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非完整链式系统的时变光滑指数镇定[1)]

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摘 要 针对非完整链式系统，导出了一种时变光滑反馈控制律，该控制律可以保证系统状态全局指数收敛到原点，克服了以往控制律非光滑或虽光滑但却只能渐近镇定系统的缺陷. 所得控制律应用于移动机器人系统的镇定.

关键词 非完整链式系统，时变光滑镇定，移动机器人
中图分类号 TP13

Smooth Time-Varying Exponential Stabilization of Nonholonomic Chained Systems

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Abstract A smooth time-varying control law is derived to stabilize nonholonomic chained systems. The proposed control law guarantees all the system states exponentially converge to the origin, and thus it is superior to the ones which stabilize the chained systems by either non-smooth exponential or smooth asymptotical control laws. The proposed control law is used to stabilize a mobile robot.

Key words Nonholonomic chained system, time-varying smooth stabilization, mobile robots

1 引言

由于可控的非完整系统不可由光滑纯状态反馈镇定，其控制问题引起了控制理论界的极大重视. 鉴于大多数实际的受一阶非完整约束的系统都可通过状态和输入变换转化为链式标准型，所以链式系统的镇定问题得到了较为深入的研究[1~8]. 现有的链式系统的镇定算法可主要归结为两类：一类为非光滑纯状态反馈控制[4~8]，其优点是可保证状态的指数收敛，缺点是要求系统的初始状态满足一定的条件，否则就要先将状态在有限时间内驱动到满

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足条件的区域,这就导致整个控制的不连续或非光滑;另一类镇定算法为时变光滑状态反馈控制[1~3],它虽可保证控制律是连续光滑的,但却不能保证系统状态的指数收敛.为克服以上两类算法的缺陷并利用它们的优点,本文给出了一种时变光滑反馈控制律,可以保证系统状态全局指数收敛到原点.

2 链式系统的时变光滑指数镇定

考虑非完整单链系统

$$\dot{x}_1 = u_1,\ \dot{x}_2 = u_2,\ \dot{x}_i = x_{i-1}u_1 \quad (i = 3,4,\cdots,n) \tag{1}$$

其中 $\boldsymbol{x}=(x_1,x_2\cdots,x_n)^{\mathrm{T}}$为系统状态,$\boldsymbol{u}=(u_1,u_2)^{\mathrm{T}}$为控制输入.

系统(1)的光滑指数反馈镇定问题可叙述为:设计时变光滑反馈控制律 $\boldsymbol{u}=\boldsymbol{u}(x,t)$使系统(1)的所有状态指数收敛到原点.为证明本文的主要定理,需要以下引理.

引理 1[9]. 考虑如下线性时变系统

$$\dot{\boldsymbol{x}} = (A_1 + A_2(t))\boldsymbol{x} \tag{2}$$

若 A_1为稳定阵,$A_2(t)$满足 1) $\lim\limits_{t\to\infty}A_2(t)=0$; 2) $\int_0^\infty \| A_2(t) \| \mathrm{d}t<\infty$. 则线性时变系统(2)全局指数稳定.下面给出本文的主要定理.

定理 1. 对链式系统(1),如下控制

$$u_1 = -k_1x_1 + \alpha(t),\quad u_2 = -\sum_{i=2}^{n}k_{2,i}\bar{x}_i = -\sum_{i=2}^{n}k_{2,i}\frac{x_i}{\alpha^{i-2}} \tag{3}$$

可以保证系统状态全局指数收敛到原点.其中

$$\alpha = \alpha_0 e^{-\lambda t}(\alpha_0 \neq 0),\quad \bar{x}_i = \frac{x_i}{\alpha^{i-2}}(i = 2,3,\cdots,n),\quad k_1 > \lambda > 0,\ c = -\frac{\lambda}{k_1} \tag{4}$$

反馈系数 $k_{2,i}(i=2,3\cdots,n)$使得如下矩阵稳定.

$$A_{1k} = \begin{bmatrix} -k_{2,2} & -k_{2,3} & -k_{2,4} & \cdots & -k_{2,n-1} & -k_{2n} \\ c & \lambda & 0 & \cdots & 0 & 0 \\ 0 & c & 2\lambda & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \ddots & (n-3)\lambda & 0 \\ 0 & 0 & 0 & \cdots & c & (n-2)\lambda \end{bmatrix} \tag{5}$$

证明. 将式(3)代入式(1)的第 1 式得到

$$\dot{x}_1 = -k_1x_1 + \alpha \tag{6}$$

由于 $k_1>0$ 且 $\alpha(t)$指数收敛于零,所以 $x_1(t)$指数收敛到零.令 $\bar{x}_1=x_1/\alpha$,则有

$$\dot{\bar{x}}_1 = -(k_1 - \lambda)\bar{x}_1 + 1 = -\bar{k}_1\bar{x}_1 + 1 \quad (\bar{k}_1 = k_1 - \lambda) \tag{7}$$

由于 $\bar{k}_1=k_1-\lambda>0$, 因此 $\bar{x}_1$指数收敛于$\frac{1}{\bar{k}_1}$.

再由 $u_1=-k_1x_1+\alpha=-k_1\alpha\bar{x}_1+\alpha$ 知,$\frac{u_1}{\alpha}=1-k_1\bar{x}_1$指数收敛于 $c=1-\frac{k_1}{\bar{k}_1}=-\frac{\lambda}{\bar{k}_1}$,即$\frac{u_1}{\alpha}-c$ 指数收敛到零.对 $\bar{x}_i=\frac{x_i}{\alpha^{i-2}}(i=2,3,\cdots,n)$两边微分并利用式(1)和式(3)可以导出

$$\dot{\bar{x}}_2 = -\sum_{i=2}^{n}k_{2,i}\bar{x}_i,\ \dot{\bar{x}}_i = \left(\frac{u_1}{\alpha} - c\right)\bar{x}_{i-1} + c\bar{x}_{i-1} + (i-2)\lambda\bar{x}_i(i = 3,4,\cdots,n) \tag{8}$$

令 $y=[\bar{x}_2,\bar{x}_3,\cdots,\bar{x}_n]^{\mathrm{T}}$，则式(8)又可写为

$$\dot{y}=(A_1-BK+A_2(t))y \tag{9}$$

其中

$$A_1=\begin{bmatrix}0 & 0 & 0 & \cdots & 0 & 0\\ c & \lambda & 0 & \cdots & 0 & 0\\ 0 & c & 2\lambda & \cdots & 0 & 0\\ \vdots & \vdots & \ddots & \ddots & \vdots & \vdots\\ 0 & 0 & 0 & \ddots & (n-3)\lambda & 0\\ 0 & 0 & 0 & \cdots & c & (n-2)\lambda\end{bmatrix},\quad B=[1\quad 0\quad\cdots\quad 0]^{\mathrm{T}},$$

$$K=[k_{2,2}\quad k_{2,3}\quad\cdots\quad k_{2,n}],\quad A_2(t)=\begin{bmatrix}0 & 0 & 0 & \cdots & 0 & 0\\ \frac{u_1}{\alpha}-c & 0 & 0 & \cdots & 0 & 0\\ 0 & \frac{u_1}{\alpha}-c & 0 & \cdots & 0 & 0\\ \vdots & \vdots & \ddots & \ddots & \vdots & \vdots\\ 0 & 0 & 0 & \frac{u_1}{\alpha}-c & 0 & 0\\ 0 & 0 & 0 & 0 & \frac{u_1}{\alpha}-c & 0\end{bmatrix}.$$

容易证明(A_1,B)可控，所以可选择 $K=[k_{2,2}\quad k_{2,3}\quad\cdots\quad k_{2,n}]$使得 $A_1-BK=A_{1k}$为稳定阵.另外，前已证明 $u_1/\alpha-c$ 指数趋于零，所以 $A_2(t)$满足引理 1 中的条件，从而系统(9)全局指数稳定，即 $\bar{x}_i(i=2,3,\cdots,n)$指数收敛于零.再由 $x_i=\alpha^{i-2}\bar{x}$ $(i=2,3,\cdots,n)$知，状态 $x_i(i=2,3,\cdots,n)$也指数收敛到零.

综上可知，控制律(3)和(4)可以保证系统(1)的所有状态 $x_i(i=1,2,\cdots,n)$指数收敛到零.

注 1. 控制律(3)和(4)中均出现时变函数 $\alpha(t)$，所以 u_1和 u_2均为时变状态反馈.又因为 $\alpha(t)\neq 0(\forall t\in[0,\infty))$，所以控制律(3)和(4)连续光滑，且计算量小，易于在实际中应用.

注 2. 和文献[4～8]中的状态反馈控制律相比，本文的控制律对系统初始状态无任何限制，因而也不需要先将系统状态驱动到某个区域，克服了纯状态反馈控制律非光滑的缺陷，其代价是控制 u_1,u_2均是时变的.

注 3. 文献[1～3]中的时变光滑控制律只能保证系统状态渐近收敛到原点，而本文给出的时变光滑控制律可保证系统状态指数收敛到原点.

3　移动小车系统的镇定

考虑移动小车的运动学简化模型

$$\dot{y}_1=v_1\cos\theta,\quad \dot{y}_2=v_1\sin\theta,\quad \dot{\theta}=v_2 \tag{10}$$

通过以下的状态和输入变换

$$x_1=\theta,\ x_2=x\cos\theta+y\sin\theta,\ x_3=y_1\sin\theta-y_2\cos\theta,\ u_1=v_2,\ u_2=v_1-v_2x_3 \tag{11}$$

可将式(10)转化为以下链式形式

$$\dot{x}_1 = u_1,\quad \dot{x}_2 = u_2,\quad \dot{x}_3 = x_2 u_1 \tag{12}$$

应用定理 1 可知,以下控制律

$$u_1 = -k_1 x_1 + \alpha,\ u_2 = -k_{2,2}x_2 - k_{2,3}\frac{x_3}{\alpha} \tag{13}$$

可保证系统(12)的状态全局指数收敛到原点,其中 $\alpha=\alpha_0 e^{-\lambda t}$, $\alpha_0\neq 0$, $k_1>\lambda>0$, $k_{2,2}$,$k_{2,3}$应使矩阵 $A_{1k}=\begin{bmatrix}-k_{2,2} & -k_{2,3}\\ c & \lambda\end{bmatrix}$稳定,即使得 A_{1k}的特征多项式 $f(s)=s^2+(k_{2,2}-\lambda)s+(ck_{2,3}-\lambda k_{2,2})$为稳定多项式.

令 $s_1>0$, $s_2>0$ 为两个正实常数,则取 $k_{2,2}$和 $k_{2,3}$满足 $k_{2,2}-\lambda=s_1+s_2$和 $ck_{2,3}-\lambda k_{2,2}=s_1 s_2$,即 $k_{2,2}=s_1+s_2+\lambda$ 和 $k_{2,3}=\dfrac{s_1 s_2+(s_1+s_2+\lambda)}{c}$时,多项式 $f(s)$具有负实根$-s_1<0$, $-s_2<0$.

取 $\lambda=1$, $k_1=2$, $\alpha_0=2$, $s_1=s_2=5$, 则有 $c=-\dfrac{\lambda}{k_1-\lambda}=-1$, $k_{2,2}=11$; $k_{2,3}=-36$.

取小车的初始状态分别为$(y_1,y_2,\theta)_0=\left(1,-1,\frac{\pi}{4}\right)$和(0,−1,0),仿真得到的状态的时间轨迹和几何路径(x-y)如图 1~4 所示.可以看出,两种初始状态下,小车的位置和姿态均指数收敛到零,特别是对第二种情况($\theta(0)=0$),文献[4~8]中的控制律均是不连续的,而本文给出的控制律是连续光滑的.

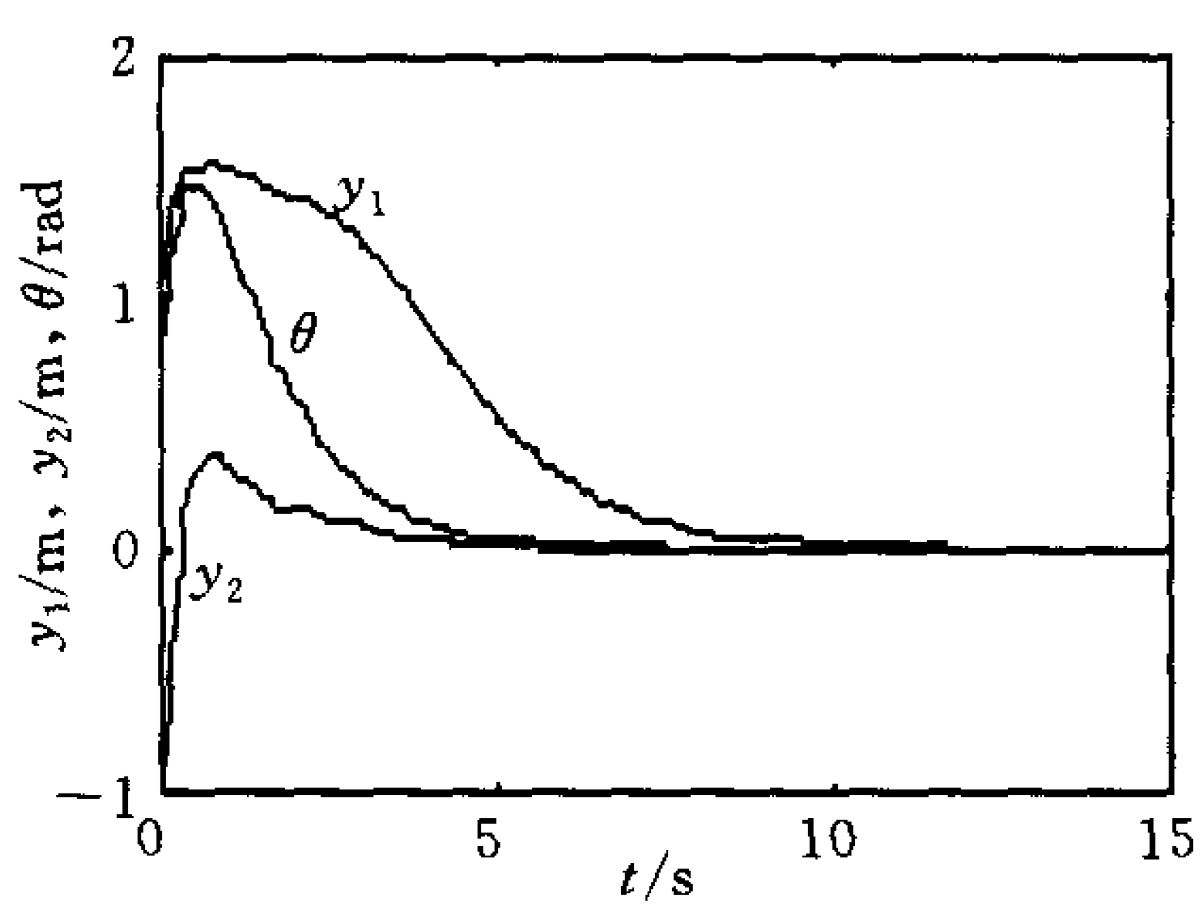

图 1 小车初始状态为 $\left(1,-1,\frac{\pi}{4}\right)$ 时的状态轨迹

Fig. 1 State trajectory of the cart with initial state $\left(1,-1,\frac{\pi}{4}\right)$

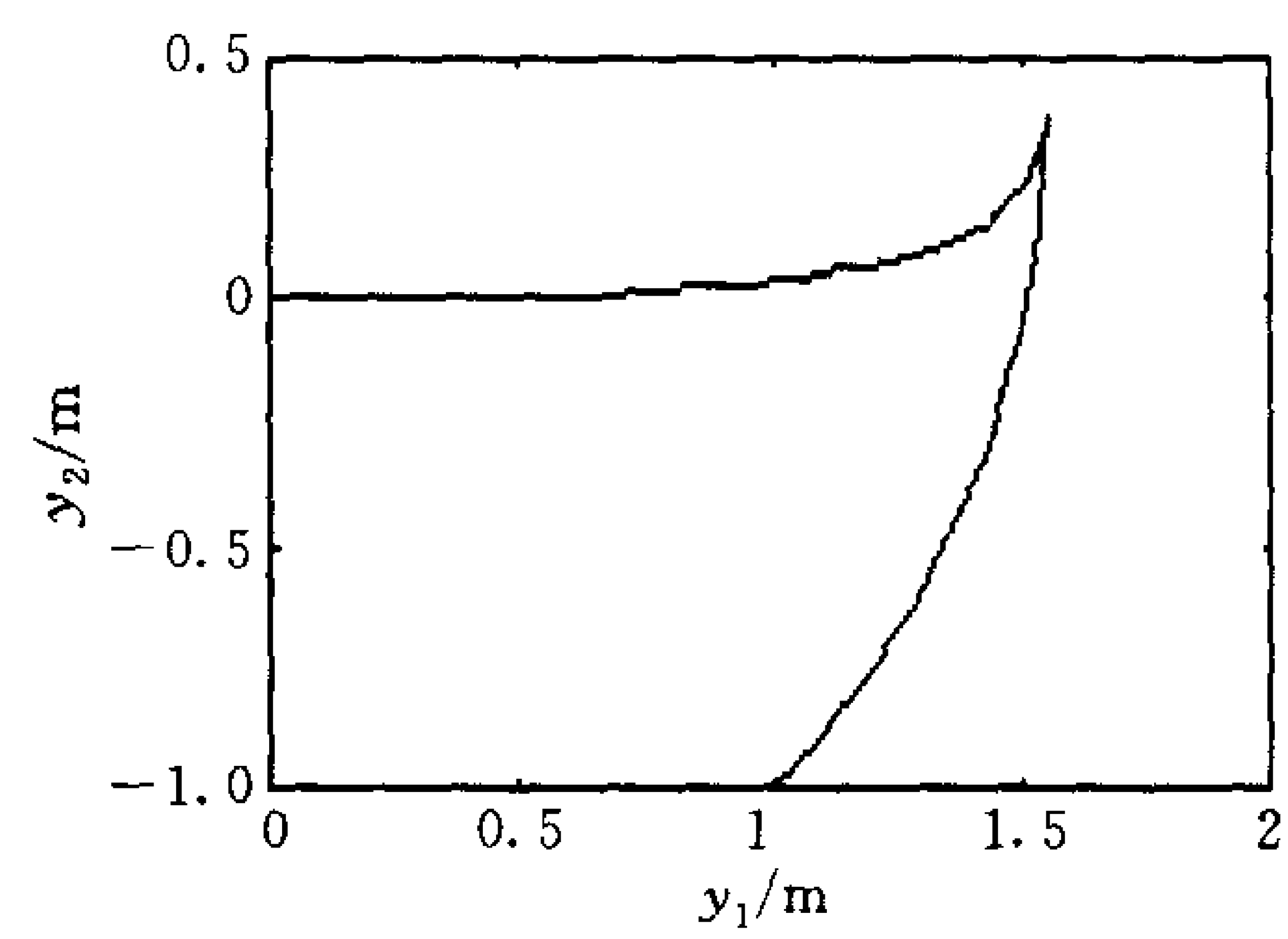

图 2 小车初始状态为 $\left(1,-1,\frac{\pi}{4}\right)$ 时的几何路径

Fig. 2 Geometric path of the cart with initial state $\left(1,-1,\frac{\pi}{4}\right)$

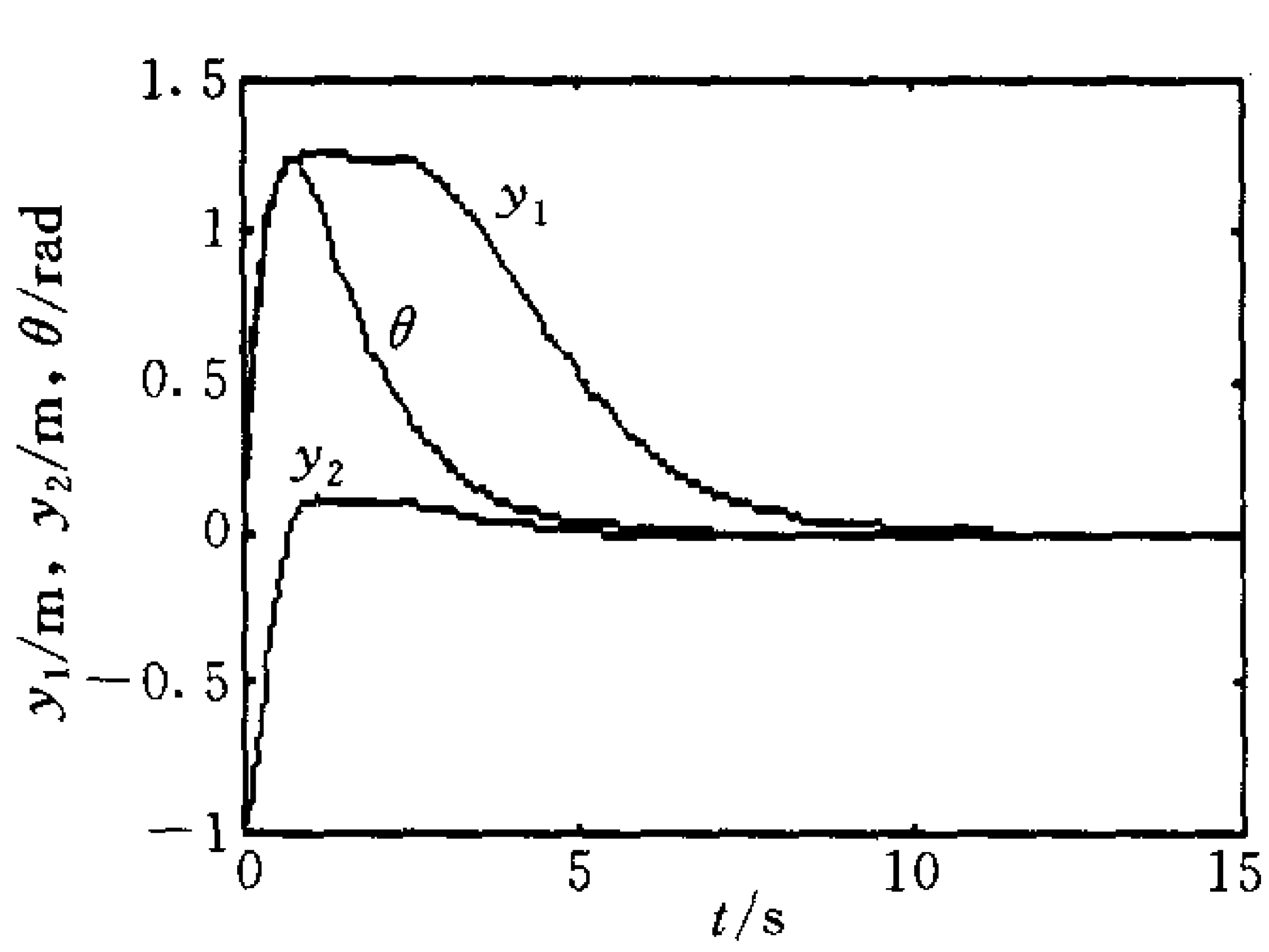

图 3 小车初始状态为(0,−1,0)时的状态轨迹

Fig. 3 State trajectory of the cart with initial state (0,−1,0)

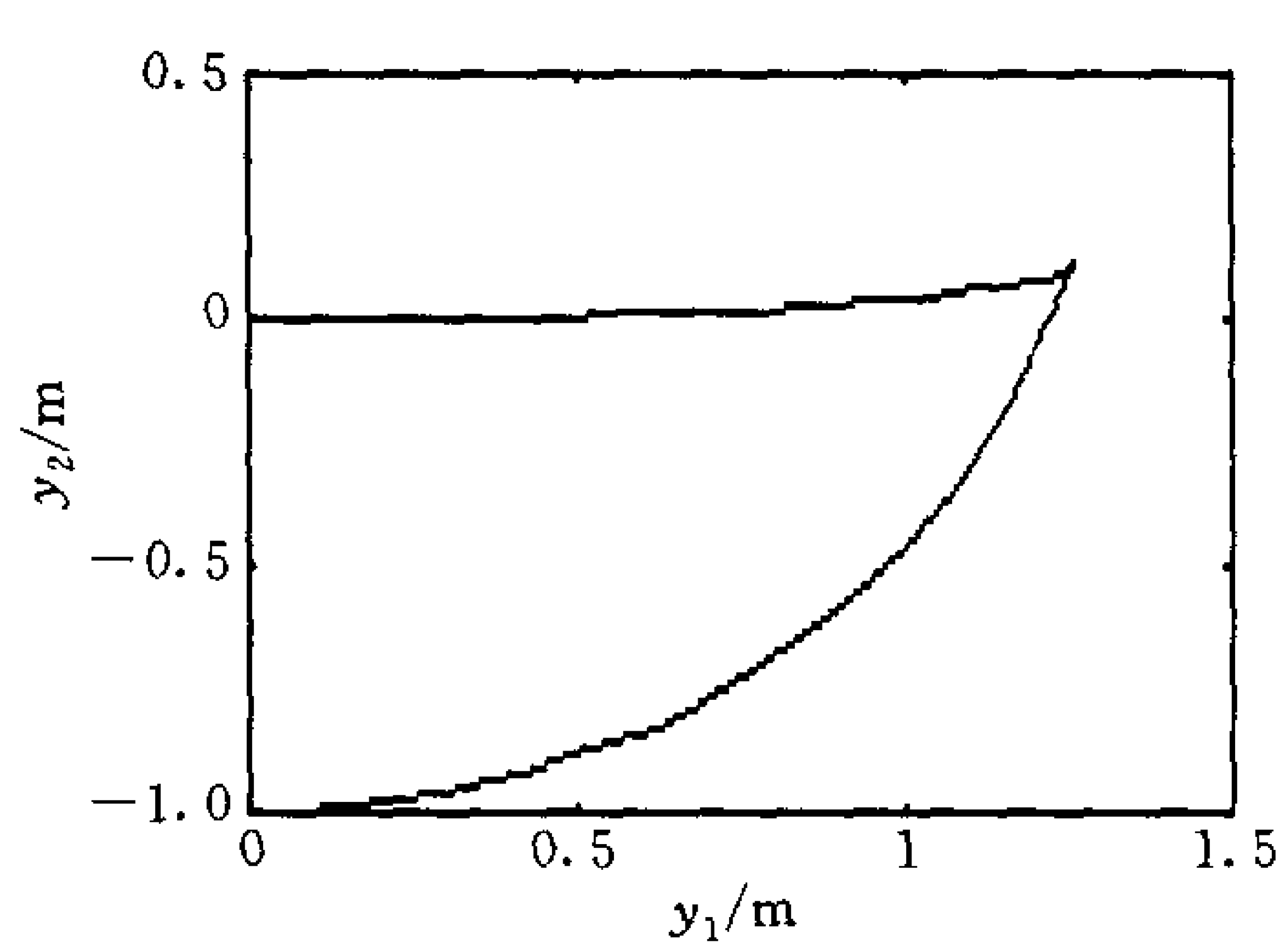

图 4 小车初始状态为(0,−1,0)时的几何路径

Fig. 4 Geometric path of the cart with initial state (0,−1,0)

4 结论

针对非完整链式系统，导出了一种时变光滑控制律，该控制律可以保证系统的所有状态全局指数收敛到原点，克服了以往控制律非光滑或虽光滑但却不能保证系统状态指数收敛的缺陷. 所得控制律用于移动机器人系统的镇定，验证了算法的有效性. 值得指出的是，本文给出的控制律虽然是针对非完整单链系统导出的，但它可以很容易地推广到非完整多链系统和高阶非完整链式系统的情况.

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