模型参考自适应控制的慢漂移分析

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摘　　要

近年来，对完全模型匹配的 MRAC 已有许多研究，尤为重视 MRAC 的慢漂移不稳定[1-7]，但其机理并非很清楚.

本文用均值平衡位置内摄动分析法，分析了有正态零均值噪音及有未建模动态时 MRAC 的慢漂移. 指出用 Ляпунов 稳定性理论或超稳定性理论设计的自适应控制器中，完成相乘项和平方项的积分，是 MRAC 慢漂移及慢漂移不稳定的根源. 在 MRAC 中唯取缔积分器及相乘项才能有转机.

关键词：模型参考自适应控制，慢漂移，摄动分析.

一、精确建模时 MRAC 的慢漂移

现分析有两个可调参数的 MRAC（图 1）系统. 设对象为

$$y = G(s)u + d, \tag{1}$$

$$G(s) = \frac{b}{s+a} \cdot \Delta(s). \tag{2}$$

式中 d 为确定性扰动或噪音；$\Delta(s)$ 为未建模动态.

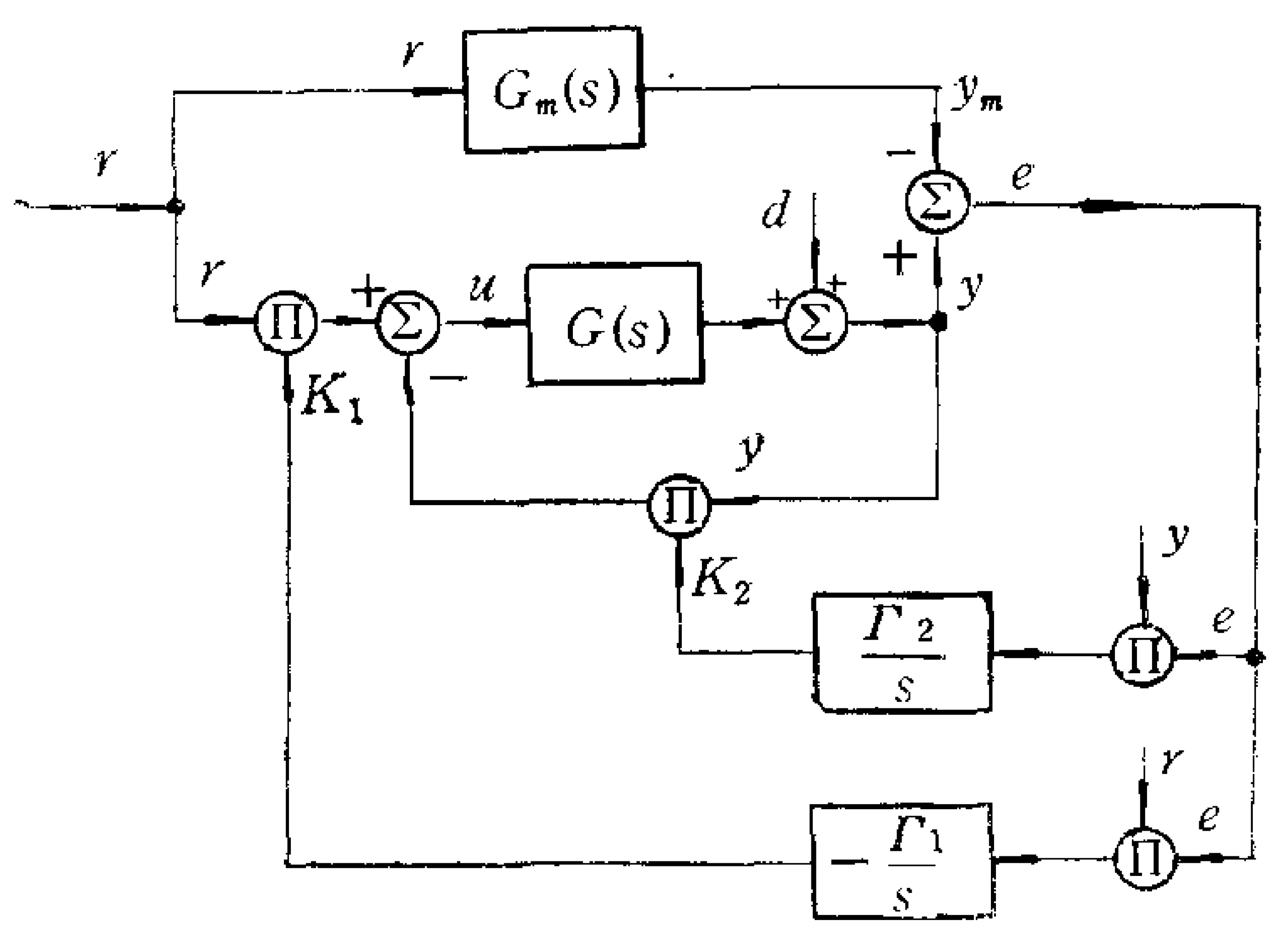

图 1　完全模型匹配之 MRAC 受扰动

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设参考模型为

$$y_m = G_m(s)r, \tag{3}$$

$$G_m(s) = \frac{a_m}{s + a_m}. \tag{4}$$

当 $\Delta(s)=1$，$d=0$ 时，用 Ляпунов 稳定性理论设计 MRAC，得

$$u = K_1 r - K_2 y,\quad \dot{K}_1 = -\Gamma_1 r e, \qquad (5),(6)$$

$$\dot{K}_2 = \Gamma_2 y e,\quad e = y - y_m. \qquad (7),(8)$$

以下为讨论方便，取 $\Gamma_1=\Gamma_2=1$. 分三种情况讨论：

1) 阶跃输入，扰动或噪音 $d=0$

由式(1),(3),(8)，当 $t\to\infty$ 时，有

$$\frac{\bar{K}_1 G(0)}{1+\bar{K}_2 G(0)} r - G_m(0)r = 0. \tag{9}$$

式中 $\bar{K}_1,\bar{K}_2$ 各为 K_1,K_2 之稳态值. 由式(9)得

$$\bar{K}_1 - \bar{K}_2 = \frac{1}{G(0)}. \tag{10}$$

由式(6),(7),(8)得

$$K_1(t)+K_2(t) = K_1(0)+K_2(0)+\int_0^t [y_m(\tau)-r(\tau) + e(\tau)]e(\tau)d\tau. \tag{11}$$

对于参考模型，$\exists t_0$，当 $t>t_0$ 时，$y_m(t)\approx r(t)$，故

$$K_1(t)+K_2(t) = A + \int_{t_0}^t e^2(\tau)d\tau. \tag{12}$$

式中 $A = K_1(0)+K_2(0)+\int_0^{t_0}[y_m(\tau)-r(\tau)+e(\tau)]e(\tau)d\tau$，故

$$K_1(t)+K_2(t) < C,\quad \bar{K}_1+\bar{K}_2 = \bar{K}. \qquad (13),(14)$$

式中 $\bar{K}$ 为 $K_1(t)+K_2(t)$ 当 $t\to\infty$ 时存在之极限.

式(10)说明平衡位置为直线，式(10)，(14)之唯一解确定了停留于平衡直线上的某点，无慢漂移.

2) 阶跃输入，恒值小扰动 $d=d_0$

$$\frac{\bar{K}_1 G(0)r + d_0}{1+\bar{K}_2 G(0)} - G_m(0)r = e, \tag{15}$$

$$\bar{K}_1 - \bar{K}_2 = \frac{1}{G(0)} - \frac{d_0}{G(0)r}. \tag{16}$$

式(16) 说明平衡位置也为直线. 同样可近似得式(14). 式(14)，(16)可唯一确定 $\bar{K}_1,\bar{K}_2$，无慢漂移. 见图 2.

3) 阶跃输入，小扰动 $d=d(t)$ 为正态零均值噪音(协方差为 R)

$$\frac{\bar{K}_1 G(0)r + d}{1+\bar{K}_2 G(0)} - G_m(0)r = e,\quad \bar{K}_1 - \bar{K}_2 = \frac{1}{G(0)} - \frac{d}{G(0)r}. \qquad (17),(18)$$

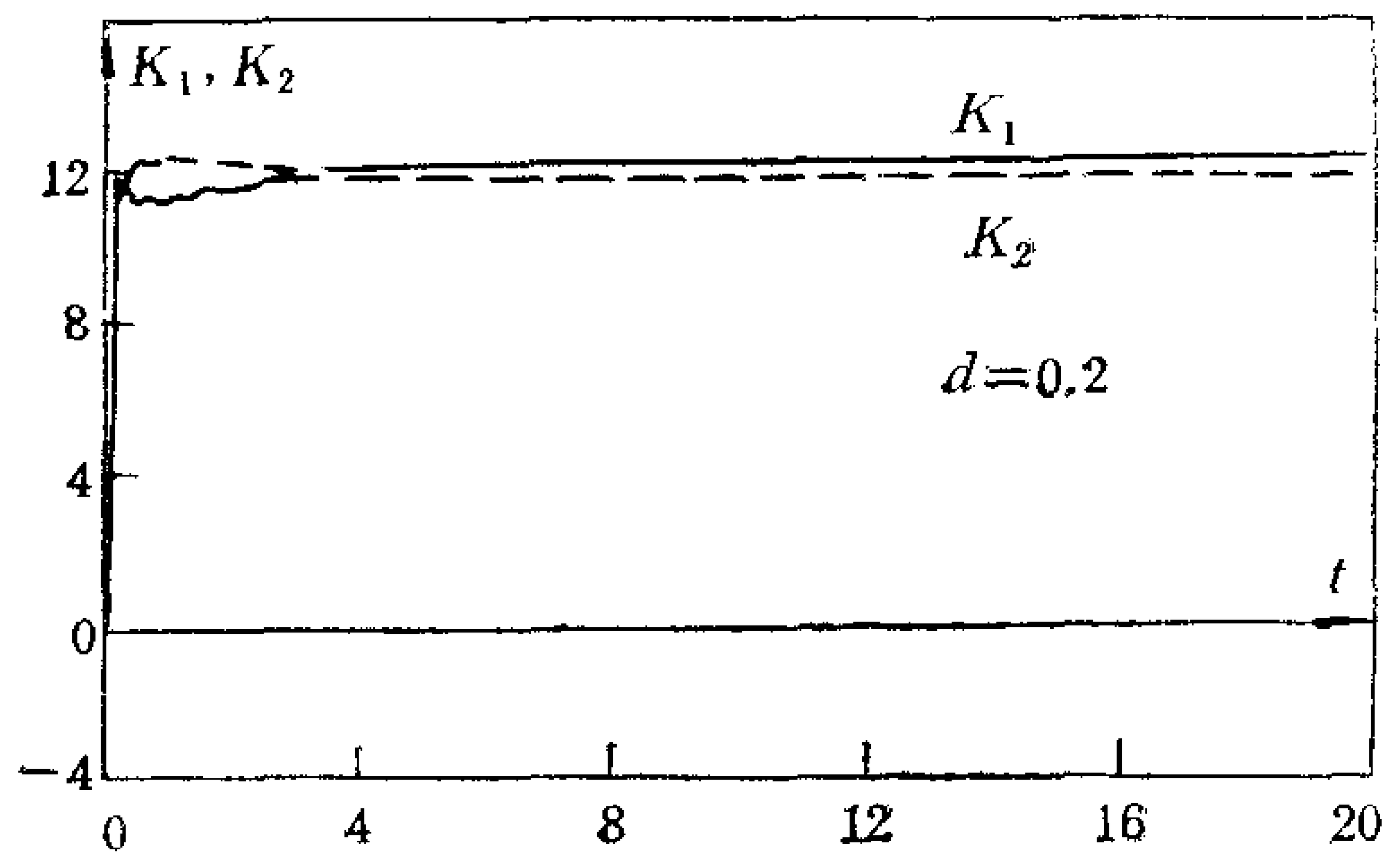

图 2 阶跃输入,恒值小扰动 $d=d_0$, 无慢漂移

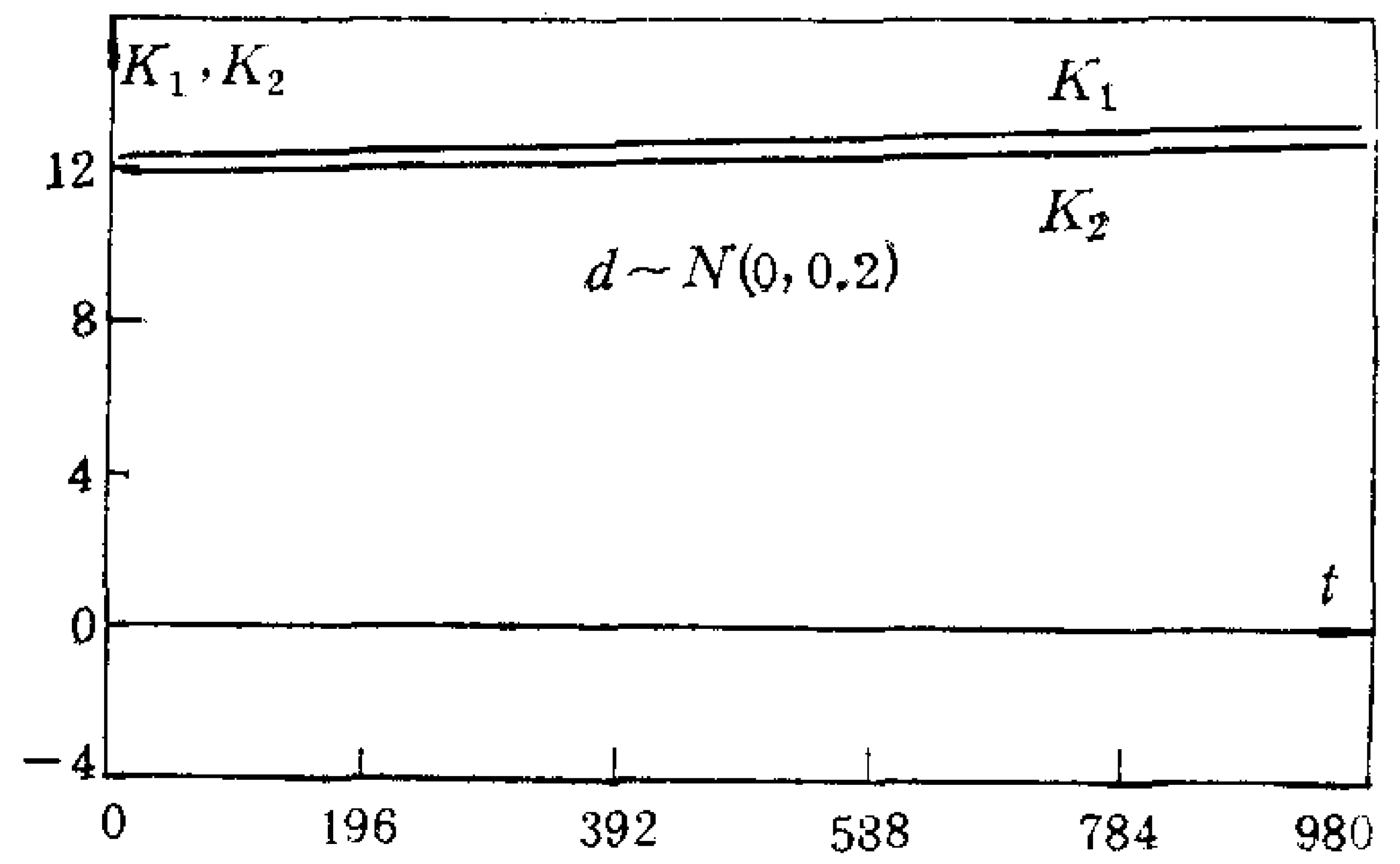

图 3 阶跃输入, 小扰动 $d=d(t)$ 为正态零均值噪音,有慢漂移

取均值
$$E\left[\bar{K}_1-\bar{K}_2-\frac{1}{G(0)}\right]=0. \tag{19}$$

现考虑在均值平衡位置内 $\bar{K}_1$, $\bar{K}_2$ 的摄动 $\delta K_1, \delta K_2$,

$$E\left[(\bar{K}_1+\delta K_1)-(\bar{K}_2+\delta K_2)-\frac{1}{G(0)}\right]=0. \tag{20}$$

即 $E[\delta K_1-\delta K_2]=0$, $E[\dot{K}_1-\dot{K}_2]=0$, $E[-re-ye]=0$. (21),(22),(23)

但
$$y=\frac{\bar{K}_1G(0)r}{1+\bar{K}_2G(0)}+\frac{d}{1+\bar{K}_2G(0)}=y_0+\eta, \tag{24}$$

$$e=y-y_m=y_0-y_m+\eta=e_0+\eta \tag{25}$$

代入式(24),得

$$E[e]=-\frac{E[\eta^2]}{r+y_0}=-\frac{ktrR}{r+y_0}\approx-\frac{ktrR}{2r}, \tag{26}$$

$$E[K_1]=-r\int_0^t E[e]dt\approx-\frac{ktrR}{2}t, \tag{27}$$

$$E[K_2]=E[K_1]-\frac{1}{G(0)}. \tag{28}$$

式中 $k=\frac{1}{[1+K_2G(0)]^2}$. 由式(27), (28)可见: *a*) $E[K_1]$ 及 $E[K_2]$ 将近似随 t 线性增长,即沿均值平衡位置漂移; *b*) $E[K_1]$ 与 $E[K_2]$ 相差一常量 $1/G(0)$ (图 3).

二、有未建模动态时 MRAC 的慢漂移

设系统无扰动、无噪音,有未建模动态; 并阶跃迭加小正弦, 即 $r=r_0+r_1\sin\omega t$, $r_1\ll r_0$.

若记 $\mathscr{L}^{-1}$ 为 Laplace 反变换, $[\cdot]_{ss}$ 表示取稳态响应分量,则稳态响应为

$$y=\frac{\bar{K}_1G(0)r_0}{1+\bar{K}_2G(0)}+\left[\mathscr{L}^{-1}\left(\frac{K_1G(s)}{1+K_2G(s)}\cdot\frac{\omega}{s^2+\omega^2}\right)\right]_{ss}=y_0+\mu, \tag{29}$$

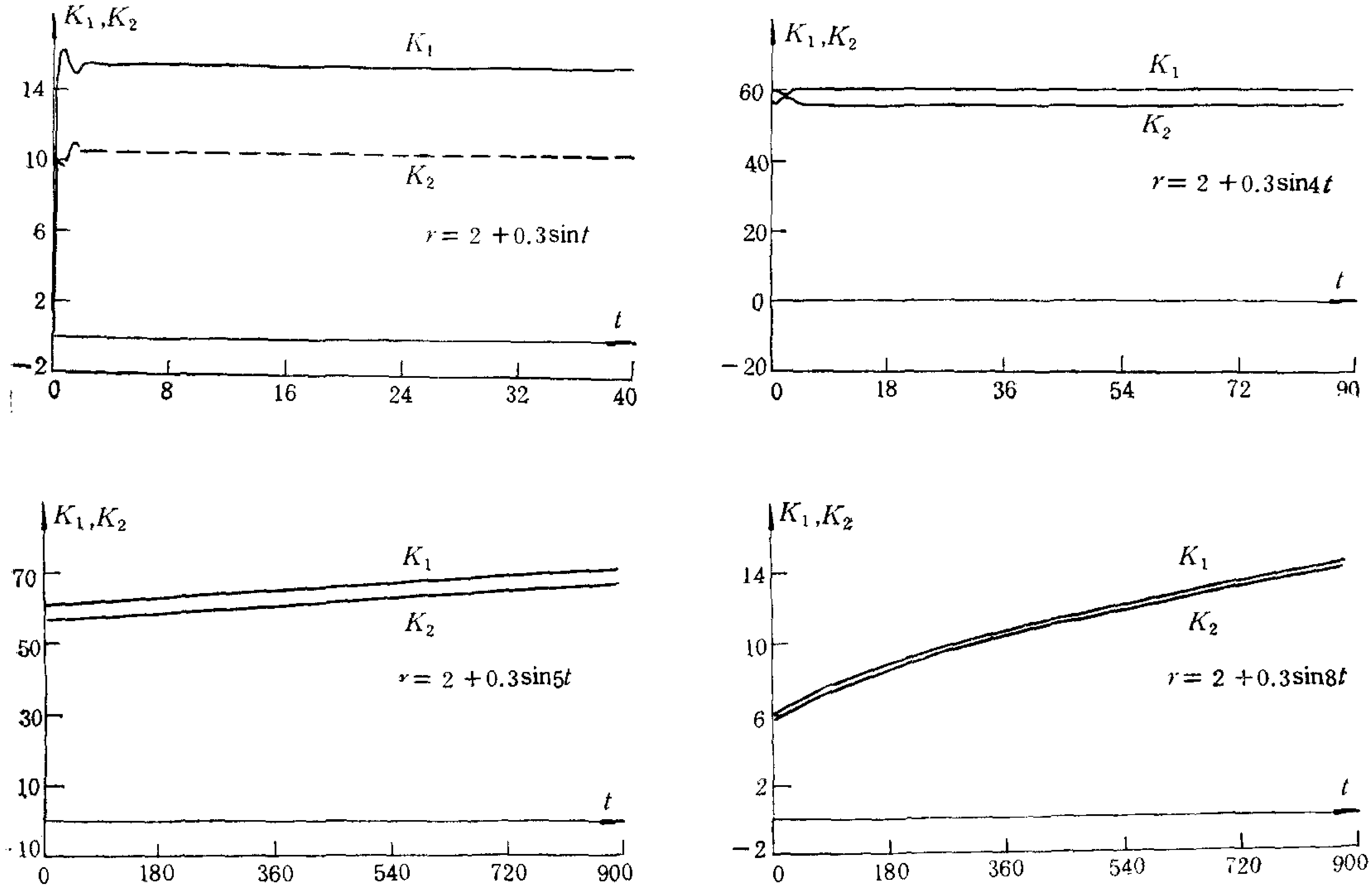

图 4　有未建模动态时，MRAC 的慢漂移

$$y_m = r_0 + \left[\mathscr{L}^{-1}\left(G_m(s)\frac{\omega}{s^2+\omega^2}\right)\right]_{ss} = r_0 + \mu_m. \tag{30}$$

同样可推得

$$E[e] = -\frac{E[\mu^2] - E[\mu\mu_m]}{r + y_0} = -\frac{M}{r+y_0} \approx -\frac{M}{2r}. \tag{31}$$

式中 $M = E[\mu^2] - E[\mu\mu_m]$,

$$E[K_1] = -r\int_0^t E[e]\,dt \approx \frac{M}{2}t, \tag{32}$$

$$E[K_2] = E[K_1] - \frac{1}{G(0)} \tag{33}$$

变化规律如图 4 所示. 当 ω 较小,正弦项不起作用,故不发生慢漂移. 由此看来，有未建模动态时,输入端迭加小摄动信号,非但不能起持续激励作用，反而引起慢漂移和慢漂移不稳定,这与理想情况下的持续激励大相径庭.

三、结　　论

(1) 研究 MRAC 慢漂移的均值平衡位置内摄动分析法,实质来源于各门学科(包括经典控制理论)中都曾使用过的方法：在静特性上用摄动法研究平衡点的稳定性. 不过，在 MRAC 中,平衡位置不是点,而是直线;出现的现象不仅是稳定和不稳定,而是慢漂移

和慢漂移不稳定。

(2) 用 Ляпунов 稳定性理论或超稳定性理论设计的自适应控制器，由于需完成相乘项的积分，从而形成了平方项的积分，这是慢漂移及慢漂移不稳定的根源。最彻底的解决办法只有取缔积分器及相乘项。目前，线性模型跟随以及变结构系统的广泛使用是这一看法的佐证。

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A SLOW-DRIFT ANALYSIS OF MRAC

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ABSTRACT

A perturbation analysis is introduced to analyze the slow-drift of MRAC in the mean balancing position with normal zero mean noise and unmodeled dynamics. It is pointed out that the adaptive controller based on Lyapunov stability theory (or the hyperstability theory) completes the integration of the square term caused by the integration of the multiplication term, and the integration of the square term is exactly the source of the slow-drift and its instability. In the MRAC, it can be shown that the integrator and multiplication term must be canceled to obtain a favourable property.

Key words: MRAC; slow-drift; perturbation analysis.