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广义离散随机线性系统的马尔可夫估计

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摘　　要

本文讨论了带有脉冲模的广义离散随机线性系统 $Ex_k = \phi x_{k-1} + \Gamma w_{k-1}$, $y_k = Hx_k + v_k$ 的马尔可夫状态估计问题，得到了这个估计的递推算法.

关键词: 广义离散随机线性系统，马尔可夫估计，递推算法.

一、引　　言

广义离散随机线性系统的状态估计问题是一个具有重要理论意义和应用前景的研究课题，它是研究广义系统随机控制问题的必要前提，但是，由于广义系统的非因果性，寻求这类系统的状态估计问题是比较困难的. 在文[1]中，按照广义线性系统的快、慢子系统的分解形式给出了状态估计的一种递推算法，并在文[2]中证明了该算法的全局渐近稳定性. 然而，由于在这个递推算法中包括多次高阶矩阵的求逆运算，给实际应用带来了许多不便和困难. 在文[3]中，讨论了广义离散随机线性系统的次优估计问题，得到的次优状态估计的递推算法避免了文[1]中包含的高阶矩阵求逆问题，但没有考虑模型噪声与量测噪声的相关性. 本文从广义线性系统的输入-输出可逆变换给出的标准分解结构出发，讨论了广义离散随机线性系统的马尔可夫状态估计，它既避免了文[1]中递推算法的复杂性，又较文[3]中给出的递推算法更为精确.

已知广义离散随机线性系统

$$Ex_k = \phi x_{k-1} + \Gamma w_{k-1}, \tag{1.1}$$

$$y_k = Hx_k + v_k, \tag{1.2}$$

其中 $x_k \in R^n$, $w_k \in R^r$, $y_k \in R^m$ 和 $v_k \in R^m$ 分别表示系统的状态矢量、模型噪声矢量、量测输出矢量和量测噪声矢量. E、ϕ、Γ 和H分别为 $n\times n$、$n\times n$、$n\times r$ 和 $m\times n$ 阶常值矩阵.

假设1. 系统(1.1)—(1.2)具有脉冲模，并且在确定性意义下是完全能控和完全能观测的.

假设2. 系统(1.1)—(1.2)是正则的，即对任意 $z\in \mathscr{C}$, $\det(zE-\phi) \not\equiv 0$, 其中$\mathscr{C}$表示复平面，$\det(\cdot)$ 表示矩阵的行列式，并且 $\mathrm{rank}E=\beta<n$, $\mathrm{rank}\phi=n$.

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假设 3. $\{\boldsymbol{w}_k\}$ 和 $\{\boldsymbol{v}_k\}$ 都是均值为零，并具有有限方差的独立随机序列，即对 k，$j=0,1,\cdots$ 有

$$E\{\boldsymbol{w}_k\}=0,\ E\{\boldsymbol{v}_k\}=0,$$
$$E\{\boldsymbol{w}_k \boldsymbol{w}_j^T\}=Q_k\delta_{k,j}, E\{\boldsymbol{v}_k \boldsymbol{v}_j^T\}=R_k\delta_{k,j},$$
$$E\{\boldsymbol{w}_k \boldsymbol{v}_j^T\}=0,$$

并且 $E\{\boldsymbol{x}_0 \boldsymbol{v}_j^T\}=0$. 其中，$E\{\cdot\}$ 表示期望算子，T 表示矩阵或矢量的转置，Q_k 和 R_k 分别是 $r\times r$ 和 $m\times m$ 阶对称正定对称矩阵，并且

$$\delta_{k,j}=\begin{cases}1, & k=j,\\ 0, & k\neq j.\end{cases}$$

下面，介绍两个引理.

引理 1[4]. 已知广义系统(1.1)—(1.2)，则存在可逆矩阵 $P\in R^{n\times n}$、$G\in R^{n\times n}$、$F\in R^{r\times r}$ 和 $S\in R^{m\times m}$，使得系统(1.1)—(1.2)受限等价于下列系统：

$$\boldsymbol{\phi}_1(k)=\phi_{11}\boldsymbol{\phi}_1(k-1)+\phi_{13}\boldsymbol{\phi}_3(k-1)+\Gamma_{11}\boldsymbol{\xi}_1(k-1)+\Gamma_{12}\boldsymbol{\xi}_2(k-1), \tag{1.3}$$
$$0=\boldsymbol{\phi}_2(k-1)+\Gamma_{22}\boldsymbol{\xi}_2(k-1), \tag{1.4}$$
$$0=\phi_{31}\boldsymbol{\phi}_1(k-1)+\Gamma_{33}\boldsymbol{\xi}_3(k-1), \tag{1.5}$$
$$\boldsymbol{z}_1(k)=H_{11}\boldsymbol{\phi}_1(k)+\boldsymbol{\eta}_1(k), \tag{1.6}$$
$$\boldsymbol{z}_2(k)=H_{21}\boldsymbol{\phi}_1(k)+H_{22}\boldsymbol{\phi}_2(k)+\boldsymbol{\eta}_2(k), \tag{1.7}$$
$$\boldsymbol{z}_3(k)=H_{33}\boldsymbol{\phi}_3(k)+\boldsymbol{\eta}_3(k). \tag{1.8}$$

其中

$$\begin{bmatrix}\boldsymbol{\phi}_1(k)\\ \boldsymbol{\phi}_2(k)\\ \boldsymbol{\phi}_3(k)\end{bmatrix}=G^{-1}\boldsymbol{x}(k),\quad \begin{bmatrix}\boldsymbol{z}_1(k)\\ \boldsymbol{z}_2(k)\\ \boldsymbol{z}_3(k)\end{bmatrix}=S\boldsymbol{y}(k),$$

$$\begin{bmatrix}\boldsymbol{\xi}_1(k)\\ \boldsymbol{\xi}_2(k)\\ \boldsymbol{\xi}_3(k)\end{bmatrix}=F^{-1}\boldsymbol{w}(k),\quad \begin{bmatrix}\boldsymbol{\eta}_1(k)\\ \boldsymbol{\eta}_2(k)\\ \boldsymbol{\eta}_3(k)\end{bmatrix}=S\boldsymbol{v}(k),$$

$$PEG=\begin{bmatrix}I_{n_1} & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{bmatrix},\quad P\phi G=\begin{bmatrix}\phi_{11} & 0 & \phi_{13}\\ 0 & I_{n_2} & 0\\ \phi_{31} & 0 & 0\end{bmatrix},$$

$$P\Gamma F=\begin{bmatrix}\Gamma_{11} & \Gamma_{12} & 0\\ 0 & \Gamma_{22} & 0\\ 0 & 0 & \Gamma_{33}\end{bmatrix},\quad SHG=\begin{bmatrix}H_{11} & 0 & 0\\ H_{21} & H_{22} & 0\\ 0 & 0 & H_{33}\end{bmatrix}.$$

引理 2[4]. 广义系统(1.1)—(1.2)是完全能控的充分必要条件是 $(\phi_{11},[\phi_{13}\Gamma_{11}\Gamma_{12}])$ 是能控对，并且 Γ_{22} 和 Γ_{33} 都是可逆矩阵. 广义系统(1.1)—(1.2)是完全能观测的充分必要条件是 $(\phi_{11},[\phi_{13}^T\ H_{11}^T\ H_{12}^T]^T)$ 是完全能观测对，并且 H_{22} 和 H_{33} 都是可逆矩阵.

本文总假设广义系统(1.1)—(1.2)是完全能控和完全能观测的.

二、马尔可夫递推估计算法

本节将从受限等价于广义系统(1.1)—(1.2)的系统(1.3)—(1.8)出发来讨论广义离散

随机线性系统状态的马尔可夫估计，即寻找 $\phi_1(k)$、$\phi_2(k)$ 和 $\phi_3(k)$ 的马尔可夫估计．

首先研究关于 $\phi_1(k)$ 的马尔可夫估计．

由于系统(1.1)—(1.2)是完全能控和完全能观测的，因此依引理 2 可知，H_{22} 和 H_{33} 都是可逆矩阵．于是由(1.4)和(1.8)式可知

$$\boldsymbol{\phi}_2(k) = -\Gamma_{22}\boldsymbol{\xi}_2(k), \tag{2.1}$$

$$\boldsymbol{\phi}_3(k) = H_{33}^{-1}\boldsymbol{z}_3(k) - H_{33}^{-1}\boldsymbol{\eta}_3(k). \tag{2.2}$$

令

$$\boldsymbol{z}_k = \begin{bmatrix} 0 \\ \boldsymbol{z}_1(k) \\ \boldsymbol{z}_2(k) \end{bmatrix}, \quad M = \begin{bmatrix} \phi_{31} \\ H_{11} \\ H_{21} \end{bmatrix}, \quad \bar{\boldsymbol{\eta}}_k = \begin{bmatrix} \Gamma_{33}\boldsymbol{\xi}_3 k \\ \boldsymbol{\eta}_1(k) \\ \boldsymbol{\eta}_2(k) - H_{22}\Gamma_{22}\boldsymbol{\xi}_2(k) \end{bmatrix},$$

$$\bar{\boldsymbol{\xi}}_k = \Gamma_{11}\boldsymbol{\xi}_1(k) + \Gamma_{12}\boldsymbol{\xi}_2(k) - \phi_{13}H_{33}^{-1}\boldsymbol{\eta}_3(k).$$

将(2.1)和(2.2)式分别代入(1.7)和(1.3)式得

$$\boldsymbol{\phi}_1(k) = \phi_{11}\boldsymbol{\phi}_1(k-1) + \phi_{13}H_{33}^{-1}\boldsymbol{z}_3(k) + \bar{\boldsymbol{\xi}}_{k-1}, \tag{2.3}$$

$$\boldsymbol{z}_k = M\boldsymbol{\phi}_1(k) + \bar{\boldsymbol{\eta}}_k. \tag{2.4}$$

依假设可知 $\{\bar{\boldsymbol{\xi}}_k\}$ 和 $\{\bar{\boldsymbol{\eta}}_k\}$ 都是零均值和有限方差的随机序列，即

$$E\{\bar{\boldsymbol{\xi}}_k\bar{\boldsymbol{\xi}}_j^T\} = \bar{Q}_k\delta_{k,j}, \quad E\{\bar{\boldsymbol{\eta}}_k\bar{\boldsymbol{\eta}}_j^T\} = \bar{R}_k\delta_{k,j},$$

其中

$$\begin{aligned}\bar{Q}_k = {} & \Gamma_{11}Q_{11}(k)\Gamma_{11}^T + \Gamma_{12}Q_{22}(k)\Gamma_{12}^T + \phi_{13}H_{33}^{-1}R_{33}(k)H_{33}^{-T}\phi_{13}^T + \Gamma_{11}Q_{12}\Gamma_{12}^T \\ & + \Gamma_{12}Q_{21}(k)\Gamma_{11}^T,\end{aligned}$$

$$\bar{R}_k = \begin{bmatrix} \Gamma_{33}Q_{33}(k)\Gamma_{33}^T & 0 & -\Gamma_{33}Q_{32}(k)\Gamma_{22}^T H_{22}^T \\ 0 & R_{11}(k) & 0 \\ -H_{22}\Gamma_{22}Q_{23}(k)\Gamma_{33}^T & 0 & R_{22}(k) + H_{22}\Gamma_{22}Q_{22}(k)\Gamma_{22}^T H_{22}^T \end{bmatrix},$$

$$Q(k) = \begin{bmatrix} Q_{11}(k) & Q_{12}(k) & Q_{13}(k) \\ Q_{21}(k) & Q_{22}(k) & Q_{23}(k) \\ Q_{31}(k) & Q_{32}(k) & Q_{33}(k) \end{bmatrix},$$

$$R(k) = \begin{bmatrix} R_{11}(k) & R_{12}(k) & R_{13}(k) \\ R_{21}(k) & R_{22}(k) & R_{23}(k) \\ R_{31}(k) & R_{32}(k) & R_{33}(k) \end{bmatrix}.$$

不难说明 $\bar{Q}_k$ 和 $\bar{R}_k$ 都是对称正定矩阵．

由(2.3)式可得

$$\boldsymbol{\phi}_1(i) = \phi_{11}^{i-k}\boldsymbol{\phi}_1(k) - \sum_{j=i}^{k-1}\phi_{11}^{i-j-1}\bar{\boldsymbol{\xi}}_j - \sum_{j=i}^{k-1}\phi_{11}^{i-j-1}\phi_{13}H_{33}^{-1}\boldsymbol{z}_3(j),$$

$$i = 1, 2, \cdots, k-1, k. \tag{2.5}$$

将(2.5)式代入(2.4)式得

$$\boldsymbol{z}_i = M\phi_{11}^{i-k}\boldsymbol{\phi}_1(k) - \sum_{j=i}^{k-1} M\phi_{11}^{i-j-1}\bar{\boldsymbol{\xi}}_j - \sum_{j=i}^{k-1} M\phi_{11}^{i-j-1}\phi_{13}H_{33}^{-1}\boldsymbol{z}_3(j) + \bar{\boldsymbol{\eta}}_i.$$

令

$$\bar{\boldsymbol{y}}_{k-1}=\begin{bmatrix}\boldsymbol{z}_1+\sum_{j=1}^{k-1}M\phi_{11}^{-j}\phi_{13}H_{33}^{-1}\boldsymbol{z}_3(j)\\ \boldsymbol{z}_2+\sum_{j=2}^{k-1}M\phi_{11}^{1-j}\phi_{13}H_{33}^{-1}\boldsymbol{z}_3(j)\\ \vdots\\ \boldsymbol{z}_{k-1}+M\phi_{11}^{-1}\phi_{13}H_{33}^{-1}\boldsymbol{z}_3(k-1)\end{bmatrix},\quad \boldsymbol{Y}_k=\begin{bmatrix}\bar{\boldsymbol{Y}}_{k-1}\\ \cdots\\ \boldsymbol{z}_k\end{bmatrix},$$

$$\boldsymbol{\varepsilon}(i)=\bar{\boldsymbol{\eta}}_i-\sum_{j=i}^{k-1}M\phi_{11}^{i-j-1}\boldsymbol{\xi}_j,$$

$$\boldsymbol{\varepsilon}_{K,k-1}=\begin{bmatrix}\boldsymbol{\varepsilon}(1)\\ \boldsymbol{\varepsilon}(2)\\ \vdots\\ \boldsymbol{\varepsilon}(k-1)\end{bmatrix},\quad \boldsymbol{\varepsilon}_{k,k}=\begin{bmatrix}\boldsymbol{\varepsilon}_{k,k-1}\\ \cdots\cdots\\ \bar{\boldsymbol{\eta}}_k\end{bmatrix},$$

$$D_{k,k-1}=\begin{bmatrix}M\phi_{11}^{1-k}\\ M\phi_{11}^{2-k}\\ \vdots\\ M\phi_{11}^{-1}\end{bmatrix},\quad D_{k,k}=\begin{bmatrix}D_{k,k-1}\\ \cdots\cdots\\ M\end{bmatrix}.$$

于是有

$$\bar{\boldsymbol{Y}}_{k-1}=D_{k,k-1}\boldsymbol{\phi}_1(k)+\boldsymbol{\varepsilon}_{k,k-1},\tag{2.6}$$

$$\boldsymbol{Y}_k=D_{k,k}\boldsymbol{\phi}_1(k)+\boldsymbol{\varepsilon}_{k,k}.\tag{2.7}$$

取性能指标

$$J_{k-1}=(\bar{\boldsymbol{Y}}_{k-1}-D_{k,k-1}\boldsymbol{\phi}_1(k))^T\Sigma_{k,k-1}^{-1}(\bar{\boldsymbol{Y}}_{k-1}-D_{k,k-1}\boldsymbol{\phi}_1(k)),$$

$$J_k=(\boldsymbol{Y}_k-D_{k,k}\boldsymbol{\phi}_1(k))^T\Sigma_{k,k}^{-1}(\boldsymbol{Y}_k-D_{k,k}\boldsymbol{\phi}_1(k)).$$

依定义可知，使 J_{k-1} 达到极小的 $\boldsymbol{\phi}_1(k)$ 的估计值 $\hat{\boldsymbol{\phi}}_1(k|k-1)$ 称为 $\boldsymbol{\phi}_1(k)$ 的马尔可夫预测估计,使 J_k 达到极小的 $\boldsymbol{\phi}_1(k)$ 的估计值 $\hat{\boldsymbol{\phi}}_1(k|k)$ 称为 $\boldsymbol{\phi}_1(k)$ 的马尔可夫滤波估计值. 不难计算

$$\hat{\boldsymbol{\phi}}_1(k|k-1)=(D_{k,k-1}^T\Sigma_{k,k-1}^{-1}D_{k,k-1})^{-1}D_{k,k-1}^T\Sigma_{k,k-1}^{-1}\bar{\boldsymbol{Y}}_{k-1},\tag{2.8}$$

$$\hat{\boldsymbol{\phi}}_1(k|k)=(D_{k,k}^T\Sigma_{k,k}^{-1}D_{k,k})^{-1}D_{k,k}^T\Sigma_{k,k}^{-1}\boldsymbol{Y}_k.\tag{2.9}$$

这里，$\Sigma_{k,k-1}=E\{\boldsymbol{\varepsilon}_{k,k-1}\boldsymbol{\varepsilon}_{k,k-1}^T\}$，$\Sigma_{k,k}=E\{\boldsymbol{\varepsilon}_{k,k}\boldsymbol{\varepsilon}_{k,k}^T\}$. 因为

$$\Sigma_{k,k}=\begin{bmatrix}\Sigma_{k,k-1} & 0\\ 0 & \bar{R}_k\end{bmatrix},$$

故(2.9)式可改写为

$$\boldsymbol{\phi}_1=(k|k)=[D_{k,k-1}^T\Sigma_{k,k-1}^{-1}D_{k,k-1}+M^T\bar{R}_k^{-1}M]^{-1}[D_{k,k-1}^T\Sigma_{k,k-1}^{-1}\bar{\boldsymbol{Y}}_{k-1}+M^T\bar{R}_k^{-1}\boldsymbol{z}_k]\tag{2.10}$$

令

$$P_{k,k}=(D_{k,k}^T\Sigma_{k,k}^{-1}D_{k,k})^{-1},$$

$$P_{k,k-1}=(D_{k\ k-1}^T\Sigma_{k,k-1}^{-1}D_{k,k-1})^{-1},$$

那么容易推出

$$P_{k,k}=[I-K_kM]P_{k,k-1},\tag{2.11}$$

$$K_k = P_{k,k-1}M^T[MP_{k,k-1}M^T + \bar{R}_k]^{-1}. \tag{2.12}$$

进而由(2.8),(2.10),(2.11)和(2.12)式不难得到

$$\hat{\Phi}_1(k|k) = [I - K_kM]\hat{\Phi}_1(k|k-1) + K_kZ_k. \tag{2.13}$$

另一方面,由 $D_{k,k}$、$D_{k,k-1}$、$\varepsilon_{k,k-1}$ 和 $\varepsilon_{k,k}$ 的定义知

$$D_{k,k-1} = D_{k-1,k-1}\phi_{11}^{-1}, \tag{2.14}$$

$$\varepsilon_{k,k-1} = \varepsilon_{k-1,k-1} - D_{k,k-1}\bar{\xi}_{k-1}. \tag{2.15}$$

于是由(2.15)式可以求得

$$\Sigma_{k,k-1} = \Sigma_{k-1,k-1} + D_{k,k-1}\bar{Q}_{k-1}D^T_{k,k-1} - D_{k,k-1}C_{k,k-1} - C^T_{k,k-1}D^T_{k,k-1}, \tag{2.16}$$

其中

$$C_{k,k-1} = E\{\bar{\xi}_{k-1}\varepsilon^T_{k-1,k-1}\} = [0 \quad C_{k-1}]$$

而

$$\begin{aligned} C_{k-1} &= E\{\bar{\xi}_{k-1}\bar{\eta}^T_{k-1}\} \\ &= [\Gamma_{11}Q_{13}(k-1)\Gamma^T_{33} + \Gamma_{12}Q_{23}(k-1)\Gamma^T_{33} - \phi_{13}H_{33}^{-1}R_{31}(k-1) \\ &\quad - \phi_{13}H_{33}^{-1}R_{32}(k-1) - \Gamma_{11}Q_{12}(k-1)\Gamma^T_{22}H^T_{22} \\ &\quad - \Gamma_{12}Q_{22}(k-1)\Gamma^T_{22}H^T_{22}. \end{aligned} \tag{2.17}$$

令 $L = \Sigma_{k-1,k-1} + D_{k,k-1}\bar{Q}_{k-1}D^T_{k,k-1} - D_{k,k-1}C_{k,k-1}$,

利用矩阵求逆公式得

$$\Sigma^{-1}_{K,K-1} = L^{-1} + L^{-1}C^T_{k,k-1}(I - D^T_{k,k-1}L^{-1}C^T_{K,K-1})^{-1}D^T_{k,k-1}L^{-1}. \tag{2.18}$$

用 $D^T_{k,k-1}$ 左乘、$D_{k,k-1}$ 右乘(2.18)式两边得

$$D^T_{k,k-1}\Sigma^{-1}_{k,k-1}D_{k,k-1} = (I - D^T_{k,k-1}L^{-1}C^T_{K,K-1})^{-1}D^T_{K,K-1}L^{-1}D_{K,K-1}, \tag{2.19}$$

$$D^T_{K,K-1}\Sigma^{-1}_{K,K-1} = (I - D^T_{K,K-1}L^{-1}C_{K,K-1})^{-1}D^T_{K,K-1}L^{-1}. \tag{2.20}$$

因此由(2.19)和(2.20)式推知

$$(D^T_{K,K-1}\Sigma^{-1}_{K,K-1}D_{K,K-1})^{-1}D^T_{K,K-1}\Sigma^{-1}_{K,K-1} = (D^T_{K,K-1}L^{-1}D_{K,K-1})^{-1}D^T_{K,K-1}L^{-1}. \tag{2.21}$$

又令

$$M_{K-1} = \Sigma_{K-1,K-1} + D_{K,K-1}\bar{Q}_{K-1}D^T_{K,K-1},$$

则

$$L = M_{K-1} - D_{K,K-1}C_{K,K-1}.$$

再次利用矩阵求逆公式得

$$L^{-1} = M^{-1}_{K-1} + M^{-1}_{K-1}D_{K,K-1}(I - C_{K,K-1}M^{-1}_{K-1}D_{K,K-1})^{-1}C_{K,K-1}M^{-1}_{K-1}. \tag{2.22}$$

用 $D^T_{K,K-1}$ 左乘 $D_{K,K-1}$ 右乘(2.22)式的两边得出

$$D^T_{K,K-1}L^{-1}D_{K,K-1} = D^T_{K,K-1}M^{-1}_{K-1}D_{K,K-1}(I - C_{K,K-1}M^{-1}_{K-1}D_{K,K-1})^{-1}, \tag{2.23}$$

$$\begin{aligned} D^T_{K,K-1}L^{-1} = D^T_{K,K-1}M^{-1}_{K-1} + D^T_{K,K-1}M^{-1}_{K-1}D_{K,K-1}(I - C_{K,K-1}M^{-1}_{K-1}D_{K,K-1})^{-1} \\ \cdot C_{K,K-1}M^{-1}_{K-1}. \end{aligned} \tag{2.24}$$

于是由(2.23)和(2.24)式可得

$$\begin{aligned} &(D^T_{K,K-1}L^{-1}D_{K,K-1})^{-1} \cdot D^T_{K,K-1}L^{-1} \\ &= C_{K,K-1}M^{-1}_{K-1} + (I - C_{K,K-1}M^{-1}_{K-1}D_{K,K-1})(D^T_{K,K-1}M^{-1}_{K-1}D_{K,K-1})^{-1}D^T_{K,K-1}M^{-1}_{K-1}. \end{aligned} \tag{2.25}$$

从(2.21)和(2.25)式可知

$$(D^T_{K,K-1}\Sigma^{-1}_{K,K-1}D_{K,K-1})^{-1}D^T_{K,K-1}\Sigma^{-1}_{K,K-1}$$

$$= C_{K,K-1}M_{K-1}^{-1} + (I - C_{K,K-1}M_{K-1}^{-1}D_{K,K-1})(D_{K,K-1}^TM_{K-1}^{-1}D_{K,K-1})^{-1}D_{K,K-1}^TM_{K-1}^{-1}. \tag{2.26}$$

依 M_{K-1} 的定义可知

$$M_{K-1}^{-1} = \Sigma_{K-1,K-1}^{-1} - \Sigma_{K-1,K-1}^{-1}D_{K,K-1}(\bar{Q}_{k-1}^{-1} + D_{k,k-1}^T\Sigma_{k-1,k-1}^{-1}D_{k,k-1})^{-1}D_{K,K-1}^T\Sigma_{K-1,K-1}^{-1}. \tag{2.27}$$

用 $D_{K,K-1}^T$ 左乘、$D_{K,K-1}$ 右乘(2.27)式得

$$D_{K,K-1}^TM_{K-1}^{-1}D_{K,K-1} = \bar{Q}_{k-1}^{-1}(D_{K,K-1}^T\Sigma_{k-1,k-1}^{-1}D_{k,k-1} + \bar{Q}_{k-1}^{-1})^{-1}D_{k,k-1}^T\Sigma_{k-1,k-1}^{-1}D_{K,K-1}, \tag{2.28}$$

$$D_{K,K-1}^TM_{K-1}^{-1} = \bar{Q}_{k-1}^{-1}(D_{K,K-1}^T\Sigma_{K-1,K-1}^{-1}D_{K,K-1} + \bar{Q}_{k-1}^{-1})^{-1}D_{K,K-1}^T\Sigma_{K-1,K-1}^{-1}, \tag{2.29}$$

由(2.28)和(2.29)式推得

$$(D_{K,K-1}^TM_{K-1}^{-1}D_{K,K-1})^{-1}D_{K,K-1}^TM_{K-1}^{-1} = (D_{K,K-1}^T\Sigma_{K-1,K-1}^{-1}D_{K,K-1})^{-1}D_{K,K-1}^T\Sigma_{K-1,K-1}^{-1}.$$

再由(2.14)式并利用 $P_{K,K}$ 的定义得

$$(D_{K,K-1}^TM_{K-1}^{-1}D_{K,K-1})^{-1}D_{K,K-1}^TM_{K-1}^{-1} = \phi_{11}P_{K-1,K-1}\cdot D_{K-1,K-1}\Sigma_{K-1,K-1}^{-1}. \tag{2.30}$$

此外，用 $C_{K,K-1}$ 左乘、$D_{K,K-1}$ 右乘(2.27)式两边得

$$C_{K,K-1}M_{K-1}^{-1}D_{K,K-1} = C_{K,K-1}\Sigma_{K-1,K-1}^{-1}D_{K,K-1} - C_{K,K-1}\Sigma_{K-1,K-1}^{-1}D_{K,K-1}(\bar{Q}_{k-1}^{-1} + D_{k,k-1}^T\Sigma_{k-1,k-1}^{-1}D_{k,k-1})^{-1}D_{k,k-1}^T\Sigma_{K-1,K-1}^{-1}D_{K,K-1}$$
$$= C_{K,K-1}\Sigma_{K-1,K-1}^{-1}D_{K,K-1}(\bar{Q}_{k-1}^{-1} + D_{k,k-1}^T\Sigma_{k-1,k-1}^{-1}D_{K,K-1})^{-1}\bar{Q}_{k-1}^{-1}. \tag{2.31}$$

再由(2.14)式并利用 $P_{K,K}$ 的定义经简单计算可得

$$C_{K,K-1}M_{K-1}^{-1}D_{K,K-1} = C_{K,K-1}\Sigma_{K-1,K-1}^{-1}D_{K,K-1}(\phi_{11}P_{K-1,K-1}\phi_{11}^T + \bar{Q}_{k-1})^{-1}. \tag{2.32}$$

同时

$$C_{K,K-1}M_{K-1}^{-1} = C_{K,K-1}\Sigma_{K-1,K-1}^{-1} - C_{K,K-1}\Sigma_{K-1,K-1}^{-1}D_{K,K-1}(\bar{Q}_{k-1}^{-1} + D_{K,K-1}^T\Sigma_{K-1,K-1}^{-1}D_{K,K-1})^{-1}D_{k,k-1}^T\Sigma_{K-1,K-1}^{-1}. \tag{2.33}$$

于是由(2.25),(2.31)—(2.33)式得

$$(D_{K,K-1}^T\Sigma_{K,K-1}^{-1}D_{K,K-1})^{-1}D_{k,k-1}^T\Sigma_{K,K-1}^{-1} = \phi_{11}P_{k-1,k-1}D_{k-1,k-1}^T\Sigma_{K-1,K-1}^{-1} + C_{K,K-1}\Sigma_{k-1,k-1}^{-1} - C_{K,K-1}\Sigma_{k-1,k-1}^{-1}D_{k-1,k-1}P_{k-1,k-1}D_{k-1,k-1}^T\Sigma_{k-1,k-1}^{-1}. \tag{2.34}$$

再由(2.8)和(2.34)式得

$$\hat{\Phi}_1(k|k-1) = (\phi_{11} - C_{K,K-1}\Sigma_{k-1,K-1}^{-1}D_{K-1,K-1})\cdot P_{k-1,k-1}D_{k-1,K-1}^T\Sigma_{k-1,K-1}^{-1}\bar{Y}_{k-1} + C_{K,K-1}\Sigma_{K-1,K-1}^{-1}\bar{Y}_{K-1}. \tag{2.35}$$

依定义有

$$\bar{Y}_{k-1} = Y_{k-1} + D_{k-1,k-1}\phi_{11}^{-1}\phi_{13}H_{33}^{-1}z_3(k-1). \tag{2.36}$$

将(2.36)代入(2.35)式得

$$\hat{\Phi}_1(k|k-1) = \phi_{11}\hat{\Phi}_1(k-1|k-1) + \phi_{13}H_{33}^{-1}z_3(k-1) + C_{K,K-1}\Sigma_{k-1,k-1}^{-1}[Y_{k-1} - D_{k-1,k-1}\hat{\Phi}_1(k-1|k-1)]. \tag{2.37}$$

由 $C_{k,k-1} = [0\ \ C_{k-1}]$ 以及 $\Sigma_{k-1,k-1}$、Y_{k-1}、$D_{k-1,k-1}$ 的定义知

$$C_{K,K-1}\Sigma_{k-1,K-1}^{-1}[Y_{k-1} - D_{k-1,k-1}\hat{\Phi}_1(k-1|k-1)] = C_{k-1}\bar{R}_{k-1}^{-1}[z_{k-1} - M\hat{\Phi}_1(k-1|k-1)].$$

于是将上式代入(2.37)式得出

$$\hat{\Phi}_1(k|k-1) = \phi_{11}\hat{\Phi}_1(k-1|k-1) + \phi_{13}H_{33}^{-1}z_3(k-1)$$

$$+ C_{K-1}\bar{R}_{k-1}^{-1}[\boldsymbol{z}_{k-1} - M\hat{\boldsymbol{\phi}}_1(k-1|k-1)]. \qquad (2.38)$$

这里 C_{K-1} 由(2.17)式给出.

类似地计算可以得到关于 $P_{k,k-1}$ 的表达式,即

$$P_{k,k-1} = \phi_{11}P_{k-1,k-1}\phi_{11}^T + C_{K-1}K_{k-1}^T\phi_{11}^T + \phi_{11}K_{k-1}C_{K-1}^T$$
$$- C_{K-1}[MP_{k-1,k-2}M^T + \bar{R}_{k-1}]^{-1}C_{k-1}^T. \qquad (2.39)$$

这就完成了关于 $\phi_1(k)$ 的马尔可夫递推估计公式的推导.

现在改写(1.7)式为

$$z_2(k) = H_{21}\hat{\phi}_1(k|k) + H_{22}\phi_2(k) + \boldsymbol{\eta}_2(k) - H_{21}\tilde{\boldsymbol{\phi}}_1(k|k), \qquad (2.40)$$

其中 $\tilde{\boldsymbol{\phi}}_1(k|k) = \boldsymbol{\phi}_1(k) - \hat{\boldsymbol{\phi}}_1(k|k)$ 为估计误差矢量. 于是由(2.40)和 H_{22} 为可逆矩阵的性质,不难得出给定 $\boldsymbol{z}_2(k)$ 后关于 $\boldsymbol{\phi}_2(k)$ 的马尔可夫估计为

$$\hat{\boldsymbol{\phi}}_2(k|k) = H_{22}^{-1}[z_2(k) - H_{21}\hat{\boldsymbol{\phi}}_1(k|k)]. \qquad (2.41)$$

同样,由(1.8)式得

$$\hat{\boldsymbol{\phi}}_3(k|k) = H_{33}^{-1}\boldsymbol{z}_3(k). \qquad (2.42)$$

于是,归纳起来有如下定理:

定理 1. 已知系统(1.3)—(1.8)及性能指标 J_{k-1} 和 J_k, 则 $\phi_1(k)$ 的马尔可夫递推估计由下列递推公式给出:

$$\hat{\boldsymbol{\phi}}_1(k|k) = \hat{\boldsymbol{\phi}}_1(k|k-1) + K_k[\boldsymbol{z}_k - M\hat{\boldsymbol{\phi}}_1(k|k-1)],$$
$$\hat{\boldsymbol{\phi}}_1(k|k-1) = \phi_{11}\hat{\boldsymbol{\phi}}_1(k-1|k-1) + \phi_{13}H_{33}^{-1}\boldsymbol{z}_3(k-1)$$
$$+ C_{K-1}\bar{R}_{k-1}^{-1}[\boldsymbol{z}_{k-1} - M\hat{\boldsymbol{\phi}}_1(k-1|k-1)],$$
$$K_k = P_{k,k-1}M^T[MP_{k,k-1}M^T + \bar{R}_k]^{-1},$$
$$P_{k,k-1} = \phi_{11}P_{k-1,k-1}\phi_{11}^T + \bar{Q}_{k-1} + C_{K-1}K_{k-1}^T\phi_{11}^T + \phi_{11}K_{k-1}C_{k-1}^T$$
$$- C_{k-1}[MP_{k-1,k-1}M^T + \bar{R}_{k-1}]^{-1}C_{K-1}^T,$$
$$P_{k,k} = [I - K_kM]P_{k,k-1},$$
$$C_{k-1} = \Gamma_{11}Q_{13}(k-1)\Gamma_{33}^T + \Gamma_{12}Q_{23}(k-1)\Gamma_{33}^T - \phi_{13}H_{33}^{-1}R_{31}(k-1)$$
$$- \phi_{13}H_{33}^{-1}R_{32}(k-1) - \Gamma_{11}Q_{12}(k-1)\Gamma_{22}^TH_{22}^T - \Gamma_{12}Q_{22}(k-1)\Gamma_{22}^TH_{22}^T,$$

$\hat{\boldsymbol{\phi}}_1(0|0)$, $P_{0,0}$ 已知.

定理 2. 已知广义系统(1.1)—(1.2). 则 $\boldsymbol{x}_k$ 的马尔可夫估计可由下式计算:

$$\hat{\boldsymbol{x}}_{k|k} = G\begin{bmatrix}\hat{\boldsymbol{\phi}}_1(k|k)\\ \hat{\boldsymbol{\phi}}_2(k|k)\\ \hat{\boldsymbol{\phi}}_3(k|k)\end{bmatrix}.$$

三、几点说明

1) 由于假设了系统 (1.1)—(1.2) 是完全能控和完全能观测的, 因此由引理 2 可知 $(\phi_{11}, [\phi_{31}^TH_{11}^TH_{12}^T]^T)$ 是能观测对. 从而只要 k 充分大, 矩阵 $D_{k,k-1}^T\Sigma_{k,k-1}^{-1}D_{k,k-1}$ 和 $D_{k,k}^T\Sigma_{k,k}^{-1}D_{k,k}$ 都是非奇异的,因而 $\boldsymbol{\phi}_1(k)$ 的马尔可夫估计 $\hat{\boldsymbol{\phi}}_1(k|k)$ 和 $\hat{\boldsymbol{\phi}}_1(k|k-1)$ 都存在.

2) 初始条件 $\hat{\boldsymbol{\phi}}_1(0|0)$ 和 $P_{0,0}$ 可按马尔可夫估计的定义确定,只要求足够的初始数

据.

3) 由于 $\hat{\Phi}_2(k|k)$ 和 $\hat{\Phi}_3(k|k)$ 只是从 k 时刻的量测数据 $z_2(k)$ 和 $z_3(k)$ 确定出来的,而没有用到历史数据,因此定理 2 中的估计 $\hat{x}_{k|k}$ 严格说来不是精确的马尔可夫估计,而是某种近似. 但是这种近似是方便于计算的.

4) 本文的结果与文[3]中不同的是在讨论 $\Phi_1(k)$ 的估计时,考虑到了模型噪声和量测噪声的相关性,从而估计精度要高一些.

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MARKOV ESTIMATION FOR SINGULAR DISCRETE-TIME STOCHASTIC LINEAR SYSTEM

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ABSTRACT

In this paper the Markov estimation of the state, x_k, for the singular discrete-time stochastic linear system with the impulse mode

$$Ex_k = \Phi x_{k-1} + \Gamma w_{k-1} \qquad y_k = Hx_k + v_k$$

is derived by using the standard decomposition structure of the input-output invertible transformation. The state estimation is given in the rcursive form.

Key words: Singular discrete-time stochastic linear systems; Markov estimation; recursive algorithm.