有色噪声扰动下的随机控制问题研究

陈 福 祥
(武 汉 工 业 大 学)

摘　　要

本文研究了有色噪声扰动下的线性二次随机控制问题，探讨了有色噪声与白色噪声随机控制问题的等效化方法，利用准最优性能指标综合了准最优输出反馈控制，最后，利用原始系统的系数矩阵导出了带有普遍意义的准最优控制算法框图.

关于有色噪声理论，H·K·Khalil, A·H·Haddad, P·V·Kokotovic 等人在 80 年前后进行了详尽的研究[1]. 本文以工业上广泛应用的直流调速系统为例从理论上研究了有色噪声扰动下的线性二次随机控制问题，探讨了有色噪声与白色噪声随机控制问题的等效化方法，利用准最优性能指标综合了准最优输出反馈控制，最后，利用原始系统的系数矩阵导出了带有普遍意义的准最优控制算法框图.

一、随机控制问题的数学描述

有色噪声扰动下的直流调速系统的组成及原理图如图 1 所示，各物理量如图 1 所标注. 其中 $\nu_1(t)$ 为等效的有色噪声电压，其自协方差为

$$\mathrm{Cov}[\nu_1(t), \nu_1(\tau)] \neq 0, \ (t, \tau \text{ 任意}) \tag{1}$$

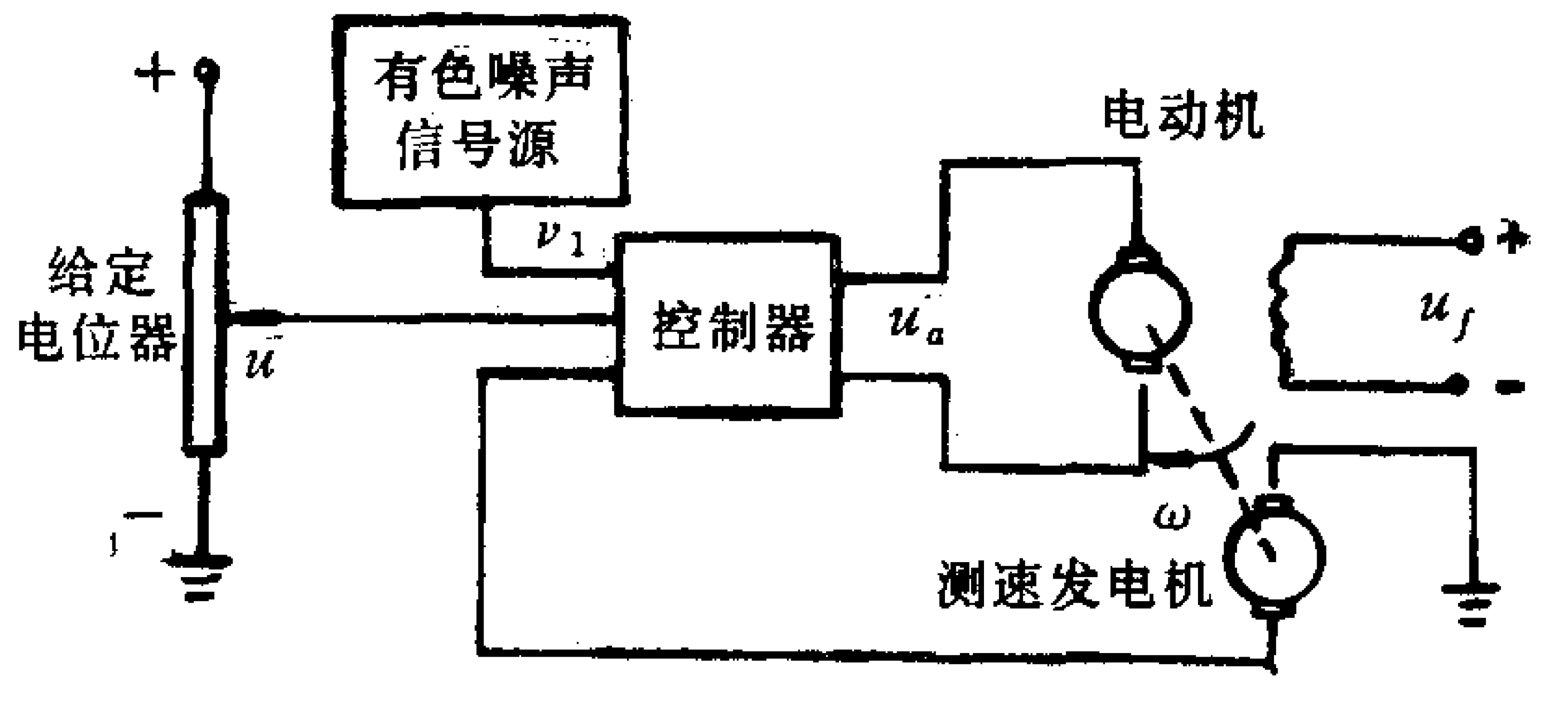

图 1

有色噪声 $\nu_1(t)$ 可以看成是由具有零均值的白色噪声 $W_1(t)$ 激励的线性系统的输出，其关系如图 2.

$$\dot{\nu}_1(t) = H(t)\nu_1(t) + W_1(t), \ \nu_1(t_0) = \nu_{10} \tag{2}$$

系统的结构图如图 3 所示.

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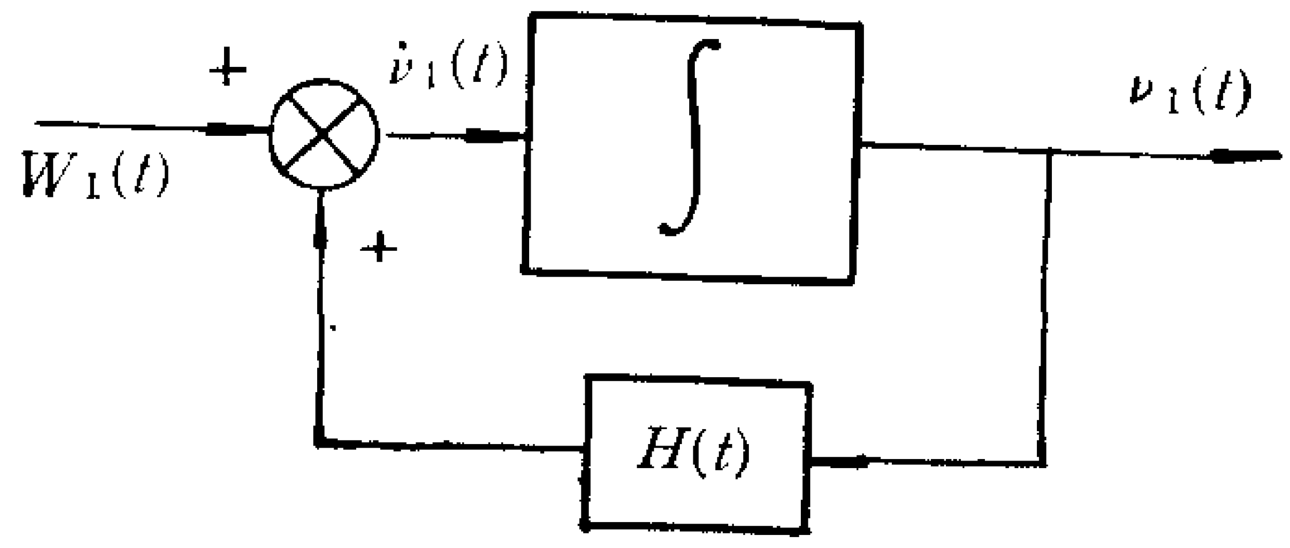

图　2

若选 $\tilde{x}_1 = u_a$, $\tilde{x}_2 = \omega$ 为状态变量，选 u 为控制变量，则可得系统的状态方程和观测方程：

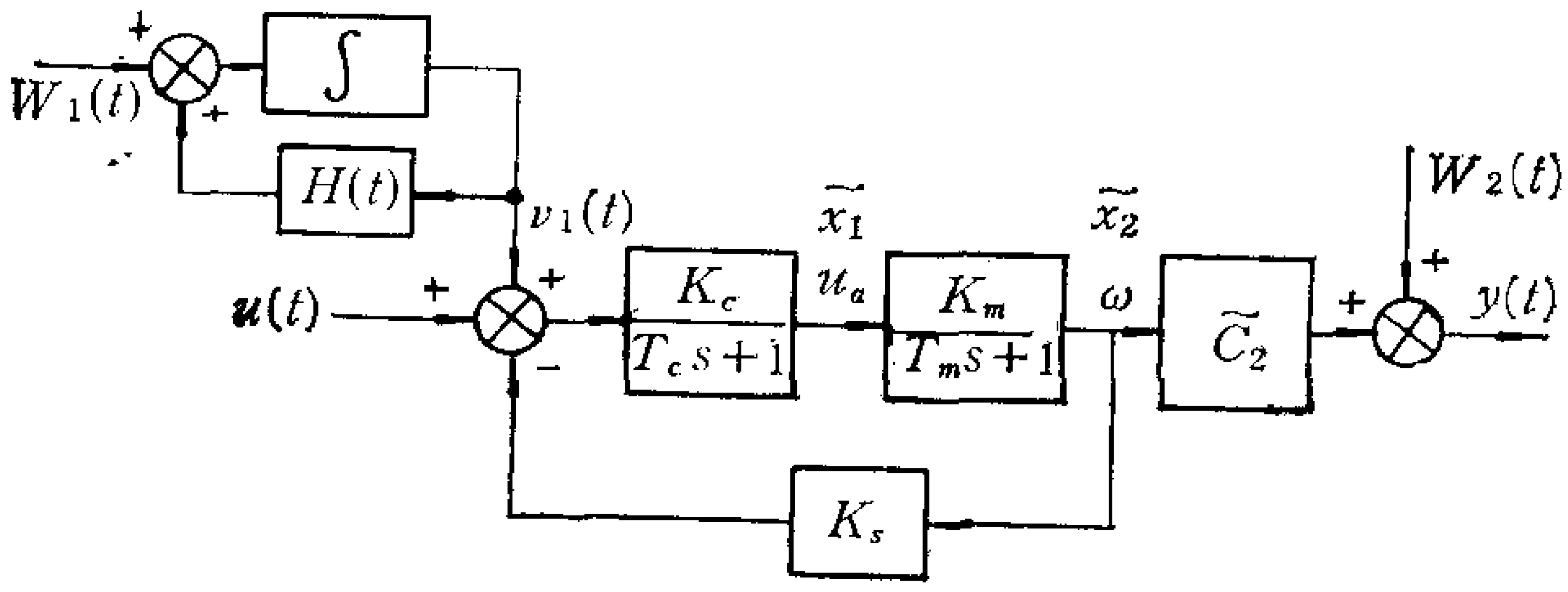

图　3

$$\begin{bmatrix} \dot{\tilde{x}}_1 \\ \dot{\tilde{x}}_2 \end{bmatrix} = \begin{bmatrix} -\dfrac{1}{T_c} & -\dfrac{K_s K_c}{T_c} \\ \dfrac{K_m}{T_m} & -\dfrac{1}{T_m} \end{bmatrix} \begin{bmatrix} \tilde{x}_1 \\ \tilde{x}_2 \end{bmatrix} + \begin{bmatrix} \dfrac{K_c}{T_c} \\ 0 \end{bmatrix} (u + \nu_1) \tag{3}$$

$$y = C_2 \tilde{x}_2 + W_2 \tag{4}$$

引入奇异摄动小参数 μ，取 $T_m = \mu T_c$, 可得

$$\dot{\tilde{x}}_1 = \tilde{A}_{11}\tilde{x}_1 + \tilde{A}_{12}\tilde{x}_2 + \tilde{B}_1 u + \tilde{G}_1 \nu_1, \quad \tilde{x}_1(t_0) = \tilde{x}_{10}, \tag{5}$$

$$\mu\dot{\tilde{x}}_2 = \tilde{A}_{21}\tilde{x}_1 + \tilde{A}_{22}\tilde{x}_2 + \tilde{B}_2 u + \tilde{G}_2 \nu_1, \quad \tilde{x}_2(t_0) = \tilde{x}_{20}, \tag{6}$$

$$y = \tilde{C}_1\tilde{x}_1 + \tilde{C}_2\tilde{x}_2 + W_2. \tag{7}$$

选复合型性能指标为

$$J = E\left\{ \frac{1}{2} \begin{bmatrix} \tilde{x}_1 \\ \tilde{x}_2 \end{bmatrix}^\tau \tilde{Q}_0 \begin{bmatrix} \tilde{x}_1 \\ \tilde{x}_2 \end{bmatrix} \Bigg|_{t=t_f} + \frac{1}{2}\int_{t_0}^{t_f} \left[\begin{bmatrix} \tilde{x}_1 \\ \tilde{x}_2 \end{bmatrix}^\tau \tilde{Q}_1 \begin{bmatrix} \tilde{x}_1 \\ \tilde{x}_2 \end{bmatrix} + u^\tau \tilde{Q}_2 u \right] dt \right\}, \tag{8}$$

$$\dot{\nu}_1 = H\nu_1 + W_1, \nu_1(t_0) = \nu_{10}. \tag{9}$$

其中 $\tilde{A}_{11} = -\dfrac{1}{T_c}$; $\tilde{A}_{12} = -\dfrac{K_s K_c}{T_c}$; $\tilde{B}_1 = \dfrac{K_c}{T_c}$; $\tilde{G}_1 = \dfrac{K_c}{T_c}$; $\tilde{A}_{21} = \dfrac{K_m}{T_c}$; $\tilde{A}_{22} = -\dfrac{1}{T_c}$; $\tilde{B}_2 = 0$; $\tilde{G}_2 = 0$; $\tilde{C}_1 = 0$; $\tilde{C}_2 = 1$;

$$\tilde{Q}_0 = \begin{bmatrix} \tilde{T}_1 & \mu\tilde{T}_{12} \\ \mu\tilde{T}_{12}^\tau & \mu\tilde{T}_2 \end{bmatrix}; \quad \tilde{Q}_1 = \begin{bmatrix} \tilde{C}_1^\tau\tilde{C}_1 & \tilde{C}_1^\tau\tilde{C}_2 \\ \tilde{C}_2^\tau\tilde{C}_1 & \tilde{C}_2^\tau\tilde{C}_2 \end{bmatrix}; \ \tilde{Q}_2 \text{ 为正定非奇异矩阵;}$$

$\tilde{T}_1$, $\tilde{T}_{12}$, $\tilde{T}_2$ 为选定参数；W_1, W_2 均为白色噪声过程．令

$$W^\tau = [W_1^\tau \vdots W_2^\tau], \tag{10}$$

$$E\{W(t)W^\tau(s)\} = \begin{bmatrix} V_1(t) & V_{12}(t) \\ V_{12}^\tau(t) & V_2(t) \end{bmatrix} \delta(t - S). \tag{11}$$

其中
$$\delta(t-S)=\begin{cases}1, & (t=S)\\ 0, & (t\neq S).\end{cases}$$

设 $\tilde{x}_1$, $\tilde{x}_2$, u, y, v_1 分别为 n_1, n_2, γ, m, l 维矢量. $\tilde{x}_{10}$, $\tilde{x}_{20}$, v_{10} 分别为具有均值 $\bar{x}_{10}$, $\bar{x}_{20}$, $\bar{v}_{10}$ 的高斯随机矢量. 这样，该问题的数学描述如 (5) 到 (9) 式所示. 试图寻找最优控制 $u^*(t)$，使(8)式取极小值.

二、将有色噪声扰动下的随机控制问题等效化为白色噪声扰动下的高斯随机控制问题

令 $x=\begin{bmatrix}\tilde{x}_1\\ v_1\end{bmatrix}$, $z=\tilde{x}_2$, 则可将(5)到(8)式改写为

$$\dot{x}=A_{11}x+A_{12}z+B_1u+G_1W_1,\quad x(t_0)=x_0, \tag{12}$$

$$\mu\dot{z}=A_{21}x+A_{22}z+B_2u+G_2W_2,\quad z(t_0)=z_0, \tag{13}$$

$$y=c_1x+c_2z+W_2, \tag{14}$$

$$J=E\left\{\frac{1}{2}\begin{bmatrix}x\\ z\end{bmatrix}^\tau Q_0\begin{bmatrix}x\\ z\end{bmatrix}\Bigg|_{t=t_f}+\frac{1}{2}\int_{t_0}^{t_f}\left[\begin{bmatrix}x\\ z\end{bmatrix}^\tau Q_1\begin{bmatrix}x\\ z\end{bmatrix}+u^\tau Q_2u\right]dt\right\}. \tag{15}$$

其中 $A_{11}=\begin{bmatrix}\tilde{A}_{11} & \tilde{G}_1\\ 0 & H\end{bmatrix}=\begin{bmatrix}-\frac{1}{T_c} & \frac{K_c}{T_c}\\ 0 & H\end{bmatrix}$; $A_{12}=\begin{bmatrix}\tilde{A}_{12}\\ 0\end{bmatrix}=\begin{bmatrix}-\frac{K_sK_c}{T_c}\\ 0\end{bmatrix}$;

$$B_1=\begin{bmatrix}\tilde{B}_1\\ 0\end{bmatrix}=\begin{bmatrix}\frac{K_c}{T_c}\\ 0\end{bmatrix};\ G_1=\begin{bmatrix}0\\ 1\end{bmatrix};\ A_{21}=[\tilde{A}_{21}\quad \tilde{G}_2]=\begin{bmatrix}\frac{K_m}{T_c} & 0\end{bmatrix};$$

$$A_{22}=\tilde{A}_{22}=-\frac{1}{T_c};\ B_2=\tilde{B}_2=0;\ G_2=\tilde{G}_2=0;\ C_1=[\tilde{C}_1\ 0]=[0\quad 0];$$

$$C_2=\tilde{C}_2=1;\ Q_0=\begin{bmatrix}T_1 & \mu T_{12}\\ \mu T_{12}^\tau & \mu T_2\end{bmatrix};\ Q_1=\begin{bmatrix}C_1^\tau C_1 & C_1^\tau C_2\\ C_2^\tau C_2 & C_2^\tau C_2\end{bmatrix};$$

Q_2 为正定非奇异矩阵.

把(12)和(13)式分别称为原系统的慢变子系统和快变子系统. 这样，就将原问题等效化为由(12)到(15)式所描述的白色噪声扰动下的随机控制问题.

三、进行低阶准最优综合

运用奇异摄动法[2,3]，对慢变子系统进行低阶综合. 令 $\mu=0$ 时，则由(13)式可得

$$z=-A_{22}^{-1}A_{21}X. \tag{16}$$

将(16)代入(12)式得降阶模型(脚注 γ 表示降阶模型的变量):

$$\dot{x}_\gamma=A_0x_\gamma+B_0u+G_1W_1,\quad x_\gamma(t_0)=x_0, \tag{17}$$

$$y=C_0x_\gamma+W_2, \tag{18}$$

$$J_\gamma=E\left\{\frac{1}{2}x_\gamma^\tau T_1x_\gamma|_{t=t_f}+\frac{1}{2}\int_{t_0}^{t_f}[x_\gamma^\tau C_0^\tau C_0x_\gamma+u^\tau Q_2u]dt\right\}. \tag{19}$$

其中

$$A_0 = A_{11} - A_{12}A_{22}^{-1}A_{21} = \begin{bmatrix} -\dfrac{1 + K_sK_cK_m}{T_c} & \dfrac{K_c}{T_c} \\ 0 & H \end{bmatrix};$$

$$B_0 = B_1 = \begin{bmatrix} \dfrac{K_c}{T_c} \\ 0 \end{bmatrix};\ C_0 = [C_1 - C_2A_{22}^{-1}A_{21}] = [K_m\ 0];\ x_r(t) = x(t)|_{\mu=0}.$$

这样，系统由 $n = (n_1 + n_2)$ 阶降低到 n_1 阶．现试图寻求一个准最优控制 $u_r(t)$，使(19)式取极小值．

由文献[1]和[2]可得线性输出反馈控制为

$$u_r = -Q_2^{-1}B_0^{\tau}N\hat{x}_r. \tag{20}$$

其中 $N(t,\mu)$ 是下列矩阵黎卡堤微分方程的解：

$$\dot{N} = -NA_0 - A_0^{\tau}N + NB_0Q_2^{-1}B_0^{\tau}N,\ N(t_f,\mu) = T_1. \tag{21}$$

$\hat{X}_r$ 是 X_r 的最优估计，它由下列滤波器方程给出：

$$\begin{cases} \dot{\hat{X}}_r = A_0\hat{X}_r + B_0u + K_r[y - C_0X_r], \\ \hat{X}_r(t_0) = E\{X(t_0)\} = \overline{X}_0. \end{cases} \tag{22}$$

K_r 是准最优线性反馈增益，它由下式给出：

$$K_r = [\lambda_rC_0^{\tau} + G_1V_{12}]V_2^{-1},\ t \in [t_0,\ t_f] \tag{23}$$

其中 λ_r 是下列矩阵黎卡堤微分方程的解：

$$\begin{aligned} \dot{\lambda}_r = {} & [A_0 - G_1V_{12}V_2^{-1}C_0]\lambda_r + \lambda_r[A_0 - G_1V_{12}V_2^{-1}C_0]^{\tau} \\ & - \lambda_rC_0V_2^{-1}C_0\lambda_r + G_1V_1G_1^{\tau} - G_1V_{12}V_2^{-1}V_{12}^{\tau}G_1^{\tau}. \end{aligned} \tag{24}$$

四、准最优控制算法实现

为了能在计算机上迅速求出准最优控制，可借助于原系统(5)到(9)式的系数矩阵来实现准最优控制算法，不过此时应做如下变换：令

$$N = \begin{bmatrix} N_1 & N_2 \\ N_2^{\tau} & N_3 \end{bmatrix},\quad \lambda_r = \begin{bmatrix} \lambda_1 & \lambda_2 \\ \lambda_2^{\tau} & \lambda_3 \end{bmatrix},$$

$$\widetilde{A}_0 = \widetilde{A}_{11} - \widetilde{A}_{12}\widetilde{A}_{22}^{-1}\widetilde{A}_{21} = -\frac{1 + K_sK_cK_m}{T_c},\ \widetilde{B}_0 = \widetilde{B}_1 = \frac{K_c}{T_c},$$

$$\widetilde{G}_0 = \widetilde{G}_1 - \widetilde{A}_{12}\widetilde{A}_{22}^{-1}\widetilde{G}_2 = \frac{K_c}{T_c},\ \widetilde{C}_0 = \widetilde{C}_1 - \widetilde{C}_2\widetilde{A}_{22}^{-1}\widetilde{A}_{21} = K_m,$$

$$\Psi_1 = \lambda_1\widetilde{C}_0^{\tau} = \lambda_1K_m,\ \Psi_2 = \lambda_2\widetilde{C}_0^{\tau} = \lambda_2K_m,$$

则准最优线性输出反馈控制由控制器方程给出：

$$u_r = Q_2^{-1}\widetilde{B}_0^{\tau}N_1\hat{X}_1 + Q_2^{-1}\widetilde{B}_0^{\tau}N_2\hat{v}_1 = Q_2^{-1}\left[\frac{K_c}{T_c}N_1\hat{X}_1 + \frac{K_c}{T_c}N_2\hat{v}_1\right]. \tag{25}$$

其中 $\hat{X}_1$ 和 $\hat{v}_1$ 分别是 X_1 和 v_1 的最优估计，且分别由如下滤波器方程给出：

$$\begin{cases} \dot{\hat{x}}_1 = \tilde{A}_0\hat{X}_1 + \tilde{G}_0\hat{\nu}_1 + \tilde{B}_0u_r + \Psi_1V_2^{-1}(y - \tilde{C}_0\hat{X}_1) \\ \quad = -\dfrac{1 + K_sK_cK_m}{T_c}\hat{X}_1 + \dfrac{K_c}{T_c}\hat{\nu}_1 + \dfrac{K_c}{T_c}u_r + \lambda_1K_mV_2^{-1}(y - K_m\hat{X}_1), \\ \hat{X}_1(t_0) = \bar{X}_{10}. \end{cases} \quad (26)$$

$$\begin{cases} \dot{\hat{\nu}}_1 = H\hat{\nu}_1 + (\Psi_2 + V_{12})V_2^{-1}(y - \tilde{C}_0\hat{X}_1) \\ \quad = H\hat{\nu}_1 + (\lambda_2K_m + V_{12})V_2^{-1}(y - K_m\hat{X}_1), \\ \hat{\nu}_1(t_0) = \bar{\nu}_{10}. \end{cases} \quad (27)$$

这样，可得准最优控制算法框图如图 4 所示.

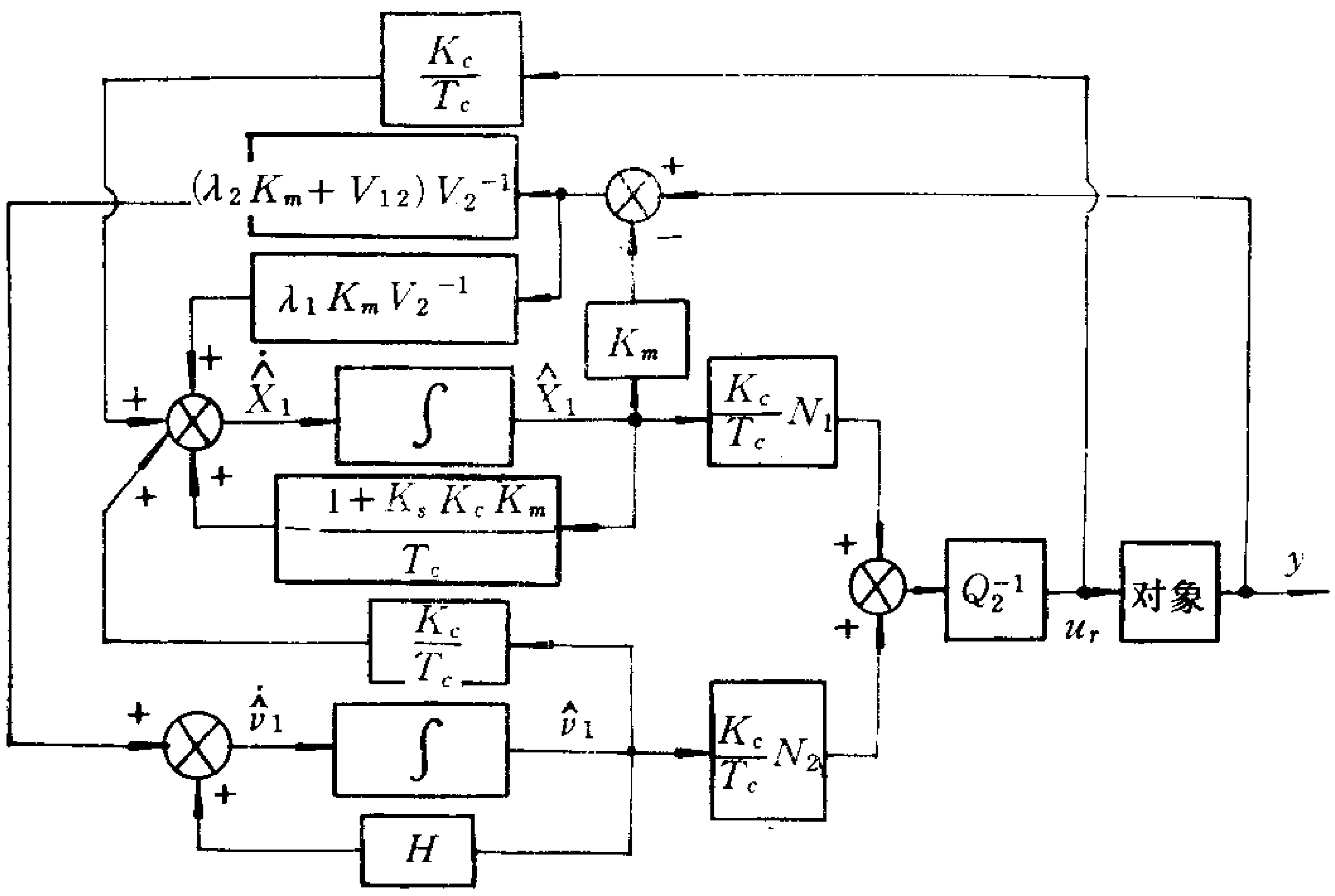

图 4

参 考 文 献

[1] P. V. Kokotovic etc., Multimodeling and Control of Large Scale Systems, Report R-848, 1979.
[2] 郑大钟，现代控制理论(第二册)，清华大学出版社，1979.
[3] A. H. 奈弗著，王辅俊译，摄动方法，上海科学技术出版社，1983.

RESEARCH ON THE STOCHASTIC CONTROL WITH DISTURBANCE OF COLOR NOISE

CHEN FUXIANG

(*Wuhan University of Technology*)

ABSTRACT

In this paper, the stochastic linear quedrats control problen under disturbance of colored noise is considered. The equalization method for describing the system with colored noise to that with white noise is investigated. The quasioptimal output feedback control is synthesized in terms of the quasi-optimal performance index. Finally, the algorithmic block-diagram of quasi-optimal control, which has a universal sense, is obtained in terms of the factor matrices of the original system.