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An Algorithm for Impulse Response Identification Based on Fixed Scale Orthogonal Wavelet Packet Transform[1)]

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Abstract Wavelet packet transform is used as an efficient tool for precisely analyzing signal property, and provides orthogonal wavelet packet bases with better time-frequency localization function. In this paper, by applying orthogonal wavelet packet bases to Eykhoff algorithm a new impulse response identification algorithm is proposed, which has better practicability , and wider application range. Simulation results show that the proposed algorithm can be applied to deterministic system and random system, and is of higher identification precision, stronger anti-noise-interference ability and better tracking system dynamic property.

Key words Wavelet packet transform, time-frequency analysis, Eykhoff algorithm, impulse response identification

1 Introduction

Impulse response identification is a kind of non-parameter identification. Its traditional identification algorithms mainly have Eykhoff algorithm and correlation analysis algorithm[1]. Correlation analysis algorithm is fitted to impulse response identification of random system, and its disadvantage lies in that correlation analysis algorithm does not trace the change of system behavior very well. Eykhoff algorithm is fitted to impulse response identification of deterministic system, and its identification principle is that impulse response function $g(t)$ is expressed as a linear combination of a group of orthogonal function bases, i. e.,

$$g(t) = \sum_{i=1}^{N} f_i(t)\xi_i = F^{\tau}(t)\xi \tag{1}$$

where $f_i(t)(i=1,2,\cdots,N)$ is a group of orthogonal function bases, and $\xi=[\xi_1,\xi_2,\cdots,\xi_N]^{\tau}$ are projecting coefficients of $g(t)$ on orthogonal function bases $F(t)$. For Eykhoff algorithm, it is important to choose a group of orthogonal function bases. However, the traditional orthogonal function bases has good time resolving power and poor frequency resolving power, or the other way round[2].

Being developed from wavelet transform, wavelet packet transform can be used as an efficient tool for precisely analyzing the localization information of signal at arbitrary moment and arbitrary frequency point[3], and can provide orthogonal wavelet packet bases with even better time resolving power and frequency resolving power. In this paper, by applying orthogonal wavelet packet bases to Eykhoff algorithm a new algorithm for impulse response identification is proposed, which has better practicability , and wider application range.

2 Basic idea for impulse response identification based on fixed scale orthogonal wavelet packet transform

In wavelet packet transform, given an orthonormal scale function $\varphi(t)$ and utilizing

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double scale difference equation:

$$\begin{cases} w_{2n}(t) = \sqrt{2}\sum_{k\in Z} h_k w_n(2t-k) \\ w_{2n+1}(t) = \sqrt{2}\sum_{k\in Z} g_k w_n(2t-k) \end{cases} \tag{2}$$

the orthogonal wavelet packet $\{w_{n,j,k}(t) := 2^{-j/2} w_n(2^{-j}t-k), n\in Z/Z^-, j\in Z, k\in Z\}$ for $\varphi(t)$ can be created, where $w_0(t)=\varphi(t)$, $\{h_k\}_{k\in Z}$ and $\{g_k\}_{k\in Z}$ are a pair of conjugate orthogonal filter coefficients deduced from $\varphi(t)$. Wavelet packet transform can precisely analyze signal properties because of its multi-layer division function for signal frequency belt.

So-called impulse response identification based on fixed scale orthogonal wavelet packet transform is as follows. The impulse response function $g(t)(\in V_{j_0})$ is projected on all orthogonal wavelet packet spaces of the largest decomposition scale, namely the spaces with the highest time-frequency resolving power. By identifying the projecting coefficients (namely wavelet packet transform coefficients) the impulse response function can be indirectly identified. For example, given the initial scale j_0, and the biggest decomposition layer $m=4$, the original signal space V_{j_0} is divided by orthogonal wavelet packet spaces $U^n_{j_0+s}(s=1,2,3,4; 0\leqslant n\leqslant 2^s-1)$ at different scale $j=j_0+s$, as shown in Fig. 1, where the shadowing spaces are orthogonal wavelet packet spaces chosen by the impulse response identification algorithm based on fixed scale wavelet packet transform.

$U^0_{j_0}(V_{j_0})$																
$U^0_{j_0+1}$								$U^1_{j_0+1}$								$s=1$
$U^0_{j_0+2}$				$U^1_{j_0+2}$				$U^2_{j_0+2}$				$U^3_{j_0+2}$				$s=2$
$U^0_{j_0+3}$		$U^1_{j_0+3}$		$U^2_{j_0+3}$		$U^3_{j_0+3}$		$U^4_{j_0+3}$		$U^5_{j_0+3}$		$U^6_{j_0+3}$		$U^7_{j_0+3}$		$s=3$
$U^0_{j_0+4}$	$U^1_{j_0+4}$	$U^2_{j_0+4}$	$U^3_{j_0+4}$	$U^4_{j_0+4}$	$U^5_{j_0+4}$	$U^6_{j_0+4}$	$U^7_{j_0+4}$	$U^8_{j_0+4}$	$U^9_{j_0+4}$	$U^{10}_{j_0+4}$	$U^{11}_{j_0+4}$	$U^{12}_{j_0+4}$	$U^{13}_{j_0+4}$	$U^{14}_{j_0+4}$	$U^{15}_{j_0+4}$	$s=4$

Fig. 1 Orthogonal wavelet packet spaces selection (namely shadowing spaces) for impulse response identification bases on fixed scale orthogonal wavelet packet transform

From multiresolution theory of wavelet transform[3], if initial scale $j_0 \in Z$ is very small, the scale space V_{j_0} can approach to the square integrabel space $L^2(R)$ very well, i. e.,

$$V_{j_0} \approx L^2(R) \tag{3}$$

Hence, for orthogonal wavelet packet transform we have

$$L^2(R) \approx V_{j_0} = U^0_{j_0} = U^0_{j_0+1} \oplus U^1_{j_0+1} = U^0_{j_0+2} \oplus U^1_{j_0+2} \oplus U^2_{j_0+2} \oplus U^3_{j_0+2} = \cdots \tag{4}$$

Function group $\bigcup_{n=0}^{2^m-1}\{2^{-(j_0+m)/2}w_n(2^{-(j_0+m)}t-k), k\in Z\}$ composes the orthogonal bases of $L^2(R)$, where m is decomposition layer of wavelet packet transform.

If system impulse response function $g(t)$ is approached by its projection on the orthogonal bases $\bigcup_{n=0}^{2^m-1}\{w_{n,j_0+m,k}(t), k\in Z\}$ of space $L^2(R)$, then $g(t)$ can be expressed as follows:

$$g(t) = \sum_{n=0}^{2^m-1}\sum_{k=0}^{k_n^{j_0+m}} p_g(n, j_0+m, k) w_{n,j_0+m,k}(t) \tag{5}$$

where projection coefficients $\{p_g(n, j_0+m, k)\}_{k\in Z}$ are wavelet packet transform coefficients of $g(t)$ in the wavelet packet space $U^n_{j_0+m}$.

Consider a single input single output linear system

$$y(t) = \int g(t)u(t-\tau)\mathrm{d}\tau \tag{6}$$

where $u(t)$, $y(t)$ and $g(t)$ are system input signal, system output signal and system impulse response function, respectively.

Substitute (5) into (6), we get

$$y(t) = \sum_{n=0}^{2^m-1}\sum_{k=0}^{k_n^{j_0+m}} p_g(n,j_0+m,k)\left(\int w_{n,j_0+m,k}(t)u(t-\tau)\mathrm{d}\tau\right) \tag{7}$$

Let

$$\zeta_{n,j_0+m,k}(t) = \int w_{n,j_0+m,k}(t)u(t-\tau)\mathrm{d}\tau = w_{n,j_0+m,k}(t) \otimes u(t) \tag{8}$$

where the sign $\otimes$ denotes the convolution operator. Thus

$$y(t) = \sum_{n=0}^{2^m-1}\sum_{k=0}^{k_n^{j_0+m}} p_g(n,j_0+m,k)\zeta_{n,j_0+m,k}(t) \tag{9}$$

Because system input $u(t)$ and output $y(t)$ are known and the chosen orthogonal wavelet packet bases are also known, from (9) the projection coefficients $\{p_g(n,j_0+m,k), n=0,\cdots,2^m-1;\ k=0,\cdots,k_n^{j_0+m}\}$ of system impulse response function $g(t)$ on the orthogonal wavelet packer bases $\bigcup_{n=0}^{2^m-1}\{w_{n,j_0+m,k}(t), k\in Z\}$ can be calculated by using least-square method. Thus the impulse response function $g(t)$ is identified indirectly, and using reconstruction algorithm of orthogonal wavelet packet transform we can get the system impulse response function $g(t)$ in the time domain.

3 Theory algorithm and fast binary tree recursion algorithm of $\zeta_{n,j_0+m,k}(t)$

First we introduce the projection operator and present a lemma as follows.

Definition 1[4]. The projection operator from space $l^2(Z)$ to space $l^2(2Z)$ is defined to be F_0 and F_1, i. e.,

$$\begin{cases} F_0(S_k)(l) = \sum_{k\in Z} h_{k-2l}S_k \\ F_1(S_k)(l) = \sum_{k\in Z} g_{k-2l}S_k \end{cases} \tag{10}$$

where $\{h_k\}$ and $\{g_k\}$ are low pass filter coefficients and high pass filter coefficients for double scale difference equation (2), respectively.

Lemma 1. Let initial scale of wavelet packet transform be j_0 and decomposition layer be $m\in Z^+$. For arbitrary $n\in\{0,1,\cdots,2^m-1\}$ at the scale $j=j_0+m$, the wavelet packet bases function $w_{n,j,l}(t)$ of wavelet packet space $U_{j_0+m}^n$ can be expressed as follows

$$w_{n,j,l}(t) = F_{e_1}\{F_{e_2}\{\cdots\{F_{e_{n_i}}\{w_{0,j-n_i,k}(t)\}(k_{n_i})\}\cdots\}(k_2)\}(l) \tag{11}$$

which is simplified to

$$w_{n,j,l}(t) = F_{e_1}F_{e_2}\cdots F_{e_{n_i}}\{w_{0,j-n_i,k}(t)\}(l) \tag{12}$$

where $\{w_{0,j-n_i,k}(t)\}_{k\in Z}$ are orthonormal bases of wavelet packet space $U_{j-n_i}^0$ (namely scale space V_{j-n_i}), and e_i is the ith coefficient for binary expression of n, i. e.,

$$n = \sum_{i=1}^{n_i} e_i 2^{i-1} \tag{13}$$

where n_i is the maximal index for binary expression of n.

Proof. Omitted.

Theorem 1 (theoretical algorithm of $\zeta_{n,j_0+m,k}(t)$). Assume that the initial scale for wavelet packet transform is j_0. For arbitrary decomposition layer $m\in Z^+$, arbitrary integer $n\in\{0,1,\cdots,2^m-1\}$ and $k\in Z$, at decomposition scale $j=j_0+m$, $\zeta_{n,j_0+m,k}(t)$ defined as (8), namely $\zeta_{n,j,k}(t)$, satisfies

$$\zeta_{n,j,k}(t) = F_{e_1}F_{e_2}\cdots F_{e_{n_i}}\{\zeta_{0,j-n_i,l}(t)\}(k) \tag{14}$$

where e_i is the ith coefficient for binary expression of n, and n_i is the maximal index for binary expression of n, i. e.,

$$n = \sum_{i=1}^{n_i} e_i 2^{i-1} \tag{15}$$

Proof. Omitted.

Theorem 1 provides a theoretical algorithm for $\zeta_{n,j_0+m,k}(t)$. However, The proposed theoretical algorithm is complicated and has much repetitious computation. Therefore, it is necessary to propose a fast discrete algorithm for $\zeta_{n,j_0+m,k}(t)$.

Owning to the specific form of orthogonal wavelet packet bases, in this paper the sampling period of time variant t is set[2]

$$T_s = 2^{j_0} \tag{16}$$

Thus time variant t is expressed as follows:

$$t = i \cdot T_s = 2^{j_0} i,\ i \in Z/Z^- \tag{17}$$

Putting (17) into (9), we get

$$y(2^{j_0} i) = \sum_{n=0}^{2^m-1} \sum_{k=0}^{k_n^{j_0+m}} p_g(n, j_0+m, k)\, \zeta_{n,j_0+m,k}(2^{j_0} i) \tag{18}$$

It must be pointed out that discrete Eq. (18) does not cause any error for identifying $p_g(n,j_0+m,k)$. In order to simplify expression, we will adopt the definition that $*(i)$ substitutes for $*(2^{j_0} i)$ in [2], i.e.,

$$*(i) = *(2^{j_0} i) \tag{19}$$

Thus (18) can be simplified to:

$$y(i) = \sum_{n=0}^{2^m-1} \sum_{k=0}^{k_n^{j_0+m}} p_g(n, j_0+m, k)\, \zeta_{n,j_0+m,k}(i) \tag{20}$$

Theorem 2 (Fast discrete binary tree recursion algorithm for $\zeta_{n,j_0+m,k}(t)$). Assume that the initial scale for wavelet packet transform is j_0. For arbitrary decomposition layer $m \in Z^+$ and arbitrary integer $n \in \{0,1,\cdots,2^m-1\}$, the discrete algorithm for $\zeta_{n,j_0+m,k}(t)$ is as follows.

1) When $j=j_0$, $\zeta_{0,j_0,0}(i) \approx 2^{j_0/2} u(i)$.

2) According to $\zeta_{n,j,0}(i)$, $\zeta_{n,j,k}(i)$ can be computed as

$$\zeta_{n,j,k}(i) = \zeta_{n,j,0}(i - 2^{j-j_0} k) \tag{21}$$

where $j = j_0+1, j_0+2, \cdots, j_0+m$, $0 \leqslant n \leqslant 2^{j-j_0}-1$.

3) According to fig. 2, $\zeta_{n,j,0}(i)$ can be computed step by step.

$$\begin{cases} \zeta_{2n,j,0}(i) = \sum_{k\in Z} h_k \zeta_{n,j-1,0}(i - 2^{j-1-j_0} k) \\ \zeta_{2n+1,j,0}(i) = \sum_{k\in Z} g_k \zeta_{n,j-1,0}(i - 2^{j-1-j_0} k) \end{cases} \tag{22}$$

where $j = j_0+1, j_0+2, \cdots, j_0+m$, $0 \leqslant n \leqslant 2^{j-j_0}-1$.

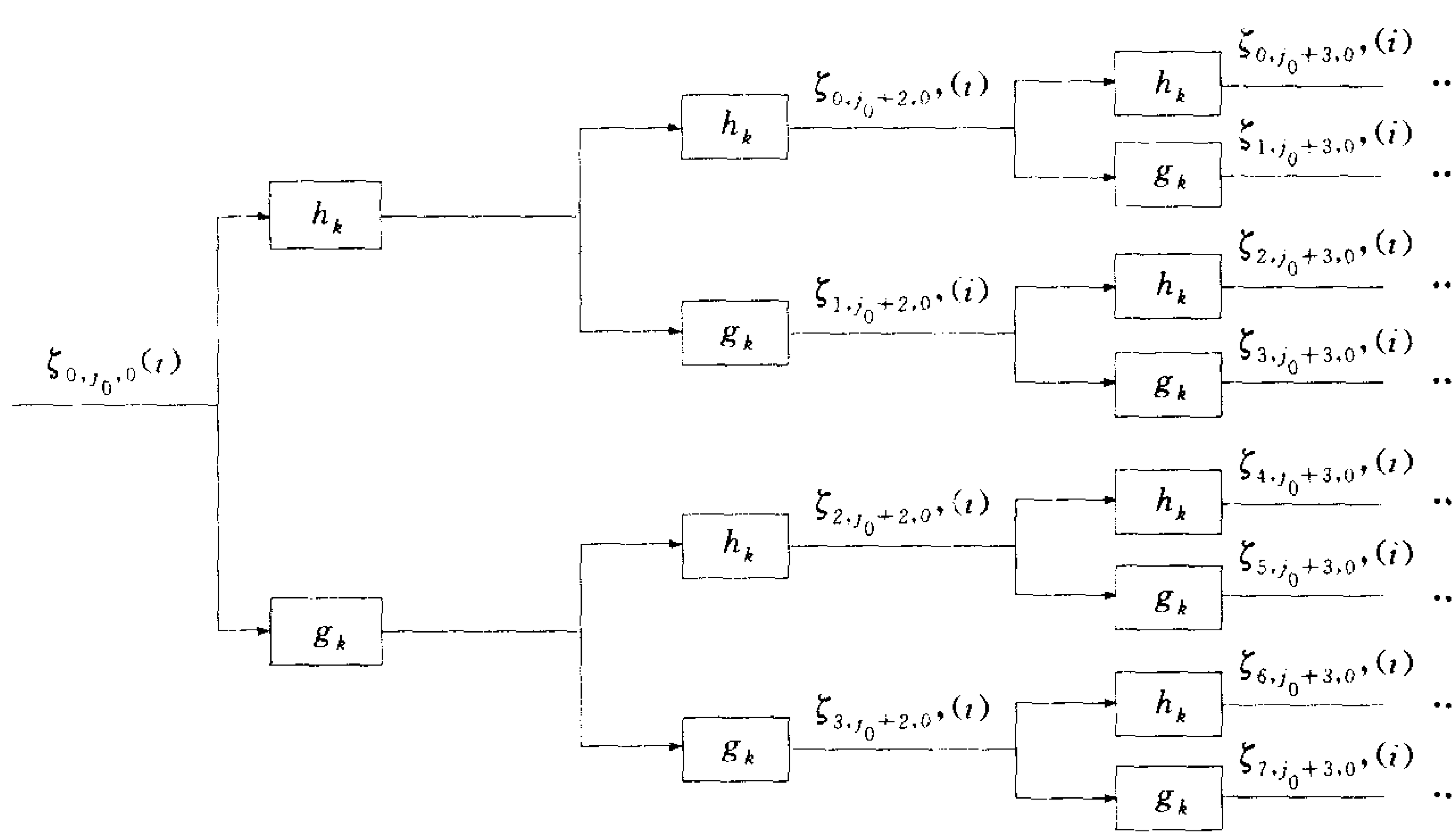

Fig. 2 Fast discrete binary tree recursion algorithm for $\zeta_{n,j_0+m,k}(i)$

Proof. Omitted.

Because the length of high pass filter group and low pass filter group of wavelet are limited, the burden of recursive computation in theorem 2 is very low, and the speed of computing $\zeta_{n,j_0+m,k}(i)$ is improved very much.

4 Some parameters discussion

For the proposed impulse response identification algorithm, some parameters such as orthogonal wavelet packet bases, initial scale j_0, decomposition layer m, and $k_n^{j_0+m}$ in (5) must to be considered. These parameters will be discussed as follows.

1) Orthogonal wavelet packet bases selection

In the proposed impulse response identification algorithm, the selected orthogonal wavelet packet bases are generated by initial scale function of Daubechies II(DB2)[5].

2) Initial scale j_0 selection

The selected j_0 directly affects the precision of the orthogonal wavelet packet series of $g(t)$ in (5). At present, there is no widely accepted criterion to select j_0, and j_0 in general takes a small negative integer.

3) Decomposition layer m selection

Given a scale space V_{j_0}, decomposition layer m does not affect the precision of impulse response identification algorithm, but it finally determines the number (i. e. $=2^m$) of frequency belt of $g(t)$ divided by wavelet packet transform.

4) $k_n^{j_0+m}$ computation in (5)

In (5), $k_n^{j_0+m}$ virtually denotes the number of wavelet packet transform coefficients of $g(t)$ in each wavelet packet space at decomposition scale $j=j_0+m$. Supposed that the number of wavelet packet transform coefficients of $g(t)$ in the scale space V_{j_0} is $k_0^{j_0}$. We have

$$k_n^{j_0+m} = 2^{-m}k_0^{j_0} \tag{23}$$

Suppose that the valid action time of impulse response function $g(t)$ is t_g. In order to ensure that $g(t)$ can be covered with its discrete form $g(i)(i\in Z^+)$, parameter $k_0^{j_0}$ must satisfy

$$2^{j_0}k_0^{j_0} \geqslant t_g \tag{24}$$

i. e. ,

$$k_0^{j_0} \geqslant 2^{-j_0}t_g \tag{25}$$

In fact, for arbitrary decomposition scale $j=j_0+s(0\leqslant s\leqslant m)$, the number $k_n^{j_0+s}$ of wavelet packet transform coefficients of $g(t)$ in each wavelet packet space $U_{j_0+s}^n(0\leqslant n\leqslant 2^s-1)$ can be computed as follows:

$$k_n^{j_0+s} = 2^{-s}k_0^{j_0} \tag{26}$$

where $0\leqslant s\leqslant m$, and $0\leqslant n\leqslant 2^s-1$.

For arbitrary $k_n^{j_0+s}$, In order to ensure that $k_n^{j_0+s}$ is an integer more than zero, $k_0^{j_0}$ must satisfy

$$\begin{cases} k_0^{j_0} \geqslant 2^{-j_0}t_g \\ k_0^{j_0} = 2^m r \end{cases} \tag{27}$$

where r is the number of wavelet packet transform coefficients of $g(t)$ in each wavelet packet space $U_{j_0+m}^n(0\leqslant n\leqslant 2^m-1)$ at decomposition scale $j=j_0+m$, i. e. , $r=k_n^{j_0+m}$.

5 Impulse response identification algorithm based on fixed scale orthogonal wavelet packet transform

Impulse response identification algorithm based on fixed scale orthogonal wavelet packet transform is proposed as follows.

Step1. Determine the initial scale j_0 and decomposition layer, and from (16) compute the sampling period T_s of $g(t)$.

Step2. Evaluate the valid action time t_g of impulse response function $g(t)$, and from (27) decide the length $k_0^{j_0}$ of discrete series of $g(t)$. From (26) the number $k_n^{j_0+s}$ of wavelet packet transform coefficients of $g(t)$ in each wavelet packet space $U_{j_0+s}^n(0\leqslant n\leqslant 2^{j_0+s}-1)$ at arbitrary decomposition scale $j=j_0+s(0\leqslant s\leqslant m)$ is computed.

Step3. Adopt orthogonal wavelet packet bases function of Daubechies II (i. e. DB2), and according to fast discrete binary tree recursion algorithm in theorem 2, compute all $\zeta_{n,j_0+s,k}(i)$ step by step, where $0\leqslant s\leqslant m$, $0\leqslant n\leqslant 2^s-1$, $0\leqslant k\leqslant 2^{-s}k_0^{j_0}-1$.

Step4. Parameter vector $\boldsymbol{X}$ construction.

Let $\boldsymbol{x}(n)(0\leqslant n\leqslant 2^m-1)$ denote a row vector composed of all wavelet packet transform coefficients $p_g(n,j_0+m,k)(0\leqslant k\leqslant 2^{-m}k_0^{j_0}-1)$ of $g(t)$ in the wavelet packet space at decomposition scale $j=j_0+m$, i. e. ,

$$\boldsymbol{x}(n) = [p_g(n,j_0+m,0),p_g(n,j_0+m,1),\cdots,p_g(n,j_0+m,2^{-m}k_0^{j_0}-1)] \tag{28}$$

Let $\boldsymbol{X}$ denote a column vector composed of all row vectors $\boldsymbol{x}(n)(0\leqslant n\leqslant 2^m-1)$ of $g(t)$ at decomposition scale $j=j_0+m$, i. e. ,

$$\boldsymbol{X} = [\boldsymbol{x}(0),\boldsymbol{x}(1),\cdots,\boldsymbol{x}(2^m-1)]^{\mathrm{T}} \tag{29}$$

where sign T denotes the matrix transpose operator.

Step5. Data vector $\boldsymbol{A}(i)$ construction.

Let $\boldsymbol{a}_n(i)(0\leqslant n\leqslant 2^m-1)$ denote a row vector composed of all $\zeta_{n,j_0+m,k}(i)(0\leqslant k\leqslant 2^{-m}k_0^{j_0}-1)$ generated by convolution in the wavelet packet space $U_{j_0+m}^n$ at decomposition scale $j=j_0+m$, i. e. ,

$$\boldsymbol{a}_n(i)=[\zeta_{n,j_0+m,0}(i),\zeta_{n,j_0+m,1}(i),\cdots,\zeta_{n,j_0+m,2^{-m}k_0^{j_0}-1}(i)] \tag{30}$$

Let $\boldsymbol{A}(i)$ denote a column vector composed of all row vectors $\boldsymbol{a}_n(i)$ $(0\leqslant n\leqslant 2^m-1)$ at decomposition scale $j=j_0+m$, i. e. ,

$$\boldsymbol{A}(i)=[\boldsymbol{a}_0(i),\boldsymbol{a}_1(i),\cdots,\boldsymbol{a}_{2^m-1}(i)]^{\mathrm{T}} \tag{31}$$

where sign T denotes the matrix transpose operator.

Step6. Based on least square method, wavelet packet transform coefficient series $\boldsymbol{X}$ of $g(t)$ at decomposition scale $j=j_0+m$ is identified.

According to the definition of $\boldsymbol{X}$ in (29) and the definition of $\boldsymbol{A}(i)$ in (31), (20) can be simplified to

$$A^{\mathrm{T}}(i)\boldsymbol{X}=y(i) \tag{32}$$

When i takes value 1, 2, $\cdots$, $k_0^{j_0}$, (32) can be changed to

$$A^{\mathrm{T}}\boldsymbol{X}=\boldsymbol{Y} \tag{33}$$

where

$$A=[\boldsymbol{A}(1),\boldsymbol{A}(2),\cdots,\boldsymbol{A}(k_0^{j_0})] \tag{34}$$

$$\boldsymbol{Y}=[y(1),y(2),\cdots,y(k_0^{j_0})]^{\mathrm{T}} \tag{35}$$

By using least square method, wavelet packet transform coefficient series $\boldsymbol{X}$ of $g(t)$ at decomposition scale $j=j_0+m$ can be identified easily.

Step7. Wavelet packet transform coefficient series $\boldsymbol{X}$ is processed.

Since wavelet packet transform coefficient series $\boldsymbol{X}$ of $g(t)$ are of specific physics meaning, $\boldsymbol{X}$ can be directly used to analyze the properties of $g(t)$ in frequency domain. On the other hand, from the identified wavelet packet transform coefficients, original impulse response function $\hat{g}(t)$ can be reconstructed by using the reconstruction algorithm of wavelet packet transform.

6 Simulation example

In order to demonstrate the validity of the proposed impulse response identification algorithm, in this section two numerical simulation examples with different conditions are given. In the simulation examples, identification error $e(i)$ and variance $\hat{\sigma}$ are defined as follows:

$$e(i)=g(i)-\hat{g}(i) \tag{36}$$

$$\hat{\sigma}=\frac{1}{L-1}\sum_{i=1}^{L}[e(i)]^2 \tag{37}$$

Simulation example adopts the transfer function in [1], i. e. ,

$$G(s)=\frac{1.2}{(8.3s+1)(6.2s+1)} \tag{38}$$

Example 1. System output without noise. Consider identifying object

$$Y(s)=\frac{1.2}{(8.3s+1)(6.2s+1)}U(s) \tag{39}$$

where input signal $u(t)$ is Gauss white noise with mean 0 and variance 1.

Let the valid action time of impulse response function $t_g=50$ seconds. In order to satisfy data length demand for wavelet packet transform at initial scale, valid action time of impulse response function virtually takes value $t_g'=52$ seconds. Let initial scale $j_0=-1$, decomposition layer $m=3$, and sampling time $t=500$ seconds of system input and output. According to the proposed impulse response identification algorithm, sampling period $t_s=2^{-1}=0.5$ seconds. Thus the length of discrete series of $g(t)$ takes value $k_0^{j_0}=104$, and sampling data length $L=1000$. We adopt orthogonal wavelet packet bases of Daubechies II. By using the proposed impulse response identification algorithm, identification result $\hat{g}(i)$ for impulse response function $g(i)$ and its identification error $e(i)$ are showed in Fig. 3.

Let system input $u(t)$ be stochastic bar signal with its amplitude being normal distribution and its period taking 40 sampling periods. By using with the proposed impulse response identification algorithm, impulse response identification results are show in Fig. 4.

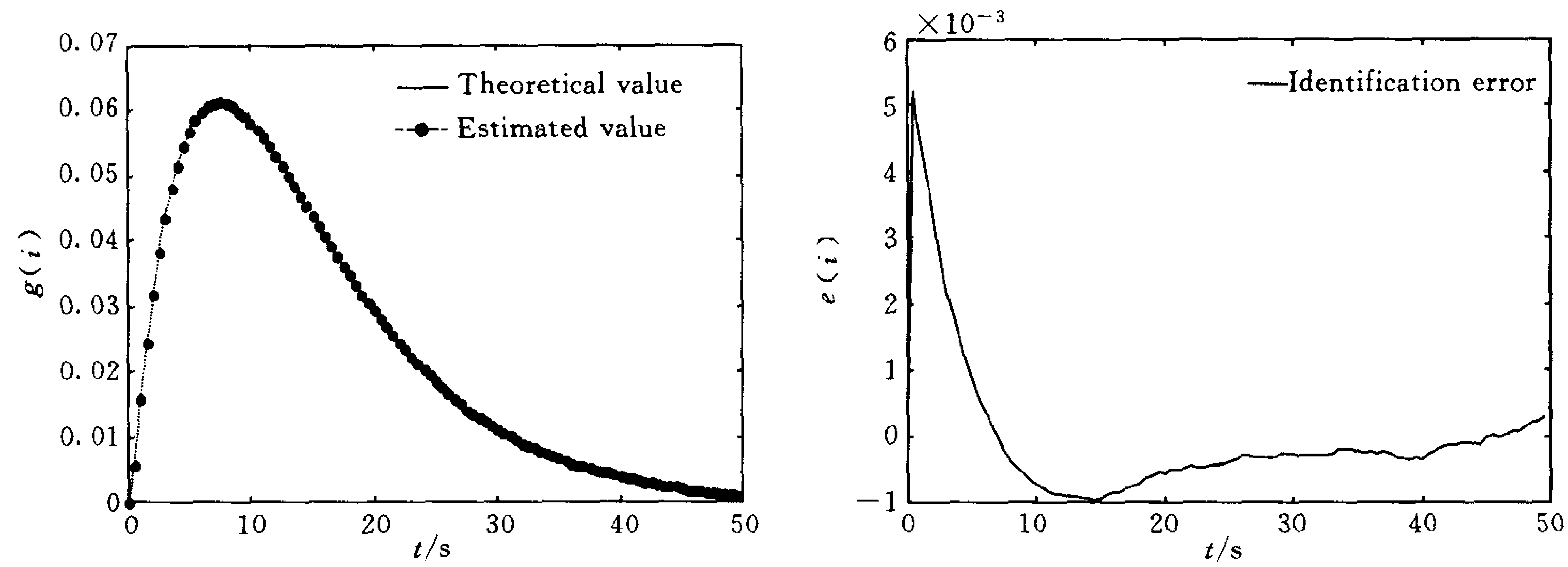

Fig. 3 Impulse response identification results for system output without noise (where $j_0=-1$)

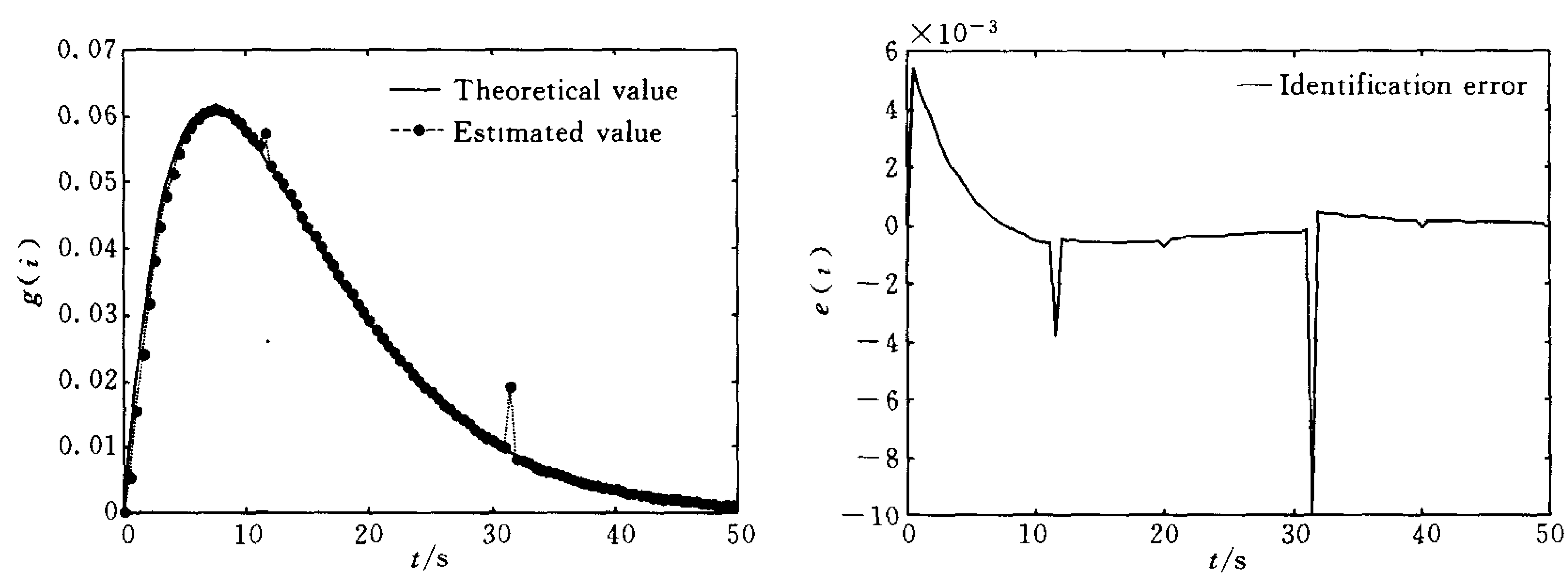

Fig. 4 Impulse response identification results for system input being stochastic bar signal (where $j_0=-1$)

In Fig. 3, the variance of identification error $e(i)$ is 1.1676e−6, which shows that impulse response identification precision for deterministic system is very high. In Fig. 4, the variance of identification error $e(i)$ is 2.3173e−6, which shows that impulse response identification precision for system input being deterministic signal is also very high.

Example 2. System output with noise. Considered identifying object

$$Y(s) = \frac{1.2}{(8.3s+1)(6.2s+1)}U(s) + \lambda N(s) \tag{40}$$

where $N(s)$ denotes Gauss white noise with mean 0 and variance 1, $u(t)$ and $n(t)$ are all white noise, $u(t)$ is not correlative with $n(t)$, λ is a parameter which limits noise. Let noise-signal-ratio be 18% and 33%, respectively.

Parameters selection is similar to example 1. Impulse response identification results for noise-signal-ratio being 18% and 33%, and their identification errors are shown in Fig. 5 and Fig. 6, respectively, where variance of identification error $e(i)$ is $\hat{\sigma}_{18}=2.4798\text{e}-6$ and $\hat{\sigma}_{33}=6.8803\text{e}-6$. Fig. 5 and Fig. 6 show that the proposed impulse response identification algorithm still is of high identification precision in spite of system output being of strong noise.

In order to compare with other impulse response identification algorithm, the impulse response identification results based on correlation analysis algorithm for noise-signal-ratio being 18% and 33%, and their identification errors are shown in Fig. 7 and Fig. 8, respectively, where variance of identification error $e(i)$ is $\hat{\sigma}_{18}=1.8038\text{e}-5$ and $\hat{\sigma}_{33}=2.2320\text{e}-5$. Compared with correlation analysis algorithm, the proposed impulse response identification algorithm in this paper is of higher identification precision.

Now, let sampling data length $L=5600$. For example 2 we adopt correlation analysis algorithm again. The impulse response identification results show that variance $\hat{\sigma}_{18}=$

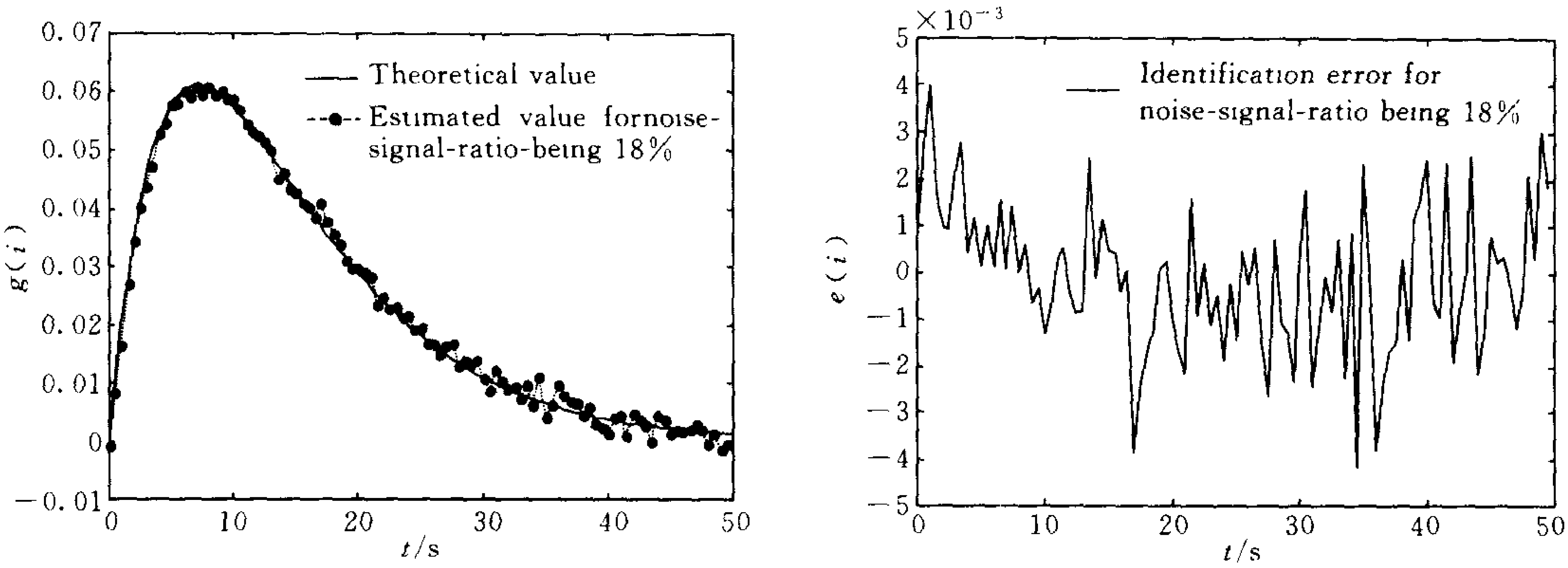

Fig. 5 Impulse response identification results for system output with noise-signal-ratio being 18% (where $j_0=-1$)

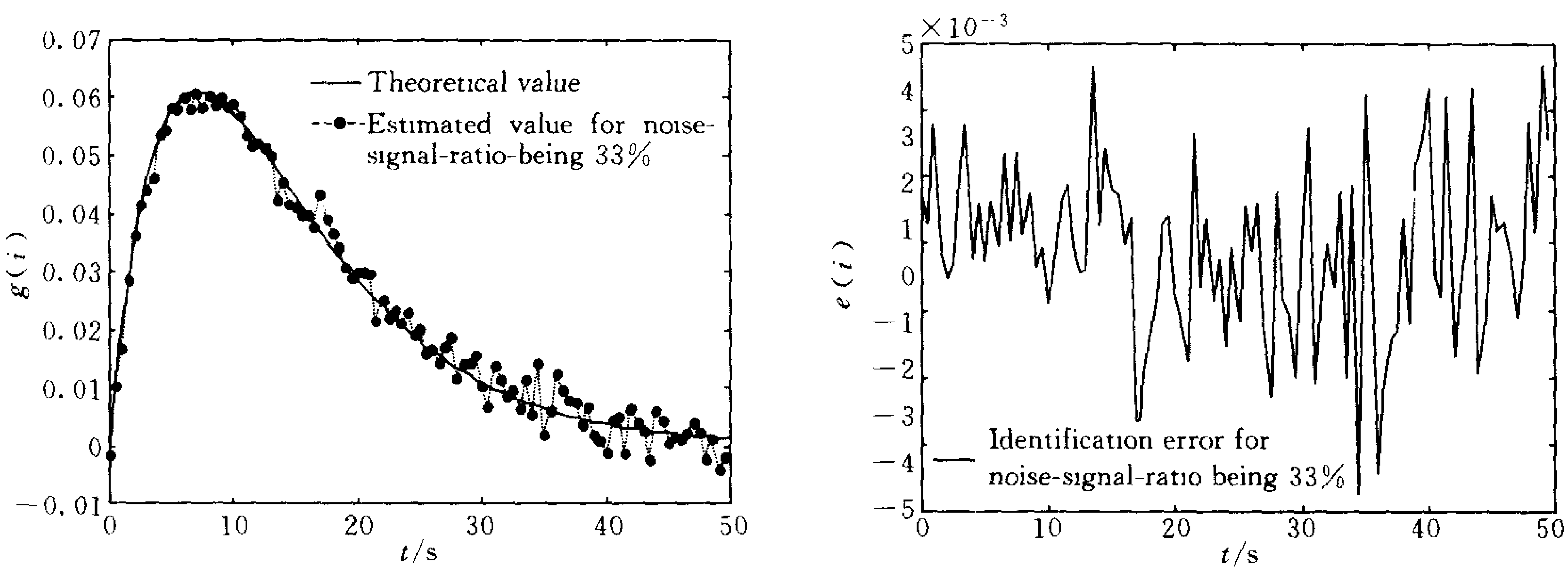

Fig. 6 Impulse response identification results for system output with noise-signal-ratio being 33% (where $j_0=-1$)

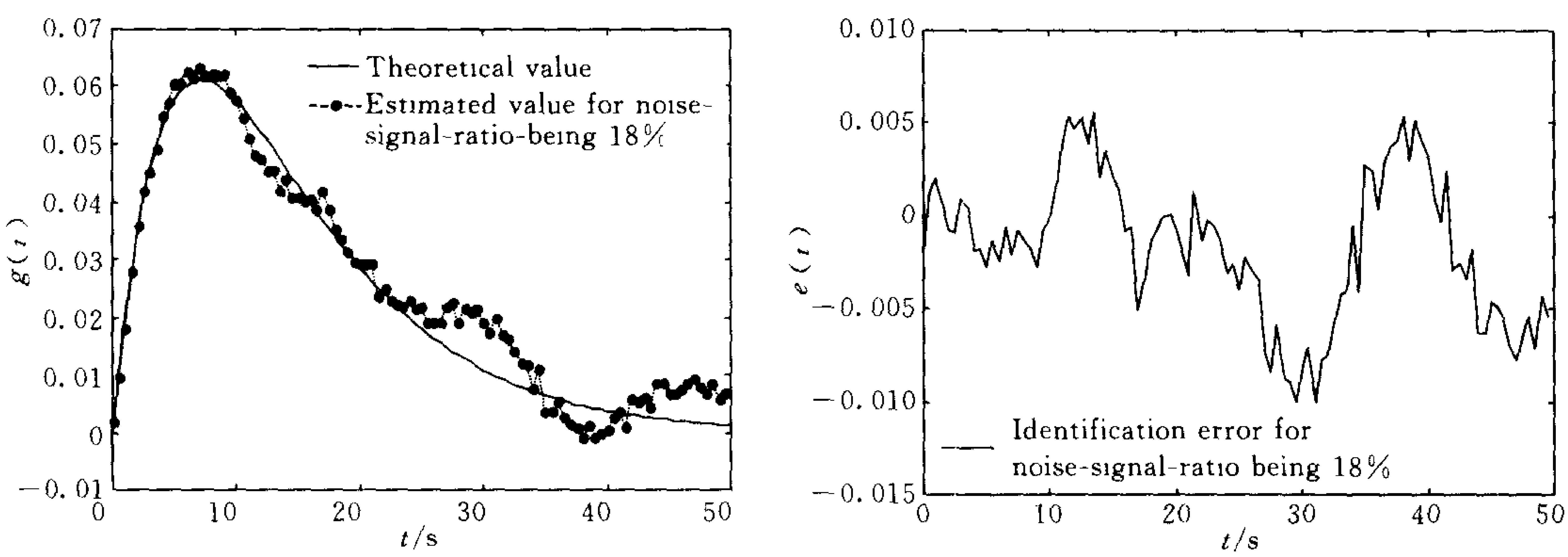

Fig. 7 Impulse response identification results based on correlation analysis algorithm for system output with noise-signal-ratio being 18%

7.0481e−6 for identification error $e(i)$ when noise-signal-ratio is 18%, and variance $\hat{\sigma}_{33}=$ 8.2515e−6 for identification error $e(i)$ when noise-signal-ratio is 33%. The impulse response identification result comparsions of the proposed algorithm in this paper and correlation analysis algorithm are listed in Table 1, where initial scale $j_0=-1$. Table 1 shows that for obtaining same identification precision the sampling data length required by the proposed algorithm is much shorter than required by correlation analysis algorithm.

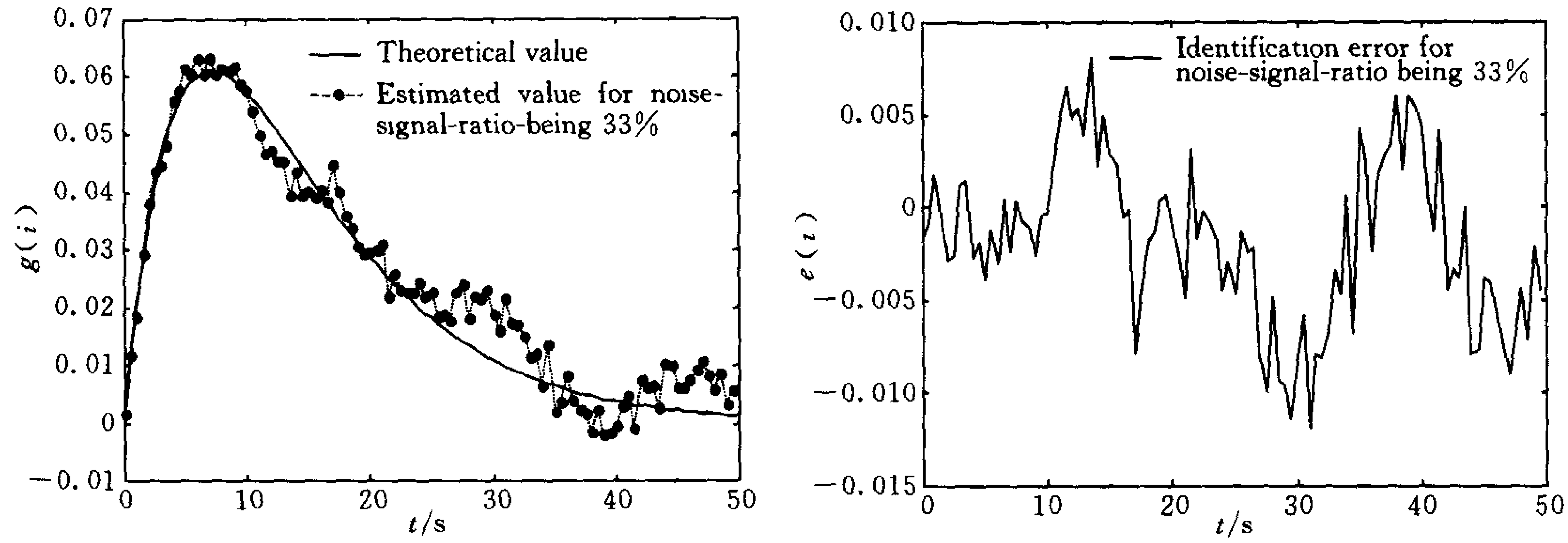

Fig. 8 Impulse response identification results based on correlation analysis algorithm for system output with noise-signal-ratio being 33%

Table 1 Impulse response identification result comparisions of two algorithms

Noise-signal-ratio (%)	The proposed impulse response identification algorithmin this paper (L=1000)	Correlation analysis algorithm (L=1000)	Correlation analysis algorithm (L=5600)
18	2.4798e−6	1.8038e−5	7.0481e−6
33	6.8803e−6	2.2320e−5	8.2515e−6

Furthermore, in order to know the effect of initial scale j_0 selection for impulse response identification precision, some numerical simulations on the condition that $j_0=0$, -2 and -3, and noise-signal-ratio takes 0%, 18% and 33% are carried out. Limited by the space of this paper, all impulse response identification result curves and their identification error curves are not presented in this paper. Only the identification errors are listed in Table. 2. It indicates that the higher identification precision , the smaller initial scale j_0, and that the proposed impulse response identification algorithm has higher identification precision and better capacity of anti-noise interference.

Table 2 Variance comparision of impulse response identification errors with different simulation conditions

j_0 \ $\hat{\sigma}$ \ Noise-signal-ratio	0%	18%	33%
0	3.8179e−6	4.5357e−6	6.3415e−6
−1	1.1676e−6	2.4798e−6	6.8803e−6
−2	3.3043e−7	2.0589e−6	6.2389e−6
−3	9.0960e−8	1.9440e−6	5.5449e−6

7 Conclusion

In this paper a new impulse response identification algorithm based on the fixed scale orthogonal wavelet packet transform is proposed . The proposed algorithm can be applied to deterministic process and random process very well. Numerical simulation is given to demonstrate the advantages of the proposed algorithm such as higher identification precision, better capacity of tracking system dynamic behavior and anti-noise interference. Compared with correlation analysis algorithm, for the same identification precision the sampling data length required by the proposed algorithm is much shorter. Furthermore, the identified wavelet packet transform coefficients of impulse response $g(t)$, which are of deterministic physics meaning, can be directly used to accurately analyze system frequency distribution.

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基于固定尺度正交小波包变换的脉冲响应辨识方法研究

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摘　要　小波包变换能作为一种有效工具对信号特性进行精细分析,同时提供具有良好时频局部性的正交小波包基.本文将 Eykhoff 脉冲响应辨识方法中的正交函数基取成正交小波基函数,提出了一种新的脉冲响应辨识方法,该方法不但保证了原来 Eykhoff 方法的有效性,而且实用性更好、实用范围更广.仿真结果表明,所提出的脉冲响应辨识算法对于确定性过程和随机过程都具有很好的辨识结果,具有辨识精度高、克服噪声干扰能力和跟踪系统动态特性变化能力强的优点.

关键词　小波包变换,时频分析,Eykhoff 方法,脉冲响应辨识

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