第 14 卷 第 6 期　　自 动 化 学 报　　Vol. 14, No. 6
1988 年 11 月　　ACTA AUTOMATICA SINICA　　Nov., 1988

具有分离变量的非线性变系数系统的全局稳定性

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摘　　要

本文应用大系统的分解理论讨论了具有分离变量的非线性变系数系统的全局稳定性。所使用的方法比较简单,与以往方法有所不同.

在自动控制理论中常常碰到非线性系统的稳定性问题. 文献[1, 2]讨论了具有两个非线性项的常系数二阶系统. 本文对含有 n^2 个非线性项且具有分离变量的变系数系统进行了讨论,得到了其零解全局稳定的充分条件.

考虑非线性大系统

$$\begin{cases} \dfrac{dx_i}{dt} = -a_{ii}(t)f_{ii}(x_i) + \sum\limits_{\substack{j=1\\ j\neq i}}^{n} a_{ij}(t)f_{ij}(x_j), \\ f_{ij}(0) = 0 \quad (i, j = 1, 2, \cdots, n), \end{cases} \tag{1}$$

假定系统(1)满足条件:

(i) $a_{ij}(t)(i, j = 1, 2, \cdots, n)$ 连续有界,

$$M = \max_{t_0 \leqslant t < \infty}[|a_{ij}(t)|, i, j = 1, 2, \cdots, n.\ i \neq j],$$

且有 $a_{ii}(t) \geqslant \alpha_i > 0 (i = 1, 2, \cdots, n)$ 对一切 $t \geqslant t_0$ 成立;

(ii) $f_{ij}(t)$ 单值连续, $x_i \cdot f_{ii}(x_i) > 0 (x_i \neq 0)$, 且有

$$|f_{i1}(x_1)| \leqslant |f_{11}(x_1)|, |f_{i2}(x_2)| \leqslant |f_{22}(x_2)|, \cdots, |f_{in}(x_n)| \leqslant |f_{nn}(x_n)|.$$

取式(1)的含 n 个子系统的系统

$$\begin{cases} \dfrac{dx_1}{dt} = -a_{11}(t)f_{11}(x_1), \\ \dfrac{dx_2}{dt} = -a_{22}(t)f_{22}(x_2), \\ \cdots\cdots\cdots, \\ \dfrac{dx_n}{dt} = -a_{nn}(t)f_{nn}(x_n). \end{cases} \tag{2}$$

本文于 1986 年 7 月 14 日收到.

对(2)式的每个子系统作李雅普诺夫函数

$$V_1(x_1)=2\int_0^{x_1}f_{11}(x_1)dx_1,\ V_2(x_2)=2\int_0^{x_2}f_{22}(x_2)dx_2,\cdots,\ V_n(x_n)=2\int_0^{x_n}f_{nn}(x_n)dx_n.$$

则有

$$\left.\frac{dV_i}{dt}\right|_{(2)}=-2a_{ii}(t)f_{ii}^2(x_i)\leqslant-2\alpha_i f_{ii}^2(x_i)<0.$$

又由已知条件 $x_i\cdot f_{ii}(x_i)>0$，可知 $V_i(i=1,2,\cdots,n)$ 是正定的． 所以系统(2)的零解是渐近稳定的．

取正定函数

$$\begin{aligned}V&=V_1(x_1)+V_2(x_2)+\cdots+V_n(x_n)\\&=2\left(\int_0^{x_1}f_{11}(x_1)dx_1+\int_0^{x_2}f_{22}(x_2)dx_2+\cdots+\int_0^{x_n}f_{nn}(x_n)dx_n\right)\end{aligned}$$

作为非线性系统(1)的李雅普诺夫函数,则有

$$\begin{aligned}\left.\frac{dV}{dt}\right|_{(1)}&=2f_{11}(x_1)\frac{dx_1}{dt}+2f_{22}(x_2)\frac{dx_2}{dt}+\cdots+2f_{nn}(x_n)\frac{dx_n}{dt}\\
&=2f_{11}(x_1)(-a_{11}(t)f_{11}(x_1)+a_{12}(t)f_{12}(x_2)+\cdots+a_{1n}(t)f_{1n}(x_n))\\
&\quad+2f_{22}(x_2)(a_{21}(t)f_{21}(x_1)-a_{22}(t)f_{22}(x_2)+\cdots+a_{2n}(t)f_{2n}(x_n))\\
&\quad+\cdots\cdots\\
&\quad+2f_{nn}(x_n)(a_{n1}(t)f_{n1}(x_1)+a_{n2}(t)f_{n2}(x_2)+\cdots-a_{nn}(t)f_{nn}(x_n))\\
&\leqslant-2(\alpha_1f_{11}^2(x_1)+\alpha_2f_{22}^2(x_2)+\cdots+\alpha_nf_{nn}^2(x_n))\\
&\quad+2|a_{12}(t)||f_{11}(x_1)f_{12}(x_2)|+\cdots+2|a_{1n}(t)||f_{11}(x_1)f_{1n}(x_n)|\\
&\quad+2|a_{21}(t)||f_{22}(x_2)f_{21}(x_1)|+2|a_{23}(t)||f_{22}(x_2)f_{23}(x_3)|+\cdots+2|a_{2n}(t)||f_{22}(x_2)f_{2n}(x_n)|\\
&\quad+\cdots\\
&\quad+2|a_{n1}(t)||f_{nn}(x_n)f_{n1}(x_1)|+2|a_{n2}(t)||f_{nn}(x_n)f_{n2}(x_2)|+\cdots\\
&\quad+2|a_{nn-1}(t)||f_{nn}(x_n)f_{nn-1}(x_{n-1})|\\
&\leqslant-2(\alpha_1f_{11}^2(x_1)+\alpha_2f_{22}^2(x_2)+\cdots+\alpha_nf_{nn}^2(x_n))\\
&\quad+M(2|f_{11}(x_1)||f_{22}(x_2)|+\cdots+2|f_{11}(x_1)||f_{nn}(x_n)|)\\
&\quad+M(2|f_{22}(x_2)||f_{11}(x_1)|+2|f_{22}(x_2)||f_{33}(x_3)|+\cdots+2|f_{22}(x_2)||f_{nn}(x_n)|)\\
&\quad+\cdots\\
&\quad+M(2|f_{nn}(x_n)||f_{11}(x_1)|+2|f_{nn}(x_n)||f_{22}(x_2)|+\cdots+2|f_{nn}(x_n)||f_{n-1n-1}(x_{n-1})|)\\
&\leqslant-2(\alpha_1f_{11}^2(x_1)+\alpha_2f_{22}^2(x_2)+\cdots+\alpha_nf_{nn}^2(x_n))\\
&\quad+M\left[\frac{1}{\sqrt{\alpha_1\alpha_2}}(\alpha_1f_{11}^2(x_1)+\alpha_2f_{22}^2(x_2))+\cdots+\frac{1}{\sqrt{\alpha_1\alpha_n}}(\alpha_1f_{11}^2(x_1)+\alpha_nf_{nn}^2(x_n))\right]\\
&\quad+M\left[\frac{1}{\sqrt{\alpha_2\alpha_1}}(\alpha_1f_{11}^2(x_1)+\alpha_2f_{22}^2(x_2))+\frac{1}{\sqrt{\alpha_2\alpha_3}}(\alpha_2f_{22}^2(x_2)+\alpha_3f_{33}^2(x_3))+\cdots\right.\\
&\quad\left.+\frac{1}{\sqrt{\alpha_2\alpha_n}}(\alpha_2f_{22}^2(x_2)+\alpha_nf_{nn}^2(x_n))\right]\\
&\quad+\cdots\cdots\end{aligned}$$

$$
\begin{aligned}
&+M\left[\frac{1}{\sqrt{\alpha_n\alpha_1}}\left(\alpha_1 f_{11}^2(x_1)+\alpha_n f_{nn}^2(x_n)\right)+\cdots\right.\\
&\left.+\frac{1}{\sqrt{\alpha_n\alpha_{n-1}}}\left(\alpha_{n-1} f_{n-1n-1}^2(x_{n-1})+\alpha_n f_{nn}^2(x_n)\right)\right]\\
=&-2\left(\alpha_1 f_{11}^2(x_1)+\alpha_2 f_{22}^2(x_2)+\cdots+\alpha_n f_{nn}^2(x_n)\right)\\
&+M\left[2(n-1)\alpha_1 f_{11}^2(x_1)\left(\frac{1}{\sqrt{\alpha_1\alpha_2}}+\frac{1}{\sqrt{\alpha_1\alpha_3}}+\cdots+\frac{1}{\sqrt{\alpha_1\alpha_n}}\right)\right.\\
&+2(n-1)\,\alpha_2 f_{22}^2\,(x_2)\left(\frac{1}{\sqrt{\alpha_2\alpha_1}}+\frac{1}{\sqrt{\alpha_2\alpha_3}}+\cdots+\frac{1}{\sqrt{\alpha_2\alpha_n}}\right)\\
&+\cdots\cdots\\
&\left.+2(n-1)\alpha_n f_{nn}^2(x_n)\left(\frac{1}{\sqrt{\alpha_n\alpha_1}}+\frac{1}{\sqrt{\alpha_n\alpha_2}}+\cdots+\frac{1}{\sqrt{\alpha_n\alpha_{n-1}}}\right)\right]\\
=&\left[2(n-1)M\left(\frac{1}{\sqrt{\alpha_1\alpha_2}}+\frac{1}{\sqrt{\alpha_1\alpha_3}}+\cdots+\frac{1}{\sqrt{\alpha_1\alpha_n}}\right)-2\right]\alpha_1 f_{11}^2(x_1)\\
&+\left[2(n-1)M\left(\frac{1}{\sqrt{\alpha_2\alpha_1}}+\frac{1}{\sqrt{\alpha_2\alpha_3}}+\cdots+\frac{1}{\sqrt{\alpha_2\alpha_n}}\right)-2\right]\alpha_2 f_{22}^2(x_2)\\
&+\cdots\cdots\\
&+\left[2(n-1)M\left(\frac{1}{\sqrt{\alpha_n\alpha_1}}+\frac{1}{\sqrt{\alpha_n\alpha_2}}+\cdots+\frac{1}{\sqrt{\alpha_n\alpha_{n-1}}}\right)-2\right]\alpha_n f_{nn}^2(x_n)\\
&<0,
\end{aligned}
$$

当

$$
\left.\begin{aligned}
&(n-1)M\left(\frac{1}{\sqrt{\alpha_1\alpha_2}}+\frac{1}{\sqrt{\alpha_1\alpha_3}}+\cdots+\frac{1}{\sqrt{\alpha_1\alpha_n}}\right)<1\\
&(n-1)M\left(\frac{1}{\sqrt{\alpha_2\alpha_1}}+\frac{1}{\sqrt{\alpha_2\alpha_3}}+\cdots+\frac{1}{\sqrt{\alpha_2\alpha_n}}\right)<1\\
&\cdots\cdots\\
&(n-1)M\left(\frac{1}{\sqrt{\alpha_n\alpha_1}}+\frac{1}{\sqrt{\alpha_n\alpha_2}}+\cdots+\frac{1}{\sqrt{\alpha_n\alpha_{n-1}}}\right)<1
\end{aligned}\right\}\tag{3}
$$

时. 因而由全局稳定性判断定理得到下面的结果:

定理. 对非线性系统(1),如果满足上面的条件 (i) 和 (ii), 同时(3)式成立, 并且有

$$
\lim_{|x_i|\to+\infty}\int_0^{x_i} f_{ii}(x_i)dx_i=+\infty,\quad \lim_{|x_i|\to 0}\int_0^{x_i} f_{ii}(x_i)dx_i=0,
$$
$$
(i=1,2,\cdots,n)
$$

则非线性系统(1)的零解是全局稳定的.

参 考 文 献

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[2] Красовский Н. Н., об устойчивости решений системы двух дифференциальных уравнений, ПММ, т. **17** (1953). вып. 6.

THE GLOBLE STABILITY OF NON-LINEAR VARIABLE COEFFICIENT SYSTEMS WITH SEPARATE VARIABLES

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Abstract

In this paper, the decomposition theory of large scale systems is used to discuss the globle stability of non-linear variable coefficient systems with separate variables. The method used is simpler than and different from the previous methods.