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Minimizing Earliness-Tardiness Penalties of Orders with Multi-Product Classes[1)]

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Abstract This paper deals with the order planning of single machine with multi-product classes and sequence-independent setup times, and proposes a mixed integer-programming model to minimize the sum of earliness/tardiness penalties of orders. It presents a pseudo-polynomial algorithm based on the filtered beam search by discussing the NP-hardness and the necessary condition of adjacent groups in an optimal sequence. The computational complexity analysis and simulation results have confirmed the effectiveness of the algorithm.

Key words Order planning, group technology, setup times, earliness/tardiness, complexity, pseudo-polynomial algorithm

1 Introduction

For enterprises that receive different sorts of orders and produce a great variety of products, if jobs of the same class are processed in groups, then redundant machine setup times and related expenses could be reasonably reduced. In [1] a comprehensive survey of scheduling problem involving setup times is provided, and the objective is job-related, while order planning has the complexity of multi-product classes and the objective is order-related[2~3]. [4] proposes a pseudo-polynomial algorithm and a dynamic programming algorithm to minimize the number of tardy orders without batch splitting and with batch splitting, respectively. Though progress in the earliness/tardiness (E/T) scheduling problem has been seen these years[5], few effective resolutions to the order planning with E/T penalties have been discussed. The problem can be stated as follows: n jobs from M orders can be classified into B product classes, and each order's due date and the job processing time are pre-determined. The assumptions are: all jobs are ready for processing at time zero and no preemption is allowed; the machine can process only one job at a particular time and no idle time is allowed; setup times are sequence-independent. In order to minimize the total earliness/tardiness of orders, jobs from different orders have to be grouped and the groups sequence should be decided. Following the three-field notation[6], the problem is $1/ST_{s_i}, GT/g$, in which ST_{s_i} represents the sequence-independent setup time, and GT means processing without group splitting. $g=\sum_{1\leqslant l\leqslant M}(\alpha_l OE_l+\beta_l OT_l)$ is the performance criteria, where OE_l and OT_l represent the earliness and tardiness of order l, respectively, α_l and β_l the earliness and tardiness penalty weights of the order, respectively.

2 Mathematical formulation

Under the group technology assumption, a group is referred to as a job sequence between two consecutive setups. Jobs from one order with processing similarities can be categorized into a group, so the total group number is $b=B\times M$. If group i has n_i jobs, the

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total job number is $n=\sum_{i=1}^{b} n_i$. The setup time of group i is s_i, J_{ij} is the jth job in group i, p_{ij}, c_{ij} and d_{ij} are the processing time, completion time and due date of job J_{ij}, respectively. $x_{ijk}=1$ if J_{ij} is in the kth position of the sequence, otherwise, 0; $y_{ik}=1$ if job in the kth position requires a setup time of s_i, otherwise, 0. The completion time and the due date of group i are C_i and d_i, respectively. So the mixed integer-programming model can be formulated as follows:

$$\min \sum_{1\leq l\leq M} (\alpha_l OE_l + \beta_l OT_l) \tag{1}$$

$$\text{s.t.} \quad \sum_{k=1}^{n} x_{ijk} = 1,\ i=1,2,\cdots,b,\ j=1,2,\cdots,n_i \tag{2}$$

$$\sum_{i=1}^{b}\sum_{j=1}^{n_i} x_{ijk} = 1,\ k=1,2,\cdots,n \tag{3}$$

$$\sum_{j=1}^{n_i} x_{ijk} - \sum_{j=1}^{n_i} x_{ij,k-1} \leq y_{ik},\ i=1,2,\cdots,b,\ k=2,3,\cdots,n \tag{4}$$

$$C_{ij} = \begin{cases} \sum_{i=1}^{b}\sum_{j=1}^{n_i} x_{ij1}(s_i+p_{ij}), & k=1 \\ \sum_{i=1}^{b}\sum_{j=1}^{n_i} x_{ij1}s_i + \sum_{i=1}^{b}\sum_{j=1}^{n_i}\sum_{l=1}^{k} x_{ijl}p_{ij} + \sum_{i=1}^{b}\sum_{l=2}^{k} y_{il}s_i, & k\geq 2 \end{cases} \tag{5}$$

$$C_i = \max_{1\leq j\leq n_i} C_{ij},\ d_i = \sum_{i=1}^{b}\sum_{j=1}^{n_i} x_{ijk}d_{ij},\ E_i=\max\{0,d_i-C_i\},\ T_i=\max\{0,C_i-d_i\} \tag{6}$$

$$OE_l = \max\{E_i \mid d_i = Od_l\},\ OT_l=\max\{T_i \mid d_i=Od_l\},\ l=1,2,\cdots,M \tag{7}$$

$$x_{ijk}=0,1,\ y_{ik}=0,1,\ i=1,2,\cdots,b,\ j=1,2,\cdots,n_i,\ k=1,2,\cdots,n \tag{8}$$

Constraint set (2) restricts that each job be assigned to only one position, and (3) guarantees exactly one job is scheduled in the kth position. Constraint (4) establishes the requirement of a setup time s_i in the kth position provided that the job in the $(k-1)$th position is not of class i. (5) provides the job's completion time, the due date and the E/T of groups and orders are provided by (6) and (7), respectively. The decision variable x_{ijk} is defined in (8).

3 Computational complexity and optimal properties

Theorem 1. The problem $1/ST_{s_i}, GT/g$ is NP-hard with M orders and B product classes(the proof is omitted).

Define the slack time of the group $S_i=d_i-t-P_i$, where t is the earliest time the machine is free, and P_i is the processing time of the composite job $P_i=\sum_{j=1}^{n_i} p_{ij}+s_i$, define α_i and β_i as the group E/T penalty weights, respectively, and define

$$\Theta_{ij} = \begin{cases} 0, & \text{if } S_i \leq 0 \\ S_i, & \text{if } 0 < S_i < P_j \\ P_j, & \text{otherwise} \end{cases}$$

We expand the theorem in [7] and get the followings.

Theorem 2. In the optimal order planning, all adjacent pairs of groups (where group i immediately precedes group j) satisfy the following adjacent condition:

$$\beta_i P_j - \Theta_{ij}(\beta_i+\alpha_i) \geq \beta_j P_i - \Theta_{ji}(\beta_j+\alpha_j),\ i,j = 1,2,\cdots,b \tag{9}$$

Proof. Consider the adjacent groups i and j are both early in sequence Π_1. Exchanging the two groups may yield two cases in Π_2 (see Fig. 1): Ⅰ) groups i and j are both early; Ⅱ) group i tardy, group j early. Let O_{ij} and O_{ji} be the cost of the subsequences (i,j) and (j,i), respectively. So in case Ⅰ) we have

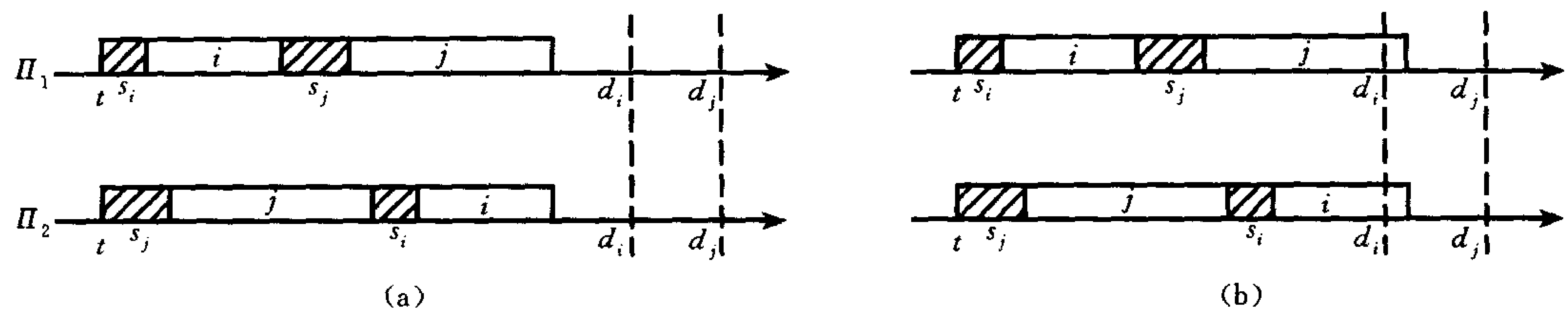

Fig. 1 Two sequences of the adjacent groups

1) $O_{ij}=\alpha_i(d_i-t-P_i)+\alpha_j(d_j-t-P_i-P_j)=\alpha_i(d_i-t-P_i)+\alpha_j(d_j-t-P_j)-\alpha_jP_i$,
$O_{ji}=\alpha_j(d_j-t-P_j)+\alpha_i(d_i-t-P_j-P_i)=\alpha_j(d_j-t-P_j)+\alpha_i(d_i-t-P_i)-\alpha_iP_j$.

Since $S_i=d_i-t-P_i\geqslant P_j$ and $S_j=d_j-t-P_j\geqslant P_i$, we have $\Theta_{ij}=P_j$ and $\Theta_{ji}=P_i$. Using these facts in inequality (9) gives $\alpha_i/P_i\leqslant\alpha_j/P_j$, therefore, $O_{ij}\leqslant O_{ji}$.

In case Ⅱ) we have

2) $O_{ij}=\alpha_i(d_i-t-P_i)+\alpha_j(d_j-t-P_i-P_j)=\alpha_i(d_i-t-P_i)+\alpha_j(d_j-t-P_j)-\alpha_jP_i$,
$O_{ji}=\alpha_j(d_j-t-P_j)+\beta_i(t+P_j+P_i-d_i)$.

Since $\Theta_{ij}=S_i$ and $\Theta_{ji}=P_i$, using these facts in inequality (9) gives $\beta_i(t+P_j+P_i-d_i)\geqslant\alpha_i(d_i-t-P_i)+\alpha_j(d_j-t-P_j)-\alpha_jP_i$, therefore, $O_{ij}\leqslant O_{ji}$.

The same procedure can be applied to other cases of groups i and j. That is,

1) In Π_1, both groups i and j are tardy. Two cases in Π_2: both groups i and j are tardy; group i tardy, group$j$ early.

2) In Π_1, group i early, group j tardy. Four cases in Π_2: both groups i and j are tardy; group i early, group j tardy; both groups i and j are early; group i tardy, group j early.

3) In Π_1, group i tardy, group j early. Only one case in Π_2: group i tardy, group j early. □

Property 1. If two adjacent groups i and j have $S_i\leqslant S_j$ and satisfy the following three conditions: 1) $H_i=\alpha_i/P_i\leqslant\alpha_j/P_j=H_j$; 2) $W_i=\beta_i/P_i\geqslant\beta_j/P_j=W_j$; 3) $\alpha_ip_j+\beta_jp_i\leqslant(\beta_j+\alpha_j)(S_j-S_i+p_j)$ or $\alpha_i+\beta_i\leqslant\alpha_j+\beta_j$. Then the sequence $i\prec j$ is the optimal group sequence in the order planning.

Property 2. In the optimal order planning, if the WSGPT (weighted shortest group processing time, i.e., $W_1\geqslant W_2\geqslant\cdots\geqslant W_b$) sequence is set and results in a schedule that does not have any early groups, then the sequence is optimal; otherwise, if we set the WLGPT (weighted longest group processing time, i.e., $H_1\leqslant H_2\leqslant\cdots\leqslant H_b$) sequence and get a schedule that does not have any tardy groups, then the sequence is optimal (Proof is referred to [7]).

4 Solution procedure

Let the improved group priority be
$$\Phi_i(S_i)=\begin{cases}W_i, & \text{if } S_i\leqslant 0\\ W_i-S_i(W_i+H_i)/k\bar{P}, & \text{if } 0\leqslant S_i\leqslant k\bar{P}\\ -H_i, & \text{otherwise}\end{cases}$$

where $\bar{P}$ is the average group processing time $\bar{P}=[\sum_{i=1}^{b}(\sum_{j=1}^{n_i}(p_{ij}+s_i)]/b$. Set parameter k to avoid clashes between multiple groups. Let U and V be two integers used for decisions in checking group priority and the objective function. The details of the algorithm are described below.

Step 1. Use the WSGPT rule. If the sequence does not have any early groups, the sequence is optimal and the algorithm stops. Otherwise, use the WLGPT rule. If the sequence does not have any tardy groups, the sequence is optimal and the algorithm stops.

Otherwise, set initial group sequence Π_0, and go to Step 2.

Step 2. Initialize the parameter $k, t, \bar{P}, b$. Let $G_0 = \{1, 2, \cdots, b\}$ be the set of groups that can be scheduled first, and $C = \{g_0\}$ be the set of nodes retained in the search.

Step 3. Let partial sequence set $A = \Phi$, and for each sequence g_i in C, calculate $\Phi_i(S_t)$ of each group in G_i at time t_{gi} and the total completion time of the partial sequence g_i. Let K be the U groups of the highest priority. If there are fewer than U groups in G_i, then select all the groups. For each group k' in K: create a new sequence $g_{ik'}$ by appending k' to the end of sequence g_i, add $g_{ik'}$ to set A, and create set $G_{ik'}$ which is set G_i excluding group k'.

Step 4. Let $C = \Phi$, calculate each sequence's objective function in set A, and select the lowest V sequences in A. If there are less than V sequences in A, select all the sequences.

Step 5. If the sequences in C are the complete sequences of groups $1, 2, \cdots, b$, then select the sequence with the lowest objective function as the best solution and the algorithm stops; otherwise, go to Step 3.

Theorem 3. In the worst case, the computational complexity of the above mentioned algorithm is bounded by $O(UV^2b^2\log b\log(UV))$.

Proof. The computational complexity of the WSGPT (or WLGPT) rule is $O(b\log b)$. The algorithm is based on the filtered beam search. At step 3, the selection of U groups of the highest priority needs a time complexity up to $O(V(b-1)\log V(b-1))$, while at Step 4, selecting V sequences with the lowest objective functions needs $O(UV\log(UV))$. There are b iterations for b-group problem, so the overall time complexity of the algorithm is bounded by $O(UV^2b^2\log b\log(UV))$. If parameters U and V are constants, the time complexity is $O(b^2\log b)$. □

5 Simulation example and conclusion

The algorithm is coded in C language. For a 3-order and 3-job class case, the processing times, setup times, E/T penalty weights and the due dates are listed in Table 1. With $k=3, U=3$, and $V=2$, the sequence result is 1-4-3-6-2-5-8-7-9 (shown in Table 2 with the starting time and E/T values of each group). Several other tests have been conducted, and all the data are generated randomly with different parameters in a discrete uniform distribution. The maximum and average deviation and optimal solution rate were compared. The results show that the algorithm can produce more accurate and efficient solutions.

Table 1 The data of 3-order and 3-job class

	Job class 1	Job class 2	Job class 3	Due date d_i	Early/tardy penalties (α_i/β_i)
Order 1	$P_1=6$	$P_2=3$	$P_3=9$	37	0.2/0.8
Order 2	$P_4=4$	$P_5=8$	$P_6=5$	43	0.3/0.7
Order 3	$P_7=10$	$P_8=4$	$P_9=6$	40	0.4/0.6
Setup times	$s_1=3$	$s_2=2$	$s_3=4$		

Table 2 The computational results

group i	t_i	E_i	T_i
1	0	28	0
2	31	1	0
3	13	11	0
4	9	30	0
5	36	0	1
6	26	12	0
7	48	0	21
8	44	0	8
9	61	0	31

The problem under this investigation suggests a new research direction. We propose a mixed integer-programming model and analyze the optimal properties and necessary conditions for order planning, which largely facilitates the solution procedure. The effectiveness of the algorithm has been testified through the complexity analysis and computational results, which may have some practical implications.

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具有多种产品类型的最小订单提前/拖期问题

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摘 要 针对单机多产品多订单有序独立机器调整时间的订单排产问题，建立了以最小化订单的提前和拖期罚值总和为优化目标的混合整数规划模型，分析了问题的NP－hard性和最优排序中临近批组的优化特性，提出一个基于过滤束搜索的拟多项式时间算法，算法复杂性分析和仿真结果验证了算法的有效性.

关键词 订单排产，成组技术，调整时间，提前/拖期，计算复杂性，拟多项式算法

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