第17卷 第6期 自 动 化 学 报 Vol. 17, No. 6
1991年11月 ACTA AUTOMATICA SINICA Nov., 1991

有滞后的中立型控制系统与无滞后控制系统在镇定理论中的等价性

谢 胜 利
（湖北荆州师专，江陵）

摘 要

本文对有滞后的中立型控制系统与无滞后控制系统在镇定理论中的等价性进行了讨论，用不等式估值方法及分析技巧给出了其等价的时滞范围，为简化相应的中立型控制系统的分析和设计提供了理论依据.

关键词：中立型控制系统，镇定理论，等价性.

一、引 言

关于有滞后的中立型控制系统，由于中立项 $\dot{\boldsymbol{X}}(t-\tau)$ 的出现，使对其的研究问题变得较为棘手. 虽然 Pandolfi[1], Rodas[2], Jacobs[3], Langenhop[4] 等人对这类系统分别进行过一些讨论，但研究结果目前尚不多见. 而在实际中，人们往往忽视时滞 τ，将有滞后的中立型控制系统视为无滞后的控制系统进行相应的分析和设计. 对这种实用方法，在什么情况下适用，什么情况下不适用，是需要有理论根据的. 本文针对镇定性进行了讨论，给出了有滞后的中立型控制系统与无滞后的控制系统在镇定理论中等价的时滞范围，为实用方法提供了理论依据.

二、结果及证明

考虑有滞后的线性定常中立型控制系统

$$\dot{\boldsymbol{X}}(t)=A_1\boldsymbol{X}(t)+A_2\boldsymbol{X}(t-\tau)+A_3\dot{\boldsymbol{X}}(t-\tau)+B_1\boldsymbol{u}(t)+B_2\boldsymbol{u}(t-\tau), \tag{1}$$

及无滞后的线性定常控制系统

$$(I-A_3)\dot{\boldsymbol{X}}(t)=(A_1+A_2)\boldsymbol{X}(t)+(B_1+B_2)\boldsymbol{u}(t). \tag{2}$$

其中 $A_i(i=1,2,3)$ 是 $n\times n$ 常数矩阵，B_1, B_2 是 $n\times m$ 常数控制矩阵，$\boldsymbol{X}\in R^n$，$\boldsymbol{u}\in R^m$.

当 $(I-A_3)$ 是非奇异矩阵时，则(2)式可变为

$$\begin{aligned}\dot{\boldsymbol{X}}(t)&=(I-A_3)^{-1}(A_1+A_2)\boldsymbol{X}(t)+(I-A_3)^{-1}(B_1+B_2)\boldsymbol{u}(t)\\&\triangleq A\boldsymbol{X}(t)+B\boldsymbol{u}(t),\end{aligned} \tag{3}$$

本文于1990年5月8日收到.

而(1)式可变为

$$\dot{X}(t)=AX(t)+Bu(t)+C(X(t-\tau)-X(t))+D(\dot{X}(t-\tau)-\dot{X}(t))+E(u(t-\tau)-u(t)). \tag{4}$$

其中 $A=(I-A_3)^{-1}(A_1+A_2),B=(I-A_3)^{-1}(B_1+B_2)$，$C=(I-A_3)^{-1}A_2,D=(I-A_3)^{-1}A_3$，$E=(I-A_3)^{-1}B_2$.

若 (A,B) 是完全可控的，则系统(3)存在最优负反馈向量函数

$$u(t)=-KX(t), \tag{5}$$

使得二次性能指标

$$J=\int_{t_0}^{+\infty}(X^T(t)QX(t)+u^T(t)Ru(t))dt, \tag{6}$$

达到最小值，且(3)式的闭环系统

$$\dot{X}(t)=(A-KB)X(t) \tag{7}$$

的零解是渐近稳定的．其中 $K=R^{-1}B^TP$，而 P 是矩阵 Riccati 方程

$$A^TP+PA-PBR^{-1}B^TP-Q=0$$

的唯一对称正定解．R 是 $m\times m$ 正定的常数矩阵，Q 是 $n\times n$ 半正定常数矩阵．

现选择(5)式为有滞后的中立型控制系统(4)的负反馈向量函数．从而相应可得 (4) 式的闭环系统

$$\dot{X}(t)=(A-KB)X(t)+(C-KE)(X(t-\tau)-X(t))+D(\dot{X}(t-\tau)-\dot{X}(t)). \tag{8}$$

记 $y(t)=X(t)e^{\varepsilon(t-t_0)}$，则(8)式可写为

$$\begin{aligned}\dot{y}(t)=&[A-KB+\varepsilon I+(C-KE-\varepsilon I)(e^{\varepsilon\tau}-1)]y(t)\\&+(C-KE-\varepsilon I)\tau e^{\varepsilon\tau}\dot{y}(t-\theta)\\&+D\tau e^{\varepsilon\tau}\ddot{y}(t-\theta)+D(e^{\varepsilon\tau}-1)\dot{y}(t)\\\triangleq&A^*(\varepsilon)y(t)+C^*(\varepsilon)\dot{y}(t-\theta)+D^*(\varepsilon)\ddot{y}(t-\theta)+\hat{D}(\varepsilon)\dot{y}(t),\end{aligned} \tag{9}$$

其中 $A^*(\varepsilon)=A-KB+\varepsilon I+(C-KE-\varepsilon I)(e^{\varepsilon\tau}-1)$，$C^*(\varepsilon)=(C-KE-\varepsilon I)\tau e^{\varepsilon\tau}$，$D^*(\varepsilon)=D\tau e^{\varepsilon\tau}$，$\hat{D}(\varepsilon)=D(e^{\varepsilon\tau}-1)$，$\dot{y}(t-\theta)=[\dot{y}_1(t-\theta_1),\cdots,\dot{y}_n(t-\theta_n)]^T,\ddot{y}(t-\theta)=[\ddot{y}_1(t-\theta_1),\cdots,\ddot{y}_n(t-\theta_n)]^T,0\leqslant\theta_i\leqslant\tau,i=1,2,\cdots,n$.

因系统(7)的零解是渐近稳定的，则矩阵 $A-KB$ 特征根的实部 $\mathrm{Re}\lambda(A-KB)<0$，记 $-\lambda=\max\{\mathrm{Re}\lambda(A-KB)\}$ 则 $-\lambda<0$．由于矩阵的特征根连续的依赖于矩阵的元素，而 $A^*(0)=A-KB$，则存在 $0<\varepsilon_1\ll 1$，使得 $\varepsilon\in(0,\varepsilon_1)$ 时，矩阵 $A^*(\varepsilon)$ 特征根的实部 $\mathrm{Re}\lambda(A^*(\varepsilon))$ 满足

$$\max\{\mathrm{Re}\lambda(A^*(\varepsilon))\}\triangleq-\lambda(\varepsilon)<0.$$

由(9)式有

$$\begin{aligned}y(t)=&e^{A^*(\varepsilon)(t-t_0)}\left[y(t_0)+\int_{t_0}^{t}e^{-A^*(s-t_0)}(C^*(\varepsilon)\dot{y}(S-\theta)+D^*(\varepsilon)\ddot{y}(S-\theta)\right.\\&\left.+\hat{D}(\varepsilon)\dot{y}(s))ds\right]\\=&e^{A^*(\varepsilon)(t-t_0)}[y(t_0)-D^*(\varepsilon)\dot{y}(t_0-\theta)-(C^*(\varepsilon)-A^*D^*(\varepsilon))y(t_0-\theta)\\&-\hat{D}(\varepsilon)y(t_0)]+D^*(\varepsilon)\dot{y}(t-\theta)+(C^*(\varepsilon)-A^*D^*(\varepsilon))y(t-\theta)\end{aligned}$$

$$+\hat{D}(\varepsilon)\boldsymbol{y}(t)$$
$$+\int_{t_0}^{t} e^{A^*(t-s)}[A^*(C^*(\varepsilon)-A^*D^*(\varepsilon))\boldsymbol{y}(s-\theta)+A^*\hat{D}(\varepsilon)\boldsymbol{y}(s)]ds. \tag{10}$$

从而

$$\begin{aligned}\|\boldsymbol{y}(t)\| \leqslant\ & e^{-\lambda(\varepsilon)(t-t_0)}[\|\boldsymbol{y}(t_0)\|+\|D^*(\varepsilon)\|\|\dot{\boldsymbol{y}}(t_0-\theta)\|+\|C^*(\varepsilon)\\&-A^*D^*(\varepsilon)\|\|\boldsymbol{y}(t_0-\theta)\|+\|\hat{D}(\varepsilon)\|\|\boldsymbol{y}(t_0)\|]+\|D^*(\varepsilon)\|\|\dot{\boldsymbol{y}}(t-\theta)\|\\&+\|C^*(\varepsilon)-A^*D^*(\varepsilon)\|\|\boldsymbol{y}(t-\theta)\|+\|\hat{D}(\varepsilon)\|\|\boldsymbol{y}(t)\|\\&+\int_{t_0}^{t} e^{-\lambda(\varepsilon)(t-s)}[\|A^*(C^*(\varepsilon)-A^*D^*(\varepsilon)\|\|\boldsymbol{y}(s-\theta)\|\\&+\|A^*\hat{D}(\varepsilon)\|\|\boldsymbol{y}(s)\|]ds\\ \leqslant\ & (1+\|C^*(\varepsilon)-A^*D^*(\varepsilon)\|+\|\hat{D}(\varepsilon)\|)M+\|D^*(\varepsilon)\|N\\&+\|D^*(\varepsilon)\|\sup_{t_0-\tau\leqslant s\leqslant t}\|\dot{\boldsymbol{y}}(s)\|+(\|C^*(\varepsilon)-A^*D^*(\varepsilon)\|\\&+\|\hat{D}(\varepsilon)\|)\sup_{t_0-\tau\leqslant s\leqslant t}\|\boldsymbol{y}(s)\|+\int_{t_0}^{t} e^{-\lambda(\varepsilon)(t-s)}[\|C^*(\varepsilon)-A^*D^*(\varepsilon)\|\\&+\|\hat{D}(\varepsilon)\|]\sup_{t_0-\tau\leqslant\theta\leqslant s}\|y(\theta)\|ds.\end{aligned} \tag{11}$$

其中$M=\sup_{t_0-\tau\leqslant t\leqslant t_0}\|\boldsymbol{y}(t)\|$，$N=\sup_{t_0-\tau\leqslant t\leqslant t_0}\|\dot{\boldsymbol{y}}(t)\|$；注意$w(t)=\sup_{t_0-\tau\leqslant s\leqslant t}\|\boldsymbol{y}(s)\|$，$v(t)=\sup_{t_0-\tau\leqslant s\leqslant t}\|\dot{\boldsymbol{y}}(s)\|$ 是不减的，则

$$\|\boldsymbol{y}(t)\|\leqslant(1+g(\varepsilon))M+\|D^*(\varepsilon)\|N+\|D^*(\varepsilon)\|v(t)+g(\varepsilon)w(t)+\frac{f(\varepsilon)}{\lambda(\varepsilon)}w(t). \tag{12}$$

其中 $g(\varepsilon)=\|C^*(\varepsilon)-A^*\hat{D}(\varepsilon)\|+\|\hat{D}(\varepsilon)\|$; $f(\varepsilon)=\|A^*(C^*(\varepsilon)-A^*\hat{D}(\varepsilon))\|+\|A^*\hat{D}(\varepsilon)\|$. 故有

$$\begin{aligned}w(t)=\sup_{t_0-\tau\leqslant s\leqslant t}\|\boldsymbol{y}(s)\|&\leqslant\sup_{t_0-\tau\leqslant s\leqslant t_0}\|\boldsymbol{y}(s)\|+\sup_{t_0\leqslant s\leqslant t}\|\boldsymbol{y}(s)\|\\&\leqslant M+(1+g(\varepsilon))M+\|D^*(\varepsilon)\|N+\|D^*(\varepsilon)\|v(t)\\&+\left(g(\varepsilon)+\frac{f(\varepsilon)}{\lambda(\varepsilon)}\right)w(t),\end{aligned} \tag{13}$$

由于(8)式可写成

$$\dot{\boldsymbol{X}}(t)=(A_1-KB_1)\boldsymbol{X}(t)+(A_2-KB_2)\boldsymbol{X}(t-\tau)+A_3\dot{\boldsymbol{X}}(t-\tau),$$

或者

$$\dot{\boldsymbol{y}}(t)=(A_1-KB_1)\boldsymbol{y}(t)+[(A_2-KB_2)e^{\varepsilon\tau}-\varepsilon I]\boldsymbol{y}(t-\tau)+A_3e^{\varepsilon\tau}\dot{\boldsymbol{y}}(t-\tau), \tag{14}$$

$$\begin{aligned}\|\dot{\boldsymbol{y}}(t)\|&\leqslant\|A_1-KB_1\|\|\boldsymbol{y}(t)\|+\|(A_2-KB_2)e^{\varepsilon\tau}-\varepsilon I\|\|\boldsymbol{y}(t-\tau)\|\\&+\|A_3\|e^{\varepsilon\tau}\|\dot{\boldsymbol{y}}(t-\tau)\|\\&\leqslant[\|A_1-KB_1\|+\|(A_2-KB_2)e^{\varepsilon\tau}-\varepsilon I\|]w(t)+\|A_3\|e^{\varepsilon\tau}v(t),\end{aligned} \tag{15}$$

所以

$$\begin{aligned}v(t)=\sup_{t_0-\tau\leqslant s\leqslant t}\|\dot{\boldsymbol{y}}(s)\|&\leqslant\sup_{t_0-\tau\leqslant s\leqslant t_0}\|\dot{\boldsymbol{y}}(s)\|+\sup_{t_0\leqslant s\leqslant t}\|\dot{\boldsymbol{y}}(s)\|\\&\leqslant N+(\|A_1-KB_1\|+\|(A_2-KB)e^{\varepsilon\tau}-\varepsilon I\|)w(t)+\|A_3\|e^{\varepsilon\tau}v(t),\end{aligned} \tag{16}$$

从而由(13),(16)式有

$$\begin{pmatrix} w(t) \\ v(t) \end{pmatrix} \leqslant \begin{pmatrix} 2+g(\varepsilon) & \|D^*(\varepsilon)\| \\ 0 & 1 \end{pmatrix} \begin{pmatrix} M \\ N \end{pmatrix} + \begin{pmatrix} g(\varepsilon)+\dfrac{f(\varepsilon)}{\lambda(\varepsilon)} & \|D^*(\varepsilon)\| \\ \|A_1-KB_1\|+\|(A_2-KB_2)e^{\varepsilon\tau}-\varepsilon I\| & \|A_3\|e^{\varepsilon\tau} \end{pmatrix} \begin{pmatrix} w(t) \\ v(t) \end{pmatrix} \tag{17}$$

当矩阵

$$G(\varepsilon)=\begin{pmatrix} g(\varepsilon)+\dfrac{f(\varepsilon)}{\lambda(\varepsilon)} & \|D^*(\varepsilon)\| \\ \|A_1-KB_1\|+\|(A_2-KB_2)e^{\varepsilon\tau}-\varepsilon I\| & \|A_3\|e^{\varepsilon\tau} \end{pmatrix} \tag{18}$$

的谱半径 $P(G(\varepsilon))<1$ 时，$(I-G(\varepsilon))^{-1}$ 存在且非负,从而

$$\begin{pmatrix} w(t) \\ v(t) \end{pmatrix} \leqslant (I-G(\varepsilon))^{-1} \begin{pmatrix} 2+g(\varepsilon) & \|D^*(\varepsilon)\| \\ 0 & 1 \end{pmatrix} \begin{pmatrix} M \\ N \end{pmatrix}, \tag{19}$$

注意

$$\begin{aligned} \begin{pmatrix} \|\boldsymbol{X}(t)\| \\ \|\dot{\boldsymbol{X}}(t)\| \end{pmatrix} &= \begin{pmatrix} \|\boldsymbol{X}(t)\|e^{\varepsilon(t-t_0)} \\ \|\dot{\boldsymbol{X}}(t)\|e^{\varepsilon(t-t_0)} \end{pmatrix} e^{-\varepsilon(t-t_0)} \leqslant \begin{pmatrix} \|\boldsymbol{y}(t)\| \\ \|\dot{\boldsymbol{y}}(t)\|+\varepsilon\|\boldsymbol{y}(t)\| \end{pmatrix} e^{-\varepsilon(t-t_0)} \\ &\leqslant \begin{pmatrix} 1 & 0 \\ \varepsilon & 1 \end{pmatrix} (I-G(\varepsilon))^{-1} \begin{pmatrix} 2+g(\varepsilon) & \|D^*(\varepsilon)\| \\ 0 & 1 \end{pmatrix} \begin{pmatrix} M \\ N \end{pmatrix} e^{-\varepsilon(t-t_0)}, \end{aligned} \tag{20}$$

和

$$\begin{pmatrix} M \\ N \end{pmatrix} \leqslant \begin{pmatrix} \sup\limits_{t_0-\tau\leqslant s\leqslant t_0}\|\boldsymbol{X}(s)\| \\ \sup\limits_{t_0-\tau\leqslant s\leqslant t_0}\|\dot{\boldsymbol{X}}(s)\|+\varepsilon\sup\limits_{t_0-\tau\leqslant s\leqslant t}\|\boldsymbol{X}(s)\| \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ \varepsilon & 1 \end{pmatrix} \begin{pmatrix} \sup\limits_{t_0-\tau\leqslant s\leqslant t_0}\|\boldsymbol{X}(s)\| \\ \sup\limits_{t_0-\tau\leqslant s\leqslant t_0}\|\dot{\boldsymbol{X}}(s)\| \end{pmatrix}, \tag{21}$$

从而

$$\begin{aligned} \begin{pmatrix} \|\boldsymbol{X}(t)\| \\ \|\dot{\boldsymbol{X}}(t)\| \end{pmatrix} &\leqslant \begin{pmatrix} 1 & 0 \\ \varepsilon & 1 \end{pmatrix} (I-G(\varepsilon))^{-1} \begin{pmatrix} 2+g(\varepsilon) & \|D^*(\varepsilon)\| \\ 0 & 1 \end{pmatrix} \\ &\quad\times \begin{pmatrix} 1 & 0 \\ \varepsilon & 1 \end{pmatrix} \begin{pmatrix} \sup\limits_{t_0-\tau\leqslant s\leqslant t_0}\|\boldsymbol{X}(s)\| \\ \sup\limits_{t_0-\tau\leqslant s\leqslant t_0}\|\dot{\boldsymbol{X}}(s)\| \end{pmatrix} e^{-\varepsilon(t-t_0)} \\ &\triangleq H(\varepsilon) \begin{pmatrix} \sup\limits_{t_0-\tau\leqslant s\leqslant t_0}\|\boldsymbol{X}(s)\| \\ \sup\limits_{t_0-\tau\leqslant s\leqslant t_0}\|\dot{\boldsymbol{X}}(s)\| \end{pmatrix} e^{-\varepsilon(t-t_0)}, \end{aligned} \tag{22}$$

因为

$$\begin{aligned} G(0) &= \begin{pmatrix} g(0)+\dfrac{f(0)}{\lambda(0)} & \|D^*(0)\| \\ \|A_1-KB_1\|+\|A_2-KB_2\| & \|A_3\| \end{pmatrix} \\ &= \begin{pmatrix} \|C-KE-(A-KB)D\|\tau+\lambda^{-1}\|(A-KB) \times [C-KE-(A-KB)D]\|\tau & \|D\|\tau \\ \|A_1-KB_1\|+\|A_2-KB_2\| & \|A_3\| \end{pmatrix}, \end{aligned} \tag{23}$$

而矩阵的谱半径连续的依赖于矩阵的元素,则当 $\rho(G(0))<1$ 时，存在$0<\varepsilon_2\ll 1$，使

$\varepsilon\in(0,\varepsilon_2)$ 时,有 $\rho(G(\varepsilon))<1$. 取 $\varepsilon_0=\min\{\varepsilon_1,\varepsilon_2\}$, 则当 $\varepsilon\in(0,\varepsilon_0)$, 有 $-\lambda(\varepsilon)<0$ 及 $\rho(G(\varepsilon))<1$. 则当 $\rho(G(0))<1$ 时,由(22)式知闭环系统(8)的零解在 C_1 空间中是渐近稳定的. 从而得到以下定理:

定理 1. 若 $\|A_3\|<1$, 且无滞后的控制系统 (3) 是完全可控的, 则当时滞 τ 使得 $\rho(G(0))<1$ 时,有滞后的中立型控制系统(1)是可镇定的.

在系统(1)中,若 $A_3=0$. 则(1)式变为

$$\dot{X}(t)=A_1X(t)+A_2X(t-\tau)+B_1u(t)+B_2u(t-\tau), \tag{24}$$

从而 $A=A_1+A_2$, $B=B_1+B_2$, $C=A_2$, $E=B_2$, $D=0$,

$$G(0)=\begin{pmatrix}(\|C-KE\|+\|(A-KB)(C-KE)\|)\tau & 0\\ \|A_1-KB_1\|+\|A_2-KB_2\| & 0\end{pmatrix}. \tag{25}$$

由定理 1 可得到

定理 2. 若 (A_1+A_2,B_1+B_2) 是完全可控的,则当时滞满足

$$\tau<(\|C-KE\|+\|(A-KB)(C-KE)\|)^{-1}$$

时,滞后控制系统(24)是可镇定的.

参 考 文 献

[1] Pandolfi, L., Stabilization of Neutral Functional Differential Equations, *J. Opt. Theory Appl.*, 20(1976), 191—204.

[2] Rodas, H. R., A Sufficient Condition for Function Space Controllability of a Linear Neutral System, *SIAM J. Control Opt.* 16(1978), 429—435.

[3] Jacobs, M. Q. and Langenhop, C. E., Controllabl Two Dimensional Neutral Systems, Mathematical Control Theory, 107—113.

[4] Jacobs, M. Q. and Langenhop, C. E., Criteria for Function Space Controllability of Linear Neutral System, *SIAM J. Control Opt.*, 14(1976), 1009—1048.

ON THE EQUIVALENCE PROBLEM OF CONTROL SYSTEMS AND NEUTRAL TYPE CONTROL SYSTEMS WITH TIME-DELAY IN THE THEORY OF STABILIZATION

Xie Shengli

(Dept. of Mathematics, Jingzhou Teacher's College, Hubei)

Abstract

In this paper, we discuss the equivalence between neutral type control systems with time-delay and control systems without time-delay in the theory of stabilization, and give the llmits of time-delay of equivalence by inequality appraisal and analytical technique. The results provide a theoretical basis for the way to simplify the prodecure of analyzing and designing relevant neutral type control systemes.

Key words: Neutral type control systems; theory of stabilization; equivalence.