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A Fairness and Liveness Control Policy of Petri Net Models for Automated Manufacturing Systems[1)]

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Abstract Based on invariants of Petri nets, an approach to the enforcement of fairness and liveness is presented for classic automated manufacturing systems. First of all, a fair net is obtained by adding some places that make the net have only one T-invariant. Then, a fair net is enforced to be live through controlling minimal siphons by P-invariants. Importantly, the concept of redundant strict minimal siphons is put forward, which can greatly simplify the synthesis and analysis of Petri net model of the system considered. Generally, the set of non-redundant strict minimal siphons is a much small subset of the set of strict minimal siphons in a Petri net, particularly in large-scale ones. The results show that all strict minimal siphons cannot be emptied if non-redundant strict minimal siphons are controlled. Examples are presented to illustrate these approaches. The results obtained can be applied to a larger class of flexible manufacturing systems and are of significance to Petri nets based scheduling problems for automated manufacturing systems.

Key words Petri net, automated manufacturing system, liveness and fairness, non-redundant SMS

1 Introduction

A variety of Petri nets[1,2] based techniques have been developed for dealing with deadlock problems arising in an FMS (Flexible Manufacturing System). The first one is called deadlock prevention. The aim of this approach is to design a system such that its corresponding Petri net model is live by restricting the number of raw materials entering into the system[3~5]. Therefore, it is logically impossible for deadlocks to occur. This method has a high conservative effect that will degrade productivity and resource utilization although liveness is guaranteed at global level. The second approach is to control the requests for resources and avoid the system in a deadlock state. If a request for a resource may lead to a deadlock, it will not be granted then [6]. The third one is to modify Petri net structures by adding some control places and make the Petri net deadlock-free or live. For instance, in [7] a control place is added for each strict minimal siphon (SMS) and therefore no siphon can be emptied. The last one is called deadlock detection and recovery. This policy will not painstakingly pursue deadlock-freeness or liveness in a system. It will permit deadlocks to occur. Once a deadlock is detected, some recovery actions can be done either manually or automatically[8,9]. Generally, high resource utilization and productivity can be reached using this policy. However, some control programs have to be pre-designed for dealing with deadlock problems when a deadlock indeed occurs. In addition, some auxiliary devices may be needed.

The deadlock control strategy proposed in this paper falls into the third category. By structure analysis for a Petri net, an algebraic technique is developed to ensure that the target net model is deadlock-free and live for many kinds of automated manufacturing systems. And furthermore, this control policy is minimally restrictive to the behavior of a

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system. The concept of RSMSs (redundant SMSs) is of significance to the design of control algorithms and controllers for FMSs. However, the deadlock control methods available do not consider the fairness problem in a control system. Fairness means starvation-freeness, which is an important criterion of progressity for a distributed system with shared resources. Consequently, a live and fair model is high-level system modeling.

2 Basic Petri net definitions and notations

A Petri net is a 3-tuple $N=(P, T, F)$ where P and T are finite, nonempty, and disjoint sets. P is the set of places and T is the set of transitions. $F\subseteq(P\times T)\cup(T\times P)$ is called flow relation or the set of directed arcs. The preset of a node $x\in P\cup T$ is defined as ${}^{\bullet}x=\{y\in P\cup T\mid(y, x)\in F\}$. The postset of a node $x\in P\cup T$ is defined as $x^{\bullet}=\{y\in P\cup T\mid(x, y)\in F\}$. The preset (postset) of a set is defined as the union of the presets (postsets) of its elements. A marking of N is mapping M: $P\rightarrow IN$, $IN=\{0,1,2,3,\cdots\}$. (N, M) is called a net system or a marked net. A transition $t\in T$ is enabled under M, in symbols $M[t\rangle$, iff $\forall p\in{}^{\bullet}t$: $M(p)>0$ holds. If $M[t\rangle$ holds the transition t may fire, resulting in a new marking M', denoted by $M[t\rangle M'$, with $M'(p)=M(p)-1$ if $p\in{}^{\bullet}t\backslash t^{\bullet}$; $M'(p)=M(p)+1$ if $p\in t^{\bullet}\backslash{}^{\bullet}t$; and otherwise $M(p)=M'(p)$, for all $p\in P$. The set of all markings reachable from a marking M_0, in symbols $R(N, M_0)$, is the smallest set in which $M_0\in R(N,M_0)$ and $M'\in R(N, M_0)$ if both $M\in R(N, M_0)$ and $M[t\rangle M'$ hold. For transition $t_1, t_2,\cdots, t_n\in T$, $\sigma=t_1t_2\cdots t_n$ is an occurrence sequence, in symbols $M_0[\sigma\rangle M_n$, iff there exist markings $M_0, M_1,\cdots, M_n$ such that $M_0[t_1\rangle\cdots[t_n\rangle M_n$ holds. $\#(\sigma, t)$ denotes the numberof times that t appears in σ. Let (N, M_0) be a net system and $N=(P, T, F)$. A transition $t\in T$ is live under M_0 iff $\forall M\in R(N, M_0)\ \exists M'\in R(N, M)$: $M'[t\rangle$ holds. N is dead under M_0 iff $\neg\exists t\in T$: $M_0[t\rangle$ holds. (N, M_0) is deadlock-free iff $\forall M\in R(N, M_0)\ \exists t\in T$: $M[t\rangle$ holds. (N, M_0) is live iff $\forall t\in T$: t is live under M_0. (N, M_0) is bounded iff $\exists k\in \mathrm{IN}\backslash\{0\}\ \forall M\in R(N, M_0)\ \forall p\in P$: $M(p)\leqslant k$ holds. $N=(P, T, F)$ is pure iff $\neg\exists(x, y)\in(P\times T)\cup(T\times P)$: $(x, y)\in F\wedge(y, x)\in F$. We assume that in the following all Petri nets are bounded and pure since the Petri net models of many physical systems are bounded and a non-pure net can be transformed into a pure one while keeping the dynamic behaviors. Again let $N=(P,T,F)$ be a net. A P-vector is a column vector $I:P\rightarrow Z$ indexed by P and a T-vector is a column vector $J: T\rightarrow Z$ indexed by T, where Z is the set of integers. The incidence matrix of N is a matrix $[N]$: $P\times T\rightarrow Z$ indexed by P and T such that $[N](p, t)=-1$ if $p\in{}^{\bullet}t\backslash t^{\bullet}$; $[N](p, t)=1$ if $p\in t^{\bullet}\backslash{}^{\bullet}t$; and otherwise $[N](p,t)=0$ for all $p\in P$ and $t\in T$. We denote column vectors where every entry equals 0(1) by $\mathbf{0}(\mathbf{1})$. I^{T} and $[N]^{\mathrm{T}}$ are the transposed versions of a vector I and a matrix $[N]$, respectively. Let I be a P-vector and J a T-vector of $N=(P, T, F)$. I is a P-invariant (place invariant) iff $I\neq 0$ and $I^{\mathrm{T}}\cdot[N]=\mathbf{0}^{\mathrm{T}}$ holds. J is a T-invariant (transition invariant) iff $J\neq 0$ and $[N]\cdot J=\mathbf{0}$ holds. $\|I\|=\{p\in P\mid I(p)\neq 0\}$ ($\|J\|=\{t\in T\mid J(t)\neq 0\}$) is called the support of $I(J)$. A nonempty set $D\subseteq P$ is a siphon iff ${}^{\bullet}D\subseteq D^{\bullet}$ holds. A siphon is minimal iff there is no siphon contained in D as a proper subset. $M(p)$ indicates the number of tokens on p under M for all $p\in P$. p is marked by M iff $M(p)>0$. A subset $D\subseteq P$ is marked by M iff at least one place in D is marked by M. The sum of tokens on all places in D is denoted by $M(D)$, $M(D)=\Sigma_{p\in D}M(p)$. Let (N, M_0) be a net system with $N=(P, T, F)$, let I be a P-invariant, and $D\subseteq P$ be a siphon of N. The siphon is controlled by the P-invariant I under M_0 iff $I^{\mathrm{T}}\cdot M_0>0$ and $I(p)<0$ for all $p\in P\backslash D$ hold, or equivalently, $I^{\mathrm{T}}\cdot M_0>0$ and $\{p\in P\mid I(p)>0\}\subseteq D$. A minimal siphon that can be emptied is called a strict minimal siphon, SMS for short. $t1$ and $t2$ are in a fair relationship iff $\exists k\in\mathrm{IN}\backslash\{0\}\ \forall M\in R(N,M_0)\ \forall\sigma\in T^{*}$, if $M[\sigma\rangle$ and $\#(\sigma,t_i)=0$, then $\#(\sigma,t_j)\leqslant k$ holds, where $i,j\in\{1,2\}$, $i\neq j$. If any two transitions are in fair relationship in N, N is a fair net. $x_1x_2\cdots x_u$ is called a path in a net, denoted as $EP(x_1, x_u)$, iff $x_i \in P \cup T$, $i \in N_u$, $x_j \in {}^{\cdot}x_{j+1}$, $j \in N_{u-1}$, where $N_u = \{1,2,\cdots,u\}$. The length of a path $EP(x_1, x_u)$, denoted as $\| EP(x_1, x_u) \|$, is defined as the number of nodes in $EP(x_1, x_u)$.

Let (N, M_0) be a net system. If I is a P-invariant of N then $\forall M \in R(N, M_0)$: $I^T \cdot M = I^T \cdot M_0$. Let $D \subseteq P$ be a siphon of N. If D is controlled by a P-invariant I under M_0, D can not be emptied, i.e., $\forall M \in R(N, M_0)$: D is marked under M. If (N, M_0) is dead the set of all unmarked places forms a siphon. If no minimal siphon of N can be emptied (N, M_0) is deadlock-free.

3 PSR: A Petri net for modeling FMSs

Definition 1. (N_i, M_0^i) is a strongly connected state machine, where $N_i = (\{p_0^i\} \cup P_S^i, T_i, F_i)$. That means $\forall t \in T_i$, $|{}^{\cdot}t| = |t^{\cdot}| = 1$ and the following are satisfied: 1) $p_0^i \notin P_S^i$, $M_0^i(p_0^i) \geqslant 1$, 2) $\forall p \in P_S^i$, $M_0^i(p) = 0$, 3) every circuit of N_i contains p_0^i.

N_i can be used to model the processing routings of part P_i. That there are ni circuits in N_i indicates part P_i has ni different processing routings in an FMS, which shows the characteristics of FMSs—flexibility. p_0^i is to model the raw materials in input buffers while $p \in P_s^i$ represents some operation performed on P_i. And hence p is called an operation place.

Definition 2. PSR (Processes with Shared Resources) is a net system (N, M_0) composed by k (N_i, M_0^i)s via shared resources, where $N = (P, T, F)$, satisfying: 1) $P = P_0 \cup P_S \cup P_R$, $P_0 = \bigcup_{i=1}^{k}\{p_0^i\}$, $P_S = \bigcup_{i=1}^{k} P_S^i$, $P_R = \bigcup_{i=1}^{k} P_R^i$, $i \in N_k$, $N_k = \{1,2,\cdots,k\}$, $\forall i \in N_k$, $\exists j \in N_k$, $P_R^i \cap P_R^j \neq \varphi$; 2) $T = \bigcup_{i=1}^{k} T_i$; 3) $F = \bigcup_{i=1}^{k} F_i$; 4) $\forall p \in P_S^i$, $M_0(p) = M_0^i(p) = 0$, $M_0(p_0^i) = M_0^i(p_0^i) \geqslant 1$, $i \in N_k$; 5) $\forall r \in P_R$, $M_0(r) \geqslant 1$; 6) $(P_0 \cup P_S) \cap P_R = \varphi$; ${}^{\cdot\cdot}(p_0^i) \cap P_R = (p_0^i)^{\cdot\cdot} \cap P_R = \varphi$; $\forall r \in P_R$, ${}^{\cdot}r \cap r^{\cdot} = \varphi$; $\forall r \in P_R$, if $\exists t \in T_i$, $r \in {}^{\cdot}t$, then $\exists t' \in T_i$, $r \in t'^{\cdot}$, $\forall r \in P_R$, $\forall i \in N_k$, $|{}^{\cdot}r \cap T_i| = |r^{\cdot} \cap T_i|$, r is called a resource place.

A PSR has the following properties: 1) there are m P-invariants $I_1, I_2, \cdots, I_m$, and $\| I_1 \| \cup \cdots \cup \| I_m \| = P$ and $m = n1 + n2 + \cdots + nk + |P_R|$; 2) a PSR is covered by P-invariants, i.e., $\forall p \in P \ \exists I \in N_m$, $p \in \| I_i \|$; 3) all P-invariants of a PSR are initially marked; 4) a PSR has n T-invariants $J_1, J_2, \cdots, J_n$, $\| J_1 \| \cup \cdots \cup \| J_n \| = T$, and $\forall i, j \in N_n, i \neq j$, $\exists t \in \| J_i \|$, $t \notin \| J_j \|$; 5) a PSR is repetitive; 6) if $D \subseteq P$ is a SMS of a PSR, one will have $M_0(D) \geqslant 2$.

PSRs can model the intrinsic feature of an FMS—flexibility, although it is a subclass of nets.

4 Fair Petri net design

Proposed in this section is a design method for fair Petri nets by adding some control places.

Definition 3. PSR $= (N_0, M_0)$ is a bounded net system with n T-invariants $J_1, J_2, \cdots, J_n$, where $N_0 = (P, T, F)$, $\| J_1 \| = \{t_{11}, t_{12}, \cdots, t_{1,C_1}\}$, $\| J_2 \| = \{t_{21}, t_{22}, \cdots, t_{2,C_2}\}, \cdots$, $\| J_n \| = \{t_{n1}, t_{n2}, \cdots, t_{n,C_n}\}$, and $T = \| J_1 \| \cup \| J_2 \| \cup \cdots \cup \| J_n \|$, $1 \leqslant C_i \leqslant |T|$. $\forall i \in N_n$, the state observer of T-invariant J_i is defined as a place p_{Ji} such that ${}^{\cdot}p_{Ji} := \{t_{i,a_i}\}$, $t_{i,a_i} \in \| J_i \|$ and $\neg \exists j \in N_n$, $j \neq i$, $t_{i,a_i} \in \| J_j \|$. The extended net system with added places $p_{J1}, p_{J2}, \cdots, p_{Jn}$ is denoted by (N_1, M_1), where $N_1 = (P_1, T_1, F_1)$, $P_1 := P \cup \{p_{J1}, p_{J2}, \cdots, p_{Jn}\}$, $T_1 := T$, $F_1 := F \cup \{(t_{1,a_1}, p_{J1}), (t_{2,a_2}, p_{J2}), \cdots, (t_{n,a_n}, p_{Jn})\}$, $\forall p \in P$, $M_1(p) = M_0(p)$, and $\forall i \in N_n$, $M_1(p_{Ji}) \geqslant 1$.

The number of tokens on p_{Ji} is increased by one if transition $t_{i,a_i} \in \| J_i \|$ fires once. Hence ni places like p_{Ji} can be used to observe the behavior of N_i. $M_1(p_{Ji}) \geqslant 1$ is a necessity for p_{Ji}. However $M_1(p_{Ji})$ will be assigned a proper number as the case may be, which will be discussed later.

Theorem 1. Let (N_1, M_1) be a net system, where $N_1=(P_1, T_1, F_1)$. If the set of arcs F' is added such that $\forall M\in R(N_1, M_1)$, $\Sigma_{i=1}^{n} M(p_{Ji})=L$, $L\in N^+$, then $N_2=(P_2, T_2, F_2)$ is a fair net, where $P_2:=P_1$, $T_2:=T_1$, $F_2:=F_1\cup F'$. (N_2, M_2) denotes the net system corresponding to N_2, where $\forall p\in P_2$, $M_2(p)=M_1(p)$.

Proof. By contradiction. Let us assume that $N_2=(P_2, T_2, F_2)$ is not a fair net. By the definition and properties of a PSR, $\exists i, j\in N_n$, $i\neq j$, $\exists t\in \| J_i \|$, $t'\in \| J_j \|$, t and t' are not in fair relationship. That is to say, $\exists$ firable transition sequence σ and a positive integer L', $\#(\sigma, t)=L'>L$ and $\#(\sigma, t')=0$ hold. By Definition 3, the state observer of T-invariant J_i is defined as p_{Ji} with ${}^\bullet p_{Ji}:=\{t_{i,a_i}\}$. Let $t_{i,a_i}:=t$. Therefore there definitely exists $M\in R(N_2, M_2)$ such that $M(p_{Ji})>L$. There is a contradiction with the fact that $\forall M\in R(N_2, M_2)$, $\Sigma_{i=1}^{n} M(p_{Ji})=L$ holds. Consequently, N_2 is a fair net. □

Theorem 2. Let (N_2, M_2) be the net system as stated in Theorem 1. Then $N_2=(P_2, T_2, F_2)$ has a unique T-invariant $J=(1,\cdots,1)^{\mathrm{T}}$, where $\| J \|=T_2=T_1=T$.

Proof. By contradiction. Suppose that N_2 has two T-invariants J_1 and J_2. By the properties of PSRs and T-invariants, there exists $\sigma\in \| J_1 \|^*$ such that $\forall L\in IN$, and we have $M_2[\sigma\rangle$, $\#(\sigma, t)\geqslant L$, and $\#(\sigma, t')=0$, where $t\in \| J_1 \|$, $t'\in \| J_2 \|$. This is obviously contradictive with the fact that N_2 is a fair net. □

Next, we will discuss how to obtain F'. For (N_2, M_2), we have $\forall M\in R(N_2, M_2)$, $\Sigma_{i=1}^{n} M(p_{Ji})=L$. Prompted by the meaning of P-invariants, one can make $\{p_{J1}, p_{J2},\cdots, p_{Jn}\}$ be the support of a P-invariant of N_2. By the definition of P-invariants, if $\{p_{J1}, p_{J2},\cdots, p_{Jn}\}$ is the support of a P-invariant, then $\forall M\in R(N_2, M_2)$, and we can get $\Sigma_{i=1}^{n} M(p_{Ji})=L$. Hence, we have

$$
[0,\cdots 0\cdots,1,\cdots,1\cdots]\cdot
\begin{array}{c}
\begin{array}{cccc} t_{11}\cdots t_{1,a_1}\cdots t_{1,C_1} & t_{21}\cdots t_{2,a_2}\cdots t_{2,C_2} & \cdots & t_{n1}\cdots t_{n,a_n}\cdots t_{n,C_n}\end{array}\\
\left[\begin{array}{cccc}
 & [N_1] & & \\ \hline
x_{11}\cdots 1\cdots x_{1,C1} & x_{21}\cdots x_{2,C2} & \cdots & x_{n1}\cdots x_{n,Cn}\\
y_{11}\cdots y_{1,C1} & y_{21}\cdots 1\cdots y_{2,C2} & \cdots & y_{n1}\cdots y_{n,Cn}\\
 & \cdots & & \\
z_{11}\cdots z_{1,C1} & z_{21}\cdots z_{2,C2} & \cdots & z_{n1}\cdots 1\cdots z_{n,Cn}
\end{array}\right]
\end{array}
\begin{array}{c} p_1\\ \cdots\\ p_{|P|}\\ p_{J1}\\ p_{J2}\\ \cdots\\ p_{Jn}\end{array}
=0^{\mathrm{T}}
$$

The above equation can be simplified to (1). The solution for F' is as a matter of fact to solve the equation system represented by (1). Note that such an equation system has generally multi-solutions. However, we always hope the number of elements in F' is small, and that the smaller the better. The reason seems quite simple. A net with fewer arcs will be easier for us to analyze than the one with more arcs when the two have the same number of places and transitions. Hence we propose the following rules for solving (1) without proof. Only one of $\varphi_{11},\cdots,\varphi_{1,C1}$, $\varphi_{21},\cdots$, $\varphi_{2,C2},\cdots$, $\varphi_{n1},\cdots,\varphi_{n,Cn}$ can be assigned to be -1 and others 0, where $\varphi\in\{x, y,\cdots,z\}$. In fact this rule ensures that $\forall i\in N_n$, $|{}^\bullet p_{Ji}|=|p_{Ji}{}^\bullet|=1$, which in most cases guarantees there are fewer SMSs in (N_2, M_2) (in next section, we use an iterative algorithm to prevent SMSs from being emptied. The smaller number of SMSs undoubtedly means the smaller number of iteration times). (1) has $n-1$ solutions.

$$
[1,1,\cdots,1]\cdot
\begin{array}{c}
\begin{array}{cccc} t_{11}\cdots t_{1,a_1}\cdots t_{1,C_1} & t_{21}\cdots t_{2,a_2}\cdots t_{2,C_2} & \cdots & t_{n1}\cdots t_{n,a_n}\cdots t_{n,C_n}\end{array}\\
\left[\begin{array}{cccc}
x_{11}\cdots 1\cdots x_{1,C1} & x_{21}\cdots x_{2,C2} & \cdots & x_{n1}\cdots x_{n,Cn}\\
y_{11}\cdots y_{1,C1} & y_{21}\cdots 1\cdots y_{2,C2} & \cdots & y_{n1}\cdots y_{n,Cn}\\
 & \cdots & & \\
z_{11}\cdots z_{1,C1} & z_{21}\cdots z_{2,C2} & \cdots & z_{n1}\cdots 1\cdots z_{n,Cn}
\end{array}\right]
\end{array}
\begin{array}{c} p_{J1}\\ p_{J2}\\ \cdots\\ p_{Jn}\end{array}
=0^{\mathrm{T}}
\qquad (1)
$$

By solving (1), every state observer p_{Ji} has an output transition t_i. In a PSR, there exists a unique p_0^i for t_i such that $p_0^i \in P_0$ and $EP(p_0^i, t_i)$ is a path. Let $M_2(p_{Ji}) := M_2(p_0^i)$ so that the behavior of the system will not be restricted due to the join of p_{J1}, p_{J2}, etc. Any of $n-1$ solutions can ensure that $\{p_{J1}, p_{J2}, \cdots, p_{Jn}\}$ is the support of some P-invariant.

5 Design of Petri nets with liveness

By the properties of a Petri net, if no minimal siphon can be emptied it will be deadlock-free. Here we present an approach to prevent a siphon from being emptied due to P-invariants based feedback control[10] and controlled siphons.

Theorem 3. Let D be an SMS of a fair net system $PSR=(N_2, M_2)$, where $N_2=(P_2, T_2, F_2)$ and $D=\{p_i, \cdots, p_j, \cdots, p_k\}$. Add place p_D such that $(0,\cdots,0,1_i,\cdots,1_j,\cdots,1_k,0,\cdots,0,-1)^{\mathrm{T}}$ becomes a P-invariant of the extended net system with p_D, (N_3, M_3), where $N_3=(P_3, T_3, F_3)$, $P_3 := P_2 \cup \{p_D\}$, $T_3 := T_2$, $\forall p \in P_2$, and $M_3(p) := M_2(p)$. Let $M_3(p_D)=M_2(D)-1$. Then D is a controlled siphon.

Proof. Let $I=(0,\cdots,0,1_i,\cdots,1_j,\cdots,1_k,0,\cdots,0,-1)^{\mathrm{T}}$. I is a P-invariant of N_3 and $\forall p \in P_3 \backslash D$, $I(p) \leqslant 0$. Furthermore, $I^{\mathrm{T}} \cdot M_3 = M_3(D) - M_3(p_D) = 1 > 0$. So D is a controlled siphon. □

Clearly, $D \cup \{p_D\}$ is the support of a P-invariant of $N_3=(P_3, T_3, F_3)$.

The solving for F_3 seems simply again. We assume $[N_3]=[[N_2]|V_D]^{\mathrm{T}}$, where V_D is a row vector due to the join of P_D into $[N_2]$. Let $I=(U,-1)^{\mathrm{T}}$, where $U=(0,\cdots,0,1_i,\cdots,1_j,\cdots,1_k,0,\cdots,0)$. One can get $U \cdot [N_2] - V_D = \mathbf{0}^{\mathrm{T}}$, and furthermore, $V_D = U \cdot [N_2]$ due to $I^{\mathrm{T}} \cdot [N_3] = \mathbf{0}^{\mathrm{T}}$.

We have already known if every SMS is controlled in a net system it will be deadlock-free. The results followed show that not all SMSs need to control if one wants a net system to be deadlock-free. Accordingly, what we need to do, in many cases, is to ensure the content of non-redundant SMSs (non-RSMSs). If every non-RSMS becomes controlled in a net system, all RSMSs are also controlled. This is an important result developed in this paper.

Definition 4. Let $N=(P, T, F)$ be a net and $D \subseteq P$ be a siphon of N. P-vector λ_D is called the characteristic P-vector of D iff $\forall p \in D$: $\lambda_D(p)=1$; otherwise $\lambda_D(p)=0$. η_D is called the characteristic T-vector of siphon D with $\eta_D^{\mathrm{T}} = \lambda_D^{\mathrm{T}} \cdot [N]$.

Obviously, $V_D = \lambda_D^{\mathrm{T}} \cdot [N_2] = \eta_D^{\mathrm{T}}$.

Definition 5. Let N be a Petri net and $D_0, D_1, D_2, \cdots, D_n (n \in IN/\{0,1\})$ be SMSs of N. η_{Di} is the characteristic T-vectors of D_i, $i \in N_n \cup \{0\}$. D_0 is called an RSMS (redundant SMS) with respect to $D_1, D_2, \cdots, D_n$ (for brevity, we call D_0 an RSMS) if $\eta_{D1} + \eta_{D2} + \cdots + \eta_{Dn} = \eta_{D0}$ holds.

Theorem 4. Let (N_0, M_0) be a net system and $D_0, D_1, D_2, \cdots, D_n$ be SMSs of N_0. D_0 is an RSMS with respect to $D_1, D_2, \cdots, D_n$. D_0 is a controlled siphon if 1) N_0 is extended by n places $p_{D1}, p_{D2}, \cdots, p_{Dn}$ such that $D_1, D_2, \cdots, D_n$ become controlled siphons, and 2) $M_0(D_0) > \sum_{i=1}^{n} M_0(D_i) - \sum_{i=1}^{n} \xi_{Di}$.

Proof. Let λ_{Di} and η_{Di} be the characteristic P-vector and T-vector of siphon D_i, respectively, $i \in N_n \cup \{0\}$. The extended net system with n places $p_{D1}, p_{D2}, \cdots, p_{Dn}$ is denoted by (N_1, M_1). And its incidence matrix is denoted by $[N_1]$. Obviously, $[N_1]=[[N_0]^{\mathrm{T}} | \eta_{D1}^{\mathrm{T}} | \eta_{D2}^{\mathrm{T}} | \cdots | \eta_{Dn}^{\mathrm{T}}]^{\mathrm{T}}$. And $M_1(p_{Di}) = M_0(D_i) - \xi_{Di}$. Note that $\eta_{Di}^{\mathrm{T}} = \lambda_{Di}^{\mathrm{T}} \cdot [N_0]$ and $\eta_{D0} = \eta_{D1} + \eta_{D2} + \cdots + \eta_{Dn}$. So $(\lambda_{D0}^{\mathrm{T}}, -1_1, -1_2, \cdots, -1_n) \cdot [N_1] = (\lambda_{D0}^{\mathrm{T}}, -1_1, -1_2, \cdots, -1_n) \cdot [N_0^{\mathrm{T}} | \eta_{D1}^{\mathrm{T}} | \eta_{D2}^{\mathrm{T}} | \cdots | \eta_{Dn}^{\mathrm{T}}]^{\mathrm{T}} = \eta_{D0}^{\mathrm{T}} - \eta_{D1}^{\mathrm{T}} - \eta_{D2}^{\mathrm{T}} - \cdots - \eta_{Dn}^{\mathrm{T}} = \mathbf{0}^{\mathrm{T}}$. According to the definition of P-invariants, $(\lambda_{D0}^{\mathrm{T}}, -1_1, -1_2, \cdots, -1_n)^{\mathrm{T}}$ is a P-invariant of N_1. Next, we will prove that D_0 is marked under M_1. $\forall M \in R(N_1, M_1)$, $(\lambda_{D0}^{\mathrm{T}}, -1_1, -1_2, \cdots, -1_n) \cdot M = (\lambda_{D0}^{\mathrm{T}}, -1_1, -1_2,$

$\cdots, -1_n) \cdot M_1 = \lambda_{D0}^{\mathrm{T}} \cdot M_0 - M_1(p_{D1}) - M_1(p_{D2}) - \cdots - M_1(p_{Dn}) = \lambda_{D0}^{\mathrm{T}} \cdot M_0 - (M_0(D_1) - \xi_{D1}) - (M_0(D_2) - \xi_{D2}) - \cdots - (M_0(D_n) - \xi_{Dn}) = M_0(D_0) - (M_0(D_1) + M_0(D_2) + \cdots + M_0(D_n) - \xi_{D1} - \xi_{Di} - \cdots - \xi_{Dn}) = M_0(D_0) - (\sum_{i=1}^{n} M_0(D_i) - \sum_{i=1}^{n} \xi_{Di})$. Due to the fact that $M_0(D_0) > \sum_{i=1}^{n} M_0(D_i) - \sum_{i=1}^{n} \xi_{Di}$ holds, we have $(\lambda_{D0}^{T}, -1_1, -1_2, \cdots, -1_n) \cdot M > 0$. That is to say, $\forall M \in R(N_1, M_1)$, $(\lambda_{D0}^{T}, -1_1, -1_2, \cdots, -1_n) \cdot M > 0$. Hence D_0 is a controlled siphon. □

Definition 6. Let (N_0, M_0) be a net system and Π be the set of SMSs of N_0. $\forall D \in \Pi$, D is called a non-RSMS if $\neg \exists D_1, D_2, \cdots, D_n \in \Pi$ such that $\eta_{D1} + \eta_{D2} + \cdots + \eta_{Dn} = \eta_{D0}$ holds, where $n \in IN \setminus \{0, 1\}$.

Note that if we use non-RSMSs to prevent SMSs from being emptied, two conditions must be satisfied for ξ_{Di}, which are 1) $1 \leqslant \xi_{Di} \leqslant M_0(D_i) - 1$ and 2) $M_0(D_0) > \sum_{i=1}^{n} M_0(D_i) - \sum_{i=1}^{n} \xi_{Di}$. The smaller ξ_D is, the more tokens p_D will have at the initial marking. That means the Petri net will have more permissive behaviour. So we strongly recommend that $\xi_D = 1$ if possible. However, in some cases ξ_D may have to be greater than 1.

Definitioin 7. Let $N = (P, T, F)$ be a net. Suppose that N has k SMSs $D_1, D_2, \cdots, D_k$, where $|P| = m$, $|T| = n$, $m, n, k \in IN$. $\lambda_{Di}(\eta_{Di})$ is the characteristic $P(T)$-vector of SMS D_i. We define $[\lambda]_{k \times m} = [\lambda_{D1} | \lambda_{D1} | \cdots | \lambda_{Dk}]^{\mathrm{T}}$ and $[\eta]_{k \times n} = [\lambda]_{k \times m} \times [N]_{m \times n} = [\eta_{D1} | \eta_{D1} | \cdots | \eta_{Dk}]^{\mathrm{T}}$. We call $[\lambda]([\eta])$ the characteristic $P(T)$-vector matrix of the SMSs of N.

Theorem 5. Let $N = (S, T, F)$ be a Petri net. The number of non-RSMSs in N is equal to the rank of $[\eta]_{k \times n}$.

Proof. We assume N has k SMSs and k' non-RSMSs $(k \geqslant k')$. Then there are $k - k'$ RSMSs in N. By Definition 5, $\eta_{Di}(i = k'+1, k'+2, \cdots, k)$ can be linearly represented by $\eta_{Dj}(j = 1, 2, \cdots, k')$. We have that the rank of $[\eta]_{k \times n}$ is k' according to the definition of the rank of a matrix. □

Due to Theorem 5, the approach becomes trivial to find non-RSMSs in Petri net N. First, we construct matrices $[\lambda]$ and $[\eta]$ for net N. Then linearly independent vectors can be found in $[\eta]$. Finally, we can find the siphons that correspond to these linearly independent vectors. These siphons are non-RSMSs.

Let (N, M_0) be a net system, V be the set of SMSs of N, and $VR \subset V$ be the set of RSMSs of N. Clearly, $VN = V \setminus VR$ is the set of non-RSMSs. By Theorem 4, not all SMSs need to consider when we use siphons to control the deadlocks in a Petri net. This is trivial since if all non-RSMSs are controlled, the RSMSs cannot be emptied. Note that the concept of RSMSs is not developed only for PSRs. It is suitable for any class of Petri nets, which is of significance to the design and analysis of control system. Next, we will propose a deadlock-free design algorithm for Petri nets.

Step 1. Get VN of (N_2, M_2).

Step 2. If $VN = \Phi$, go to Step 4, otherwise, go to Step 3.

Step 3. For every SMS, add a control place and make the SMS controlled, then go to Step 1.

Step 4. The final net system (N_3, M_3) is deadlock-free. And go to Step 5.

Step 5. Over.

Next, we will make some explanations for this algorithm. Suppose that there are k SMSs and m deadlock markings in the initial net system (we assume k SMSs are all non-RSMSs, which is certainly the worst case). Add k control places and k SMSs become controlled. By the definitions of SMSs and PSRs, we have $^{\cdot\cdot}(p_0^i) \cap P_R = (p_0^i)^{\cdot\cdot} \cap P_R = \varphi$, $\forall r \in P_R$, $^{\cdot}r \cap r^{\cdot} = \varphi$, $\forall r \in P_R$, if $\exists t \in T_i$, $r \in {}^{\cdot}t$, then $\exists t' \in T_i$, $r \in t'^{\cdot}$, $\forall r \in P_R$, $\forall i \in N_n$, $|^{\cdot}r \cap T_i| = |r^{\cdot} \cap T_i|$. And due to the properties of PSRs and Theorem 3, the number of deadlock markings in the extended net system will not be greater than that of the initial one (Note that new siphons will generate if new places are added. So the number of siphons in the extended net may excess that of the initial one. However, the number of the controlled siphons in the extended net system is much greater than that of the old one). Assuredly, the algorithm will terminate after finite iterations. Furthermore, this iteration process will have a quick convergence if we consider the existence of RSMSs.

Of course, not any Petri net can be enforced to be deadlock-free or live via adding some control places. [11] shows that only (partially) repetitive Petri nets can be enforced to be live (deadlock-free). And a PSR is repetitive (see property 5 of PSRs) so this algorithm is feasible.

6 Fairness is kept in the deadlock-free Petri nets

Theorem 6. A PSR (N_0, M_0) is enforced to be a fair net (N_1, M_1). (N_1, M_1) is enforced to be adeadlock-free net system (N, M) via adding some places to make all SMSs controlled. (N, M) is a fair net.

Proof. We have to prove that (N, M) has a unique T-invariant $J=(1,\cdots,1)^{\mathrm{T}}$ and $\| J \| = T$, where T is the set of transitions of N and N_1. The mathematical induction over the number of SMSs is employed here.

First, we assume (N_1, M_1) has only one SMS, $D=\{p_i,\cdots, p_j, \cdots, p_k\}$. Let $U=(0,\cdots,0,1_i,\cdots,1_j,\cdots, 1_k,0, \cdots,0)$. So we have $[N]=[[N_1]|U\cdot[N_1]]^{\mathrm{T}}$ and $(U\cdot[N_1])\cdot J=\mathbf{0}$ due to $[N_1]\cdot J=\mathbf{0}$. Hence $[N]\cdot J=\mathbf{0}$ holds. Next, we will prove $J=(1,\cdots,1)^{\mathrm{T}}$ is unique. By contradiction. Suppose that N has the other T-invariant J', $J'\neq J$. Since N_1 is a fair net, J is the unique T-invariant of N_1. So we have $[N_1]\cdot J=\mathbf{0}\Rightarrow [N_1]\cdot J'\neq\mathbf{0}\Rightarrow [N]\cdot J'\neq\mathbf{0}$. Hence, J' is not a T-invariant of N. And there is a contradiction with the fact that N has the other T-invariant J'. That means $J=(1,\cdots,1)^{\mathrm{T}}$ is the unique T-invariant of (N, M).

Next, we assume that the theorem holds when (N_1, M_1) has L SMSs. Following the case when (N_1, M_1) has only one SMS, we can have that the conclusion also holds when (N_1, M_1) has $L+1$ SMSs. Therefore, the theorem holds. □

Theorem 7. A fair and deadlock-free Petri net (N, M) is live.

Proof. By contradiction. We assume that (N, M) is just deadlock-free but not live. Hence, there exist a live transition t, a dead transition t', and an occurrence sequence σ such that $\forall m\in N^+$, $\#(\sigma, t)>m$ and $\#(\sigma, t')=0$ hold. This is contradictive with the fairness of net N. Consequently, (N, M) is live. □

7 Example

An FMS cell consists of four machine tools, $M1$, $M2$, $M3$, and $M4$, three robots, $R1$, $R2$, and $R3$, two input buffers, $I1$ and $I2$, and three output buffers, $O1$, $O2$, and $O3$. The cell can produce two kinds of products, $P1$ and $P2$, with the processing routings as follows. $P1$: $I1\rightarrow R1\rightarrow M3\rightarrow R3\rightarrow M4\rightarrow R3\rightarrow O1$ or $I1\rightarrow R1\rightarrow M1\rightarrow R2\rightarrow M2\rightarrow O2$ and $P2$: $I2\rightarrow M2\rightarrow R2\rightarrow M1\rightarrow O3$. The Petri net model of this cell is shown in Figure 1.

(N_0, M_0) is a PSR with three T-invariants, $\| J_1 \| =\{t_1, t_2, t_3, t_4, t_5, t_6\}$, $\| J_2 \| =\{t_1, t_7, t_8, t_9, t_{10}\}$, and $\| J_3 \| =\{t_{11}, t_{12}, t_{13}, t_{14}\}$. For J_1, we add place p_{J1} such that $^{\bullet}p_{J1}=\{t_2\}$(Note that t_1 cannot be selected as the input transition of p_{J1}). Similarly, one can get $^{\bullet}p_{J2}=\{t_7\}$and $^{\bullet}p_{J3}=\{t_{11}\}$. The extended net system (N_1, M_1) with added places p_{J1}, p_{J2} and p_{J3} is shown in Figure 2. $\{p_{J_i}|i=1, 2, 3\}$ is the support of a P-invariant of N_2. So equation (2) holds. By solving (2), we have two solutions, 1) $x_{21}=-1$, $y_{31}=-1$, $z_{12}=-1$, and other elements are zero, and 2) $x_{31}=-1$, $y_{12}=-1$, $z_{21}=-1$, and other elements are zero. The fair net system (N_2, M_2) is shown in Figure 3 if we choose the first solution, where $M_2(p_{J1})=M_0(p_1)=9$, $M_2(p_{J2})=M_0(p_{10})=8$, $M_2(p_{J3})=M_0(p_1)=9$.

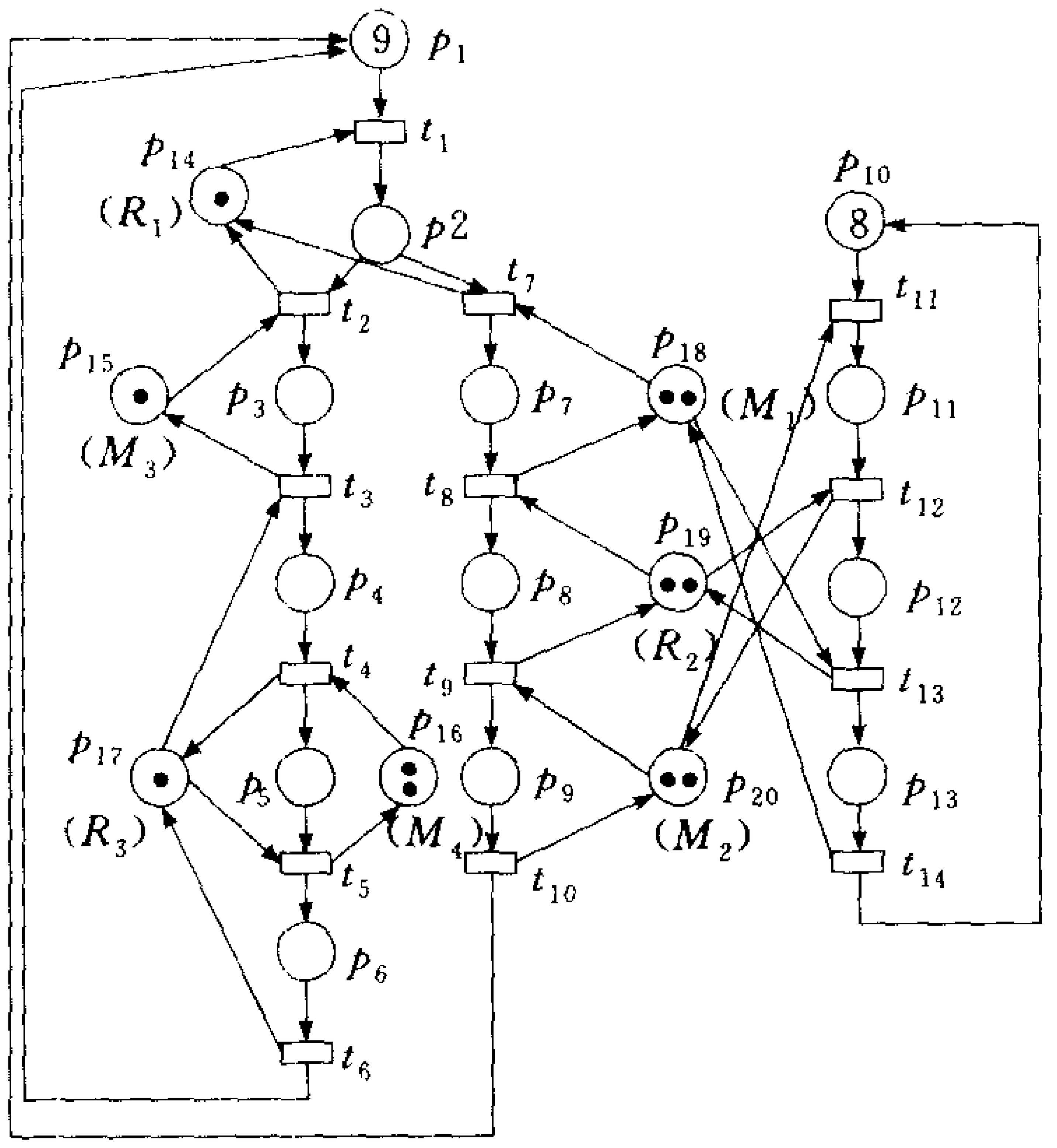

Fig. 1 Petri net model (N_0, M_0) for an FMS

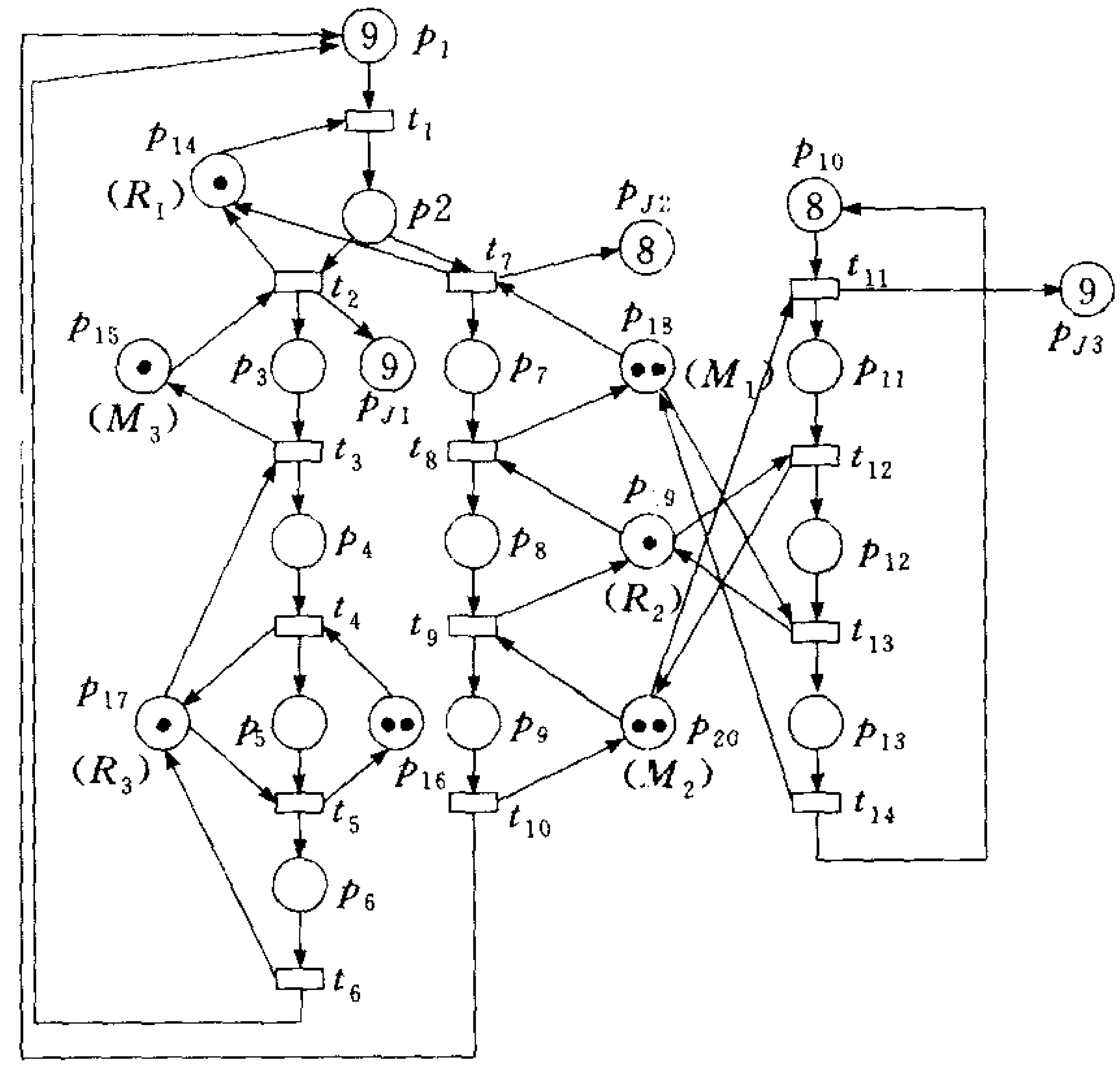

Fig. 2 Petri net model(N_1, M_1) with state observers

$$[1,1,1]\cdot\begin{array}{c}\begin{array}{cccccccccccccc} t_1 & t_2 & t_3 & t_4 & t_5 & t_6 & t_7 & t_8 & t_9 & t_{10} & t_{11} & t_{12} & t_{13} & t_{14}\end{array}\\ \left[\begin{array}{cccccccccccccc} x_{11} & 1 & x_{13} & x_{14} & x_{15} & x_{16} & x_{21} & x_{22} & x_{23} & x_{24} & x_{31} & x_{32} & x_{33} & x_{34}\\ y_{11} & y_{12} & y_{13} & y_{14} & y_{15} & y_{16} & 1 & y_{22} & y_{23} & y_{24} & y_{31} & y_{32} & y_{33} & y_{34}\\ z_{11} & z_{12} & z_{13} & z_{14} & z_{15} & z_{16} & z_{21} & z_{22} & z_{23} & z_{24} & 1 & z_{32} & z_{33} & z_{34}\end{array}\right]\end{array}=\mathbf{0}^{\mathrm{T}} \quad (2)$$

There are four SMSs in Figure 3, $D_1=\{p_6, p_{16}, p_{17}\}$, $D_2=\{p_8, p_{13}, p_{18}, p_{19}\}$, $D_3=\{p_9, p_{12}, p_{19}, p_{20}\}$, and $D_4=\{p_9, p_{13}, p_{18}, p_{19}, p_{20}\}$. We can have $\lambda^{\mathrm{T}}_{D1}=(0,0,0,0,0,1,0,0,0,0,0,0,0,0,0,1,1,0,0,0)$, $\lambda^{\mathrm{T}}_{D2}=(0,0,0,0,0,0,0,1,0,0,0,0,1,0,0,0,0,1,1,0)$, $\lambda^{\mathrm{T}}_{D3}=(0,0,0,0,0,0,0,0,1,0,0,1,0,0,0,0,0,0,1,1)$, $\lambda^{\mathrm{T}}_{D3}=(0,0,0,0,0,0,0,0,1,0,0,0,1,0,0,0,0,1,1,1)$, $\eta^{\mathrm{T}}_{D1}=(0,0,-1,0,1,0,0,0,0,0,0,0,0,0)$, $\eta^{\mathrm{T}}_{D2}=(0,0,0,0,0,0,-1,1,0,0,0,-1,1,0)$, $\eta^{\mathrm{T}}_{D3}=(0,0,0,0,0,0,0,-1,1,0,-1,1,0,0)$, and $\eta^{\mathrm{T}}_{D4}=(0,0,0,0,$

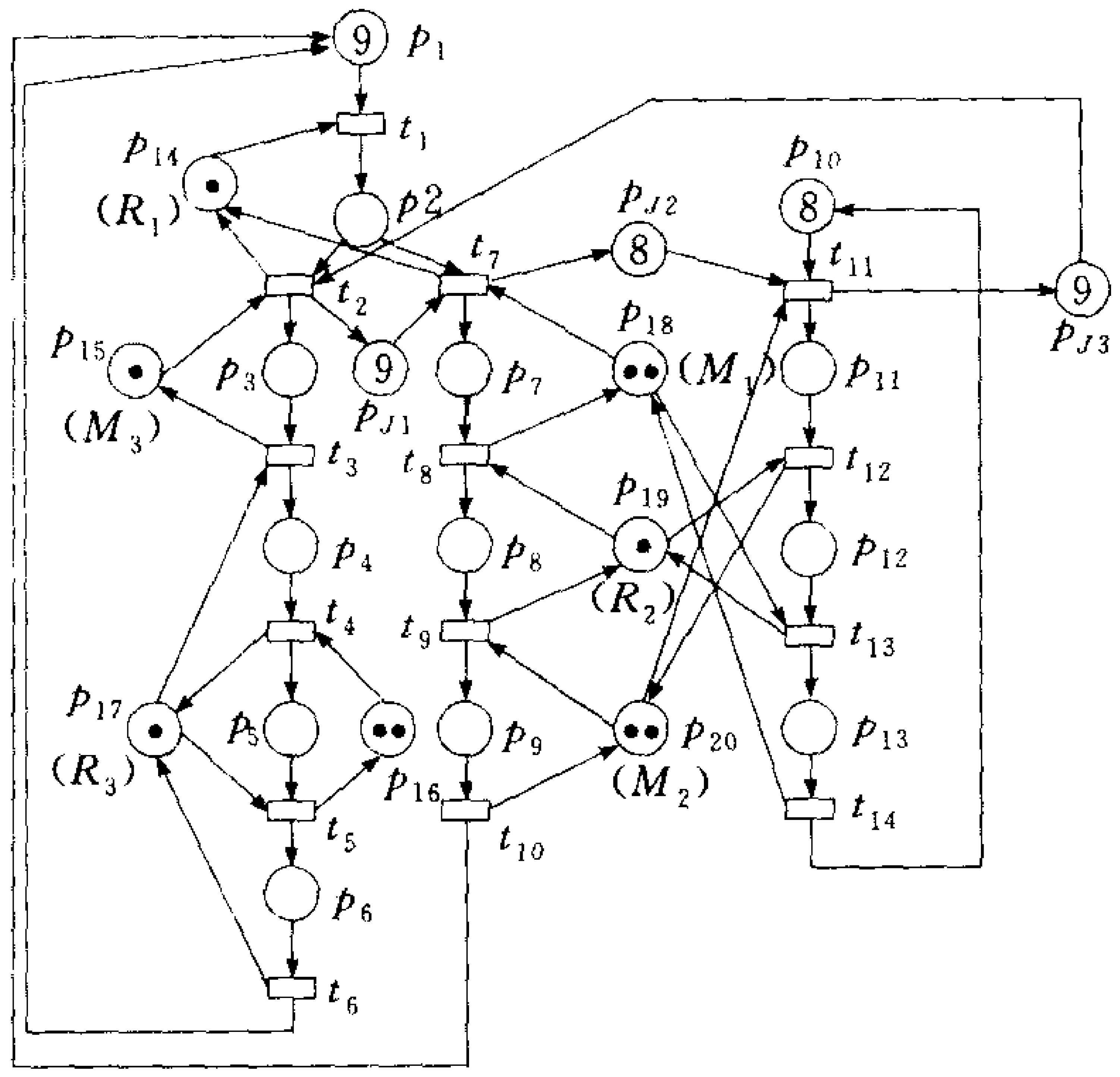

Fig. 3 A fair Petri net system(N_2, M_2)

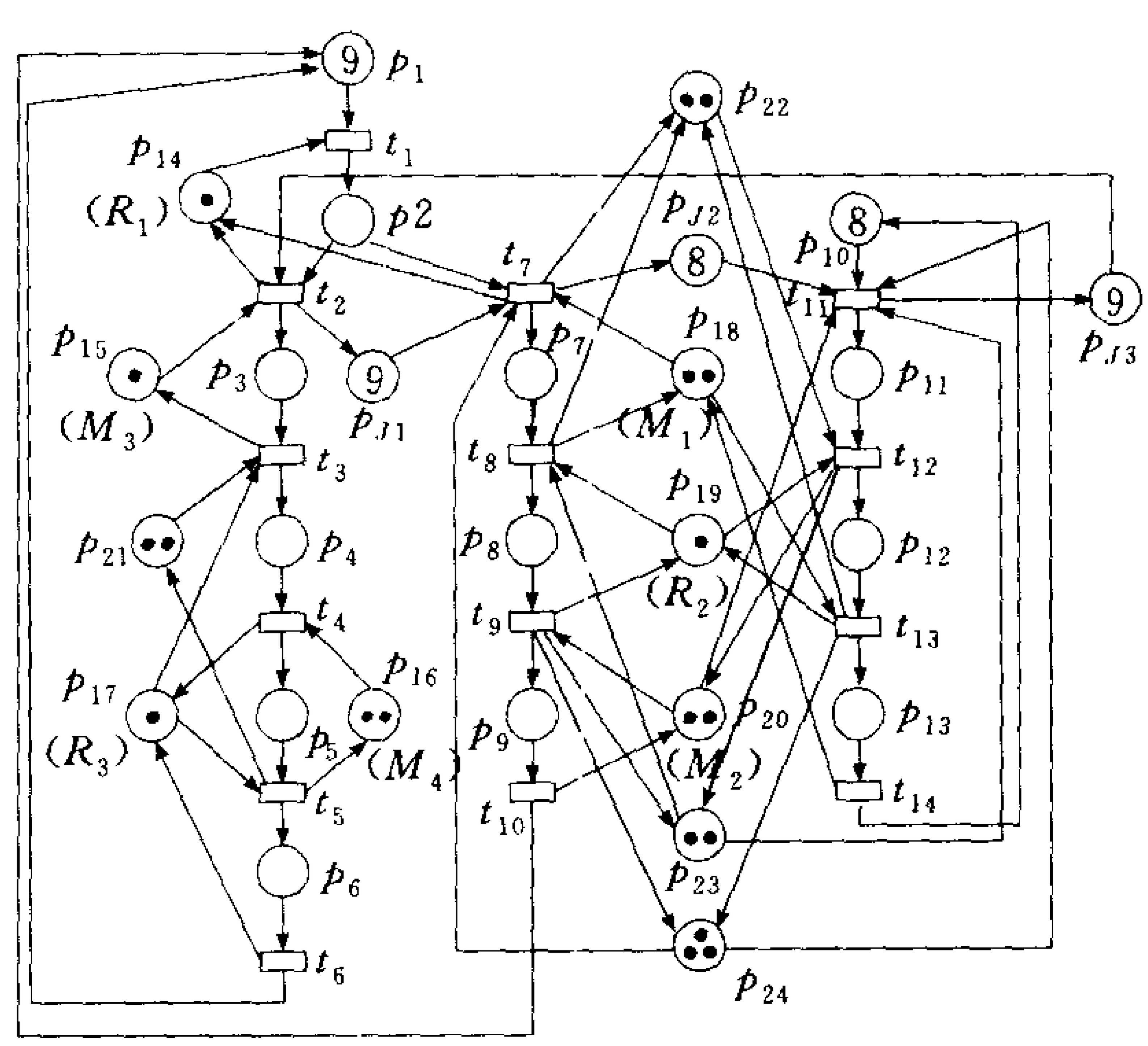

Fig. 4 A live and fair net system(N_3, M_3)

0,0,−1,0,1,0,−1,0,1,0). So we have $\eta_{D4}=\eta_{D2}+\eta_{D3}$ and $M_2(D_4)>M_2(D_2)+M_2(D_3)-2$. Hence D_4 is an RSMS with respect to D_2 and D_3. Add places p_{21}, p_{22}, and p_{23} such that D_1, D_2, and D_3 become controlled. So D_4 is controlled. The net with p_{21}, p_{22}, and p_{23} contains a new SMS, $D_5=\{p_9, p_{13}, p_{22}, p_{23}\}$. Add place p_{24} to make D_5 controlled. Now there is no SMS in the extended net and hence it is deadlock-free, as shown in Figure 4, where $M_3(p_{21})=M_2(p_{16})+M_2(p_{17})-1=2$, $M_3(p_{22})=M_2(p_{18})+M_2(p_{19})-1=2$, $M_3(p_{23})=M_2(p_{19})+M_2(p_{20})-1=2$, and $M_3(p_{24})=M_3(p_{22})+M_3(p_{23})-1=3$. The net system in Figure 4 is fair and live.

8 Conclusion and discussion

Based on the T-invariants, we develop a design method of fairness for a class of bounded Petri nets. Then a design approach for deadlock-free Petri nets is presented due to P-invariants and controlled siphons. We have also proved that a fair and deadlock-free net system is a live one. These techniques are demonstrated with an example. The results obtained show that the concept of RSMSs proposed in this paper can be of great significance to the simplification of the design and analysis of the control systems for FMSs. For instance, the Petri net model for an FMS cell[8] has 18 SMSs while the number of non-RSMSs is only six. That means 12 SMSs need not consider when we try to ensure no siphon to be emptied. For this example, 12 places and 74 arcs are saved using the concept of RSMSs. The further research interests include what kind of Petri nets can be enforced to be live via adding control places and how to guarantee no siphon to become emptied when there are uncontrolled transitions in a Petri net.

References

1 Murata T. Petri nets: Properties, analysis, and applications. *In Proceedings of the IEEE*, 1989, **77**(4):541~580

2 Reisig W. Petri Nets: An Introduction. Berlin: Springer-Verlag, 1985

3 Li Zhi-Wu. A formal design method of Petri net controllers for sequential manufacturing systems. *Acta Automatica Sinica*, 1997,**23**(2):213~219

4 Espeleta J, Martinez J. Synthesis of live models for a class of FMSs. In: Proceedings of the IEEE International Conference on Robotics and Automation. Altanta: IEEE Computer Society Press, 1993. 557~563

5 Zhou M C, DiCesare F. Parallel and sequential mutual exclusion for Petri net modeling of manufacturing systems with shared resources. *IEEE Transactions on Robotics and Automation*, 1991, **7**(4): 515~527

6 Bananszak Z, Krogh B H. Deadlock avoidance in flexible manufacturing systems with concurrently competing process flows. *IEEE Transactions on Robotics and Automation*, 1990, **6**(6):724~734

7 Ezpeleta J, Manuel J, Martinez J. A Petri net based deadlock prevention policy for flexible manufacturing systems. *IEEE Transactions on Robotics and Automation*, 1995, **11**(2):173~184

8 Viswanadham N, Narahary Y, Johnson T L. Deadlock prevention and deadlock avoidance in flexible manufacturing systems using Petri net models. *IEEE Transactions on Robotics and Automation*, 1990, **6**(6): 713~723

9 Wysk R A, Yang N, Joshi S. Detection of deadlocks in flexible manufacturing systems. *IEEE Transactions on Robotics and Automation*, 1991, **7**(6):853~889

10 Yamalidou K, Moody J, Lemmon M, Antsaklis P. Feedback control of Petri nets based on place invariants. *Automatica*, 1996, **32**(1):15~28

11 Sreevinas R S. On the existence of supervisory policies that enforce liveness in discrete event dynamic systems modeled by controlled Petri nets. *IEEE Transactions on Automatic Control*, 1996, **42**(7):928~942

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自动制造系统 Petri 网的公平活性控制策略

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摘　要　基于 Petri 网的不变式理论，针对典型的自动制造系统，提出了 Petri 模型强制公平性和活性的方法．首先，基于网论 T-不变式的概念，把系统的网模型设计为一个公平网．此后，利用 P-不变式把一个公平网设计为一个活的且公平网．同时，提出了非冗余严格极小信标的概念，大大简化了系统的分析与设计．一般说来，非冗余严格极小信标是系统严格极小信标一个小的子集，尤其对于复杂系统的网模型．研究结果表明，只要使非冗余的严格极小信标受控，则系统所有的严格极小信标就不会被清空．文中举例说明了这些控制方法的应用．研究结果适用于一大类柔性制造系统，具有相当的普遍性．这种方法对于自动制造系统的调度设计也具有一定意义和价值．

关键词　Petri 网，自动制造系统，活性和公平性，非冗余严格极小信标

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