{bmatrix} 0 \\ [B_1 \ B_{2\varphi}] \end{bmatrix} \\ \bar{C}_0 &= [C_0 \ C_0 + D_{02\bar{\varphi}} K_{\bar{\varphi}}], \bar{C}_1 = [C_1 \ C_1 + D_{12\bar{\varphi}} K_{\bar{\varphi}}] \end{aligned}$$

Then the transfer function from z_0 to \bar{w} is

$$T_{z_0\bar{w}}(s) = (C_0 + D_{02\bar{\varphi}} K_{\bar{\varphi}})[sE - (A + B_{2\bar{\varphi}} K_{\bar{\varphi}})]^{-1} [B_1 \ B_{2\varphi}] \tag{13}$$

from z_1 to \bar{w} is

$$T_{z_1\bar{w}}(s) = (C_1 + D_{12\bar{\varphi}}K_{\bar{\varphi}})[sE - (A + B_{2\bar{\varphi}}K_{\bar{\varphi}})]^{-1}[B_1 \quad B_{2\varphi}]$$

Remark 2. For descriptor systems (1), the actuators in Ω are redundant, but these actuators can improve the capability of the systems. And $\bar{\Omega}$ is a minimal set whose elements, the actuators, make the descriptor system (1) admissible.

Definition 1. For given constant scalar $\gamma > 0$, when part of the actuators of descriptor systems (1) are at fault, if the closed-loop descriptor systems (12) are admissible and $\|T_{z_0\bar{w}}(s)\|_\infty < r$ and minimize $\|T_{z_1\bar{w}}(s)\|_2$, then the controller (2) is called reliable controller of systems (1) with mixed H_2/H_∞ performance.

For closed-loop descriptor systems (12), we assume

(A₃) $(E, A, B_{2\bar{\Omega}})$ is stabilizable.

Theorem 5. Under (A₁)~(A₃), if there exist invertible matrices \bar{X}_1 and W satisfying

$$\begin{bmatrix} \bar{X}_1^T A^T + A\bar{X}_1 - 2B_{2\bar{\Omega}}B_{2\bar{\Omega}}^T & B_2 & \bar{X}_1^T C_1^T \\ & B_2^T & -I & 0 \\ & C_1\bar{X}_1 & 0 & -I \end{bmatrix} < 0 \tag{14a}$$

$$\begin{bmatrix} \bar{X}_1^T E^T & E \bar{B}_1 \\ \bar{B}_1^T & E^T \\ \bar{B}_1 & E^T \\ & W \end{bmatrix} \geq 0 \tag{14b}$$

such that $T_{z_1\bar{w}}(s)$ is strictly proper, where $\bar{B}_1 = Q_1 P_1 [B_1 \quad B_{2\varphi}]$ with matrices P_1 and Q_1 making (E, A) possess the form of (5), then the fault closed-loop descriptor systems (12) are admissible and $\|T_{z_1\bar{w}}(s)\|_2 \leq \sqrt{tr(W)}$. In this case, matrix L makes (E, A_0) admissible, and

$$K = \begin{bmatrix} K_\varphi \\ K_{\bar{\varphi}} \end{bmatrix} = \begin{bmatrix} -B_{2\varphi}^T \\ -B_{2\bar{\varphi}}^T \end{bmatrix} \bar{X}_1^{-1} \tag{15}$$

Theorem 6. Under (A₁)~(A₃), if there exists an invertible matrix \bar{X}_2 satisfying

$$\begin{bmatrix} \bar{X}_2^T A^{rmT} + A\bar{X}_2 - 2B_{2\bar{\Omega}}B_{2\bar{\Omega}}^T & B_1 & B_{2\Omega} & B_2 & \bar{X}_2^T C_0^T \\ & B_1^T & -r^2 I & 0 & 0 \\ & B_{2\Omega}^T & 0 & -r^2 I & 0 \\ & C_0\bar{X}_2 & 0 & 0 & 0 \\ & & & & -I \end{bmatrix} < 0 \tag{16a}$$

$$\bar{X}_2^T E^T = E\bar{X}_2 \geq 0 \tag{16b}$$

then the fault closed-loop descriptor systems (12) are admissible and $\|T_{z_0\bar{w}}(s)\|_\infty < r$. In this case, matrix L makes (E, A_0) admissible, and

$$K = \begin{bmatrix} K_\varphi \\ K_{\bar{\varphi}} \end{bmatrix} = \begin{bmatrix} -B_{2\varphi}^T \\ -B_{2\bar{\varphi}}^T \end{bmatrix} \bar{X}_2^{-1} \tag{17}$$

For the fault closed-loop descriptor systems (12), the reliable controller design with mixed H_2/H_∞ performance is equivalent to seeking matrices K and L such that matrices \bar{X} and W satisfy (14) and (16), and minimize the upper bound $\|T_{z_1\bar{w}}(s)\|_2$ of $tr(W)$. So, we have the following theorem.

Theorem 7. Under (A₁)~(A₃), if there exist matrices \bar{X} and W satisfying

$$\begin{bmatrix} \bar{X}^T A^T + A\bar{X} - 2B_{2\bar{\Omega}}B_{2\bar{\Omega}}^T & B_1 & B_{2\Omega} & B_2 & \bar{X}^T C_0^T \\ & B_1^T & -r^2 I & 0 & 0 \\ & B_{2\Omega}^T & 0 & -r^2 I & 0 \\ & B_2^T & 0 & 0 & -I \\ & C_0\bar{X} & 0 & 0 & 0 \end{bmatrix} < 0 \tag{18a}$$

$$\begin{bmatrix} \bar{X}^T A^T + A\bar{X} - 2B_{2\bar{\Omega}}B_{2\bar{\Omega}}^T & \bar{X}^T C_1^T & B_2 \\ & C_1\bar{X} & -I & 0 \\ & B_2^T & 0 & -I \end{bmatrix} < 0 \tag{18b}$$

$$\begin{bmatrix} \bar{X}^T E^T & E \bar{B} \\ \bar{B}_1^T & E^T \\ \bar{B}_1 & E^T \\ & W \end{bmatrix} \geq 0 \tag{18c}$$

and $T_{z_1\bar{w}}(s)$ is strictly proper, where $\bar{B}_1 = Q_1 P_1 [B_1 \ B_{2\varphi}]$, then

i) There exists a controller (2) with mixed H_2/H_∞ performance for the fault closed-loop descriptor systems (12).

ii) If (\bar{X}^*, W^*) is the optimal solution of (18), then the controller matrices with mixed H_2/H_∞ performance can be chosen as

$$K^* = \begin{bmatrix} K_\varphi^* \\ K_{\bar{\varphi}}^* \end{bmatrix} = \begin{bmatrix} -B_{2\varphi}^T \\ -B_{2\bar{\varphi}}^T \end{bmatrix} (\bar{X}^*)^{-1}$$

and L makes (E, A_0) admissible. In this case

$$\|T_{z_0\bar{w}}(s)\|_\infty < \gamma, \quad \|T_{z_1\bar{w}}(s)\|_2 \leq \sqrt{\text{tr}(W^*)}$$

Remark 3. Given $\gamma > 0$, the controller which is designed to make the H_2 capability index of the normal descriptor systems (1) optimal can even not make the descriptor systems admissible^[6]. The reliable controller designed in Theorem 7 is mixed H_2/H_∞ capability, and for $\gamma > 0$, the H_2 capability index of the normal descriptor systems is not optimal. But it can by no means pass H_2 capability index of the fault systems and it pays the price of losing the H_2 capability index of the systems.

5 Conclusion

The paper discussed the reliable controller design problem which mixes the H_2/H_∞ capability for the descriptor systems. The controller is designed by way of LMI. The designed controller can make the closed-loop descriptor systems admissible, $\|T_{z_0\bar{w}}(s)\|_\infty < r$, and optimize $\|T_{z_1\bar{w}}(s)\|_2$. But the simulation problem of this kind of descriptor systems will be studied in the future.

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