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广义离散随机线性系统的最优递推滤波方法（II）[1)]

王恩平　王朝珠
（中国科学院系统科学所）

摘　要

本文对文献[1]给出的广义离散随机线性系统最优估计误差协方差阵进行了分析，在一定条件下得到了误差协方差阵的上界和下界，继而讨论了由文献[1]给出的滤波器的稳定性.

一、引　言

文献[1]讨论了广义离散随机线性系统的最优状态估计，给出了最优状态估计的递推方法. 本文在文[1]的基础上，首先讨论了最优估计误差协方差阵的界，继而讨论了由文[1]给出的最优滤波器的稳定性.

不失一般性，设完全能观测广义离散随机线性系统为

$$\boldsymbol{x}_1(k)=\phi_1\boldsymbol{x}_1(k-1)+B_1\boldsymbol{w}_{k-1}, \tag{1.1}$$

$$N\boldsymbol{x}_2(k)=\boldsymbol{x}_2(k-1)+B_2\boldsymbol{w}_{k-1}, \tag{1.2}$$

$$\boldsymbol{y}_k=H_1\boldsymbol{x}_1(k)+H_2\boldsymbol{x}_2(k)+\boldsymbol{v}_k. \tag{1.3}$$

诸符号同文献[1].

记

$$\begin{aligned}\boldsymbol{x}_k&=\begin{bmatrix}\boldsymbol{x}_1(k)\\ \boldsymbol{x}_2(k)\end{bmatrix},\quad \phi=\begin{bmatrix}\phi_1^{-1} & 0\\ 0 & N\end{bmatrix},\\ G&=\begin{bmatrix}\phi_1^{-1}B_1\\ B_2\end{bmatrix},\quad H=[H_1,H_2].\end{aligned} \tag{1.4}$$

则系统(1.1)—(1.3)的最优递推滤波器为

$$\begin{aligned}\hat{\boldsymbol{x}}_{k|k}&=P_{k,k}\phi^TP_{k,k-1}^{-1}\hat{\boldsymbol{x}}_{k-1|k-1}+K_k\boldsymbol{y}_k,\\ K_k&=P_{k,k}H^TR_k^{-1},\\ P_{k,k}^{-1}&=\phi^TR_{k,k-1}^{-1}\phi+H^TR_k^{-1}H,\\ P_{k,k-1}&=P_{k-1,k-1}+GQ_{k-1}G^T.\end{aligned} \tag{1.5}$$

这里 $\hat{\boldsymbol{x}}_{k|k}$ 是 $\boldsymbol{x}_k$ 的线性无偏马尔可夫估计，

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$$P_{k,k}=E\{(\boldsymbol{x}_k-\hat{\boldsymbol{x}}_{k|k})(\boldsymbol{x}_k-\hat{\boldsymbol{x}}_{k|k})\}=(D_{k,k}^T\Sigma_{kk}^{-1}D_{k,k})^{-1},$$

$$D_{k,k}=\begin{bmatrix}H\phi^{k-1}\\H\phi^{k-2}\\\vdots\\H\phi\\H\end{bmatrix},\quad \varepsilon_{k,k}=\begin{bmatrix}\boldsymbol{v}_1-\sum_{j=1}^{k-1}H\phi^{j-1}G\boldsymbol{w}_j\\\boldsymbol{v}_2-\sum_{j=2}^{k-1}H\phi^{j-1}G\boldsymbol{w}_j\\\vdots\\\boldsymbol{v}_{k-1}-HG\boldsymbol{w}_{k-1}\\\boldsymbol{v}_k\end{bmatrix},$$

$$\Sigma_{k,k}=E\{\varepsilon_{k,k}\varepsilon_{k,k}^T\}.$$

由文献[1]知，只要(1.1)—(1.3)完全能观测，则 (ϕ, H) 必为能观测对，因而有

$$P_{k,k}>0,\quad P_{k+1,k}>0,\quad \forall k\geqslant n.$$

二、$P_{k,k}$ 的上界

关于 $P_{k,k}$ 的上界有如下定理：

定理 1 设存在正数 α, β, γ, δ, 使对任意 $k\geqslant n$ 都有

$$\alpha I_n\leqslant W(k-n, k-1)\leqslant \beta I_n, \tag{2.1}$$

$$\gamma I_n\leqslant M(k-n+1, k)\leqslant \delta I_n. \tag{2.2}$$

其中

$$W(k-n, k-1)=\sum_{i=k-n}^{k-1}\phi^{i-(k-n)}GQ_iG^T(\phi^T)^{i-(k-n)},$$

$$M(k-n+1, k)=\sum_{i=k-n+1}^{k}(\phi^T)^{k-i}H^TR_i^{-1}H\phi^{k-i}.$$

则对一切 $k\geqslant n$, $P_{k,k}$ 一致有上界，即存在一个正常数 $c>0$ 使

$$P_{k,k}\leqslant cI_n,\quad \forall k\geqslant n. \tag{2.3}$$

证．由(1.1)—(1.4)直接得

$$\boldsymbol{y}_i=H\phi^{k-i}\boldsymbol{x}_k+\boldsymbol{v}_i-\sum_{j=i}^{k-1}H\phi^{j-i}G\boldsymbol{w}_j\quad j=1,2\cdots k-1,$$

记

$$R(k, n)=\mathrm{diag}\{R_{k-n+1}, R_{k-n+2},\cdots R_k\},$$

$$Y_k=\begin{bmatrix}\boldsymbol{y}_{k-n+1}\\\boldsymbol{y}_{k-n+2}\\\vdots\\\boldsymbol{y}_k\end{bmatrix}.$$

取 $\boldsymbol{x}_k$ 的一个如下加权最小二乘估计

$$\hat{\boldsymbol{x}}_{k|k}^*=M^{-1}(k-n+1, k)\sum_{j=k-n+1}^{k}\phi^{k-j}H^TR_j^{-1}\boldsymbol{y}_j. \tag{2.4}$$

将(2.3)式和 $\boldsymbol{y}_k=H\boldsymbol{x}_k+\boldsymbol{v}_k$ 代入上式得

$$\tilde{x}^*_{k|k} = x_k - \hat{x}^*_{k|k} = -M^{-1}(k-n+1, k) \sum_{j=k+1-n}^{k} (\phi^T)^{k-j} H^T R_j^{-1} v_j$$

$$+ M^{-1}(k-n+1, k) \sum_{j=k-n+1}^{k-1} (\phi^T)^{k-j} H^T R_j^{-1} \sum_{i=j}^{k-1} H\phi^{i-j} G w_i. \tag{2.5}$$

由 $E\{x_k\} = E\{\hat{x}^*_{k|k}\}$ 知，$\hat{x}^*_{k|k}$ 是 x_k 的一个线性无偏估计．再由 $\hat{x}^*_{k|k}$ 的最优性得

$$P_{k,k} \leqslant E\{\tilde{x}^*_{k|k}\tilde{x}^T_{k|k}\} = \tilde{P}_{k,k}. \tag{2.6}$$

记

$$C_{k,j} = (\phi^T)^{k-j} H^T R_j^{-1} H, \tag{2.7}$$

$$T_k(n) = \mathrm{cov}\left\{\sum_{j=k-n+1}^{k-1} C_{kj} \sum_{i=j}^{k-1} \phi^{i-j} G w_i\right\}, \tag{2.8}$$

则有

$$\tilde{P}_{k,k} = M^{-1}(k-n+1, k) + M^{-1}(k-n+1, k) T_k(n) M^{-1}(k-n+1, k). \tag{2.9}$$

只要注意到 $\{w_k\}$ 的统计特性，并记 $\mu = \max(j_1, j_2)$，

$$W(\mu, k-1) = \sum_{i=\mu}^{k-1} \phi^{i-\mu} G Q_i G^T (\phi^T)^{i-\mu} = S(k, \mu) S^T(k, \mu),$$

$$g_1(j_1, j_2) = C_{kj_1} \phi^{\mu-j_1} S(k, \mu),$$

$$g_2(j_1, j_2) = S^T(k, \mu)(\phi^T)^{\mu-j_2} C^T_{kj_2},$$

由(2.8)式直接得

$$T_k(n) = \sum_{j_1=k-n+1}^{k-1} \sum_{j_2=k-n+1}^{k-1} g_1(j_1, j_2) g_2(j_1, j_2).$$

显然对任意矢量 z 有

$$\|z\|^2_{T_k(n)} = \sum_{j_1=k-n+1}^{k-1} \sum_{j_2=k-n+1}^{k-1} z^T g_1(j_1, j_2) g_2(j_1, j_2) z$$

$$\leqslant \frac{1}{2} \sum_{j_1=k-n+1}^{k-1} \sum_{j_2=k-n+1}^{k-1} \{\|g_1(j_1, j_2) z\|^2 + \|g_2(j_1, j_2) z\|^2\}$$

$$\leqslant \frac{1}{2} \sum_{j_1=k-n+1}^{k-1} \sum_{j_2=k-n+1}^{k-1} z^T C_{kj_1} \phi^{\mu-j_1} W(\mu, k-1)(\phi^T)^{\mu-j_1} C^T_{kj_1} z$$

$$+ \frac{1}{2} \sum_{j_1=k-n+1}^{k-1} \sum_{j_2=k-n+1}^{k-1} z^T C_{kj_2} \phi^{\mu-j_2} W(\mu, k-1)(\phi^T)^{\mu-j_2} C^T_{kj_2} z$$

$$= \sum_{j_1=k-n+1}^{k-1} \sum_{j_2=k-n+1}^{k-1} z^T C_{kj_1} \phi^{\mu-j_1} W(\mu, k-1)(\phi^T)^{\mu-j_1} C^T_{kj_1} z.$$

只要注意到 $W(\mu, k-1)$ 随 μ 变小而单调增加和题设，由上式直接得

$$\|z\|^2_{T_k(n)} \leqslant \sum_{j_1=k-n+1}^{k-1} \sum_{j_2=k-n+1}^{k-1} z^T C_{kj_1} \phi^{\mu-j_1} W(k-n+1\ k-1)(\phi^T)^{\mu-j_1} C^T_{kj_1} z$$

$$\leqslant \sum_{j_1=k-n+1}^{k-1} \sum_{j_2=k-n+1}^{k-1} \|(\phi^T)\|^{\mu-j_1} C^T_{kj_1} z\|^2_{W(k-n\ k-1)}$$

$$\leqslant \beta \sum_{j_1=k-n+1}^{k-1} \sum_{j_2=k-n+1}^{k-1} \|(\phi^T)^{\mu-j_1}\|^2 \| C_{kj_1}^T z\|^2.$$

由于 ϕ 为常阵，必存在正数 $d > 0$，使

$$\|(\phi^T)^{\mu-j_1}\| \leqslant d, \quad \forall j_1, j_2 \in [k-n+1, k-1],$$

从而有

$$\|z^2\|_{T_k(n)} \leqslant d\beta n \sum_{j=k-n+1}^{k-1} z^T C_{kj} C_{kj}^T z$$

$$= d\beta n \sum_{j=k-n+1}^{k-1} z^T (\phi^T)^{k-j} H^T R_j^{-\frac{1}{2}} (R_j^{-\frac{1}{2}} H H^T R_j^{-\frac{1}{2}}) R_j^{-\frac{1}{2}} H \phi^{k-j} z$$

$$\leqslant d\beta n \sum_{j=k-n+1}^{k-1} z^T (\phi^T)^{k-j} H^T R_j^{-\frac{1}{2}} (\mathrm{tr}\, M(j-n+1, j)) R_j^{-\frac{1}{2}} H \phi^{k-j} z$$

$$\leqslant d\beta n \delta z^T M(k-n+1, k) z.$$

从而得知

$$T_k(n) \leqslant d\beta\delta n M(k-n+1, k). \tag{2.10}$$

将(2.10)式代入(2.9)式得

$$\tilde{P}_{k,k} \leqslant (1 + d\beta\delta n) M^{-1}(k-n+1, k) \leqslant \frac{1 + d\beta\delta_n}{\gamma} I_n.$$

取 $c = \dfrac{1 + d\beta\delta_n}{\gamma}$，由式(2.6)和上式直接得

$$P_{k,k} \leqslant c I_n.$$

三、$P_{k,k}$ 的下界

为讨论 $P_{k,k}$ 的下界，引进(1.1)—(1.3)的共轭系统.

$$x_k^* = \phi^T x_{k-1}^* + H^T \xi_{k-1}, \tag{3.1}$$

$$z_k = G^T x_k^* + \eta_k. \tag{3.2}$$

其中

$$\left.\begin{aligned} &E\{\xi_k\} = 0, \ E\{\eta_k\} = 0, && \forall k \\ &E\{\xi_k \xi_j^T\} = R_k^{-1}\delta_{kj}, \ E\{\eta_k \eta_j^T\} = Q_{k-1}^{-1}\delta_{kj}, && \forall k, j \\ &E\{\eta_k \xi_j^T\} = 0, && \forall k, j \\ &E\{x_0^* \xi_k^T\} = 0, \ E\{x_0^* \eta_k^T\} = 0, && \forall k. \end{aligned}\right\} \tag{3.3}$$

由 Kalman 滤波器理论知道，其最优滤波器为

$$\left.\begin{aligned} &\hat{x}_{k|k}^* = (I - K_k^* G^T)\phi^T \hat{x}_{k-1|k-1}^* + K_k^* z_k, \\ &P_{k,k-1}^* = \phi^T P_{k-1,k-1}^* \phi + H^T R_{k-1}^{-1} H, \\ &P_{k,k}^* = [I - K_k^* G^T] P_{k,k-1}^*, \\ &K_k^* = P_{k,k-1}^* G (G^T P_{k,k-1}^* G + Q_{k-1}^{-1})^{-1}. \end{aligned}\right\} \tag{3.4}$$

令

$$M^*(k-n, k-1)=\sum_{j=k-n}^{k-1} \phi^{j-(k-n)} G Q_j G^T (\phi^T)^{j-(k-n)},$$

$$W^*(k-n+1, k)=\sum_{j=k-n+1}^{k} (\phi^T)^{k-j} H^T R_j^{-1} H \phi^{k-j},$$

显然,如果式(2.1),(2.2)成立,则必有

$$\alpha I_n \leqslant M^*(k-n, k-1) \leqslant \beta I_n, \tag{3.5}$$

$$\gamma I_n \leqslant W^*(k-n+1, k) \leqslant \delta I_n, \tag{3.6}$$

关于 $P_{k,k}$ 的下界有如下定理:

定理 2. 设(2.1),(2.2)式成立,则对一切 $k \geqslant 2n$, $P_{k,k}$ 一致有下界,即存在一常数 $b>0$,使

$$b I_n \leqslant P_{k,k}.$$

证. 记

$$D_k^*(n)=[G, \phi G, \cdots \phi^{n-1} G],$$

$$G_k(n)=\operatorname{diag}\{G^T, G^T, \cdots G^T\}\begin{bmatrix} 0 & \cdots\cdots & & 0 & 0 \\ H^T & \ddots & & \vdots & \vdots \\ \phi^T H^T & & \ddots & \vdots & \vdots \\ \vdots & \ddots & & 0 & \vdots \\ (\phi^T)^{n-2} H^T & \cdots & \phi^T H^T & H^T & 0 \end{bmatrix},$$

$$\boldsymbol{z}_k(n)=\begin{bmatrix} \boldsymbol{z}(k-n) \\ \boldsymbol{z}(k-n+1) \\ \vdots \\ \boldsymbol{z}(k-1) \end{bmatrix}, \quad \boldsymbol{\xi}_k(n)=\begin{bmatrix} \boldsymbol{\xi}(k-n) \\ \boldsymbol{\xi}(k-n+1) \\ \vdots \\ \boldsymbol{\xi}(k-1) \end{bmatrix},$$

$$\boldsymbol{\eta}_k(n)=\begin{bmatrix} \boldsymbol{\eta}(k-n) \\ \boldsymbol{\eta}(k-n+1) \\ \vdots \\ \boldsymbol{\eta}(k-1) \end{bmatrix}, \quad Q_k^*(n)=\operatorname{diag}\{Q(k-n-1), Q(k-n-2) \cdots Q(k-2)\},$$

$$R_k^*(n)=\operatorname{diag}\{R(k-n), R(k-n-1) \cdots R(k-1)\},$$

$$F_k(n)=[(\phi^T)^{n-1} H^T, (\phi^T)^{n-2} H^T, \cdots \phi^T H^T, H^T].$$

由(3.1),(3.2)式直接得

$$\boldsymbol{x}_k^*=(\phi^T)^n \boldsymbol{x}_{k-n}^*+F_k(n) \boldsymbol{\xi}_k(n), \tag{3.7}$$

$$\boldsymbol{z}_k(n)=D_k^{*T}(n) \boldsymbol{x}_{k-n}^*+\boldsymbol{\eta}_k(n)+G_k(n) \boldsymbol{\xi}_k(n). \tag{3.8}$$

如果记

$$B_k(n)=(\phi^T)^n M^{*^{-1}}(k-n, k-1) D_k^*(n) Q_k^*(n),$$

由 $M^*(k-n, k-1)=D_k^*(n) Q_k^*(n) D_k^{*T}(n)$ 和(3.8)式直接得

$$\hat{\boldsymbol{x}}_k^* \triangleq (\phi^T)^n M^{*^{-1}}(k-n, k-1) D_k^*(n) Q_k^*(n) \boldsymbol{z}_k(n)$$

$$= (\phi^T)^n x_{k-n}^* + B_k(n)\eta_k(n) + B_k(n)G_k(n)\xi_k(n).$$

由(3.7)式和上式得

$$\tilde{x}_k^* \triangleq x_k^* - \hat{x}_k^* = [F_k(n) - B_k(n)G_k(n)]\xi_k(n) - B_k(n)\eta_k(n).$$

显然，$\hat{x}_k^*$ 是 x_k^* 的一个无偏估计．由上式且利用 $\hat{x}_{k|k}^*$ 的最优性直接得

$$\begin{aligned} P_{k,k}^* \leqslant P_k^* \triangleq E\{\tilde{x}_k^*\tilde{x}_k^{*T}\} &= F_k(n)R_k^{*^{-1}}(n)F_k^T(n) + B_k(n)Q_k^{*^{-1}}(n)B_k^T(n) \\ &+ B_k(n)G_k(n)R_k^{*^{-1}}(n)G_k^T(n)B_k^T(n) - B_k(n)G_k(n)R_k^{*^{-1}}(n)F_k^T(n) \\ &- F_k(n)R_k^{*^{-1}}(n)G_k^T(n)B_k^T(n). \end{aligned}$$

只要注意到，

$$\begin{aligned} &F_k(n)R_k^{*^{-1}}(n)G_k^T(n)B_k^T(n) + B_k(n)G_k(n)R_k^{*^{-1}}(n)F_k^T(n) \\ &\leqslant F_k(n)R_k^{*^{-1}}(n)F_k^T(n) + B_k(n)G_k(n)R_k^{*^{-1}}(n)F_k^T(n), \end{aligned}$$

从而有

$$\begin{aligned} P_{k,k}^* \leqslant\ & 2W^*(k-n, k-1) + (\phi^T)^n M^{*^{-1}}(k-n, k-1)\phi^n \\ &+ 2B_k(n)G_k(n)R_k^{*^{-1}}(n)G_k^T(n)B_k^T(n). \end{aligned} \tag{3.9}$$

已知

$$\begin{aligned} &B_k(n)G_k(n)R_k^{*^{-1}}(n)G_k^T(n)B_k^T(n) \\ &\quad = (\phi^T)^n M^{*^{-1}}(k-n, k-1)\operatorname{cov}(D_k^*(n)Q_k^*(n)G_k(n)\xi_k(n)) \\ &\quad \cdot M^{*^{-1}}(k-n, k-1)\phi^n, \\ &D_k^*(n)Q_k^*(n)G_k(n)\xi_k(n) = \sum_{j=k-n+1}^{k-1} \bar{C}_{kj} \sum_{i=k-n}^{j-1} (\phi^T)^{j-1-i}H^T\xi_i, \\ &\bar{C}_{kj} = \phi^{j-(k-n)}GQ_jG^T, \end{aligned} \tag{3.10}$$

由 $\{\xi_k\}$ 的统计特性易知

$$\begin{aligned} T_k^*(n) &\triangleq \operatorname{Cov}(D_k^*(n)Q_k^*(n)G_k(n)\xi_k(n)) \\ &= \sum_{j_1=k-n+1}^{k-1} \sum_{j_2=k-n+1}^{k-1} \bar{C}_{kj}(\phi^T)^{j_1-\bar{\mu}} \sum_{i=k-n}^{\bar{\mu}-1} (\phi^T)^{\bar{\mu}-1-i}H^TR_i^{-1}H\phi^{\bar{\mu}-1-i}\phi^{j_2-\bar{\mu}}\bar{C}_{kj_2}^T, \end{aligned}$$

其中 $\bar{\mu} = \min(j_1, j_2)$.

记

$$\begin{aligned} &\sum_{i=k-n}^{\bar{\mu}-1} (\phi^T)^{\bar{\mu}-1-i}H^TR_i^{-1}H\phi^{\bar{\mu}-1-i} = W^*(k-n, \bar{\mu}-1) = \bar{S}(k, \bar{\mu}-1)\cdot \\ &\qquad \bar{S}^T(k, \bar{\mu}-1), \\ &\bar{g}_1(j_1, j_2) = \bar{C}_{kj_1}(\phi^T)^{j_1-\bar{\mu}}\bar{S}(k, \bar{\mu}-1), \\ &\bar{g}_2(j_1, j_2) = \bar{S}^T(k, \bar{\mu}-1)\phi^{j_2-\bar{\mu}}\bar{C}_{kj_2}^T, \end{aligned}$$

则

$$T_k^*(n) = \sum_{j_1=k-n+1}^{k-1} \sum_{j_2=k-n+1}^{k-1} \bar{g}_1(j_1, j_2)\bar{g}_2(j_1, j_2).$$

用类似于第二节所用的方法，且注意到 $k \geqslant 2n$，可得

$$T_k^*(n) \leqslant (n-1)n^2\delta\beta M^*(k-n, k-1).$$

将上式代入(3.10)式中得

$$B_k(n)G_k(n)R_k^{*^{-1}}(n)G_k^T(n)B_k^T(n) \leqslant (n-1)n^2\delta\beta(\phi^T)^m M^{*^{-1}}(k^{-n}, k-1)$$
$$\leqslant \frac{(n-1)n^3\delta\beta}{\alpha}\operatorname{tr}[(\phi^T)^m\phi^m]I_n. \tag{3.11}$$

当 $k \geqslant 2n$ 时，由(3.5)，(3.6)式得

$$W^*(k-n, k-1) \leqslant \delta I_n,$$
$$(\phi^T)^m M^{*^{-1}}(k-n, k-1)\phi^m \leqslant \frac{n}{\alpha}\operatorname{tr}[(\phi^T)^m\phi^m]I_n.$$

从(3.9)和(3.11)式得知，必存在一 $q > 0$，使

$$P_{k,k}^* \leqslant qI_n. \tag{3.12}$$

从(3.4)式易知，只要取 $P_{0,6}^* > 0$，且注意到 $\operatorname{Rank}[\phi^T, H^T] = n$，则对一切 $k \geqslant 0$ 都有 $P_{k,k}^* > 0$，$P_{k,k-1}^* > 0$。因而得

$$P_{k\bar{k}}^{*^1} = P_{k,k-1}^{*^{-1}} + GQ_{k-1}G^{-1}. \tag{3.13}$$

由(1.5)，(3.4)和(3.12)式得

$$P_{k,k}^{-1} = \phi^T P_{k,k-1}^{-1}\phi + H^T R_k^{-1} H, \quad P_{k+1,k}^* = \phi^T P_{k,k}^*\phi + H^T R_k^{-1} H,$$
$$P_{k,k-1} = P_{k-1,k-1} + GQ_{k-1}G^T, \quad P_{k,k}^{*^{-1}} = P_{k,k-1}^{*^{-1}} + GQ_{k-1}G^T.$$

于是有

$$(P_{k,k}^{-1} - P_{k+1,k}^*) = \phi^T(P_{k,k-1}^{-1} - P_{k,k}^*)\phi,$$
$$(P_{k,k-1} - P_{k,k}^{*^{-1}}) = (P_{k-1,k-1} - P_{k,k-1}^{*^{-1}}).$$

只要取 $P_{0,0} = P_{10}^{*^{-1}} = [\phi^T P_{0,0}^*\phi + H^T R_0 H]$，则对一切 $k \geqslant 0$ 有

$$P_{k,k}^{-1} = P_{k+1,k}^*, \quad P_{k,k}^* = P_{k,k-1}^{-1}. \tag{3.14}$$

由于 ϕ 为常阵，则由(3.4)和(3.12)式可知，存在常数 $b > 0$，使

$$P_{k+1,k}^* \leqslant b^{-1}I_n.$$

从而有

$$bI_n \leqslant P_{k+1,k}^{*-1} = P_{k,k}.$$

四、最优滤波器的稳定性

为讨论最优滤波器的稳定性，我们考察最优递推方程(1.5)的齐次方程

$$\hat{x}_{k|k} = \Psi_{k,k-1}\hat{x}_{k-1|k-1}, \tag{4.1}$$

其中

$$\Psi_{k,k-1} = P_{k,k}\phi^T P_{k,k-1}^{-1}. \tag{4.2}$$

众所周知，方程(4.1)大范围一致渐近稳定的充要条件是，存在 $\mu > 0$，$\lambda > 0$，使得对任意的 $k \geqslant j$ 皆有

$$\|\Psi_{k,j}\| \leqslant \mu e^{-\lambda(k-j)}.$$

为了证明式(4.1)的稳定性，我们先讨论式(3.4)的齐次方程

$$\hat{x}_{k|k}^* = \Psi_{k,k-1}^*\hat{x}_{k-1|k-1}^*, \tag{4.3}$$

其中

$$\Psi_{k,k-1}^* = [I - K_k^* G^T]\phi^T.$$

仍设 $P_{0,0} > 0$，由式(3.4)和(3.14)直接得

$$\Psi^*_{k,k-1} = P^*_{k,k} P^{*-1}_{k,k-1} \phi^T = P^{-1}_{k,k-1} P_{k-1,k-1} \phi^T.$$

从而有

$$\Psi_{k,j} = P_{k+1,k} \Psi^*_{k+1,j+1} P^{-1}_{j+1,j}.$$

依第二、三两节证明过程知，当 $k \geqslant j \geqslant 2n$ 时，必有

$$P_{k+1,k} \leqslant \alpha_0 I_n, \quad P^{-1}_{j+1,j} \leqslant \beta_0 I_n.$$

其中 $\alpha_0 > 0$，$\beta_0 > 0$，综上所述可得

$$\|\Psi_{k,j}\| \leqslant \alpha_0 \beta_0 \|\Psi^*_{k+1,j+1}\|.$$

由此可知，如果(4.3)式是大范围一致渐近稳定的，必存在 $\mu_0 > 0$，$\lambda > 0$ 使

$$\|\Psi^*_{k+1,j+1}\| \leqslant \mu_0 e^{-\lambda(k-j)}, \quad \forall k \geqslant j. \tag{4.4}$$

于是有

定理 3. 设对任意 $k \geqslant n$，式(2.1)，(2.2)成立． 则式(4.1)必是大范围一致渐近稳定的．

证．依题设并用文献[2]所提供的方法，可证明式(4.3)是大范围一致渐近稳定的，从而式(4.4)成立．只要取 $\mu = \mu_0 \alpha_0 \beta_0$，得

$$\|\Psi_{k,j}\| \leqslant \mu e^{-\lambda(k-j)}, \quad k \geqslant j \geqslant 2n.$$

即式(4.1)是大范围一致渐近稳定的．

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OPTIMUM RECURRENCE FILTERING METHOD FOR SINGULAR DISCRETE STOCHASTIC LINEAR SYSTEMS (II)

Wang Enping　　Wang Chaozhu

(Institute of Systems Science, Academia Sinica)

Abstract

In this paper, properties of the error covariance matrix of the optimal estimate given in [1] are analyzed for singular discrete stochastic linear systems. Under some conditions, upper and lower bounds of the matrix have been obtained. In view of the above results, the stability of the optimal filter in [1] has been discussed.