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区域水环境经济系统的多目标递阶规划[1)]

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摘　　要

本文针对文献[1]提出的以水系水质规划为基础的多子区域多目标水环境经济规划问题,提出了它的相互作用式逐步折衷递阶优化解法,并以两个子区域和五个污水集中处理厂的系统为例进行计算机仿真,说明其有效性。

关键词: 环境经济,多目标优化,递阶算法.

一、等价 ε-约束问题的建立

使用文献[3]的相互作用式折衷序贯优化方法,将文献[1]提出的多目标优化问题化为单目标问题求解,在与决策者对话的每一步将优先改进的一个目标保留在目标函数中,其余目标均化作 ε-约束,形成单目标 ε-约束问题WEP($\boldsymbol{\varepsilon}$).为便于形成求解 WEP($\boldsymbol{\varepsilon}$) 的递阶算法,参照文献[2]在该问题中引入预测向量 $\boldsymbol{v}_t^p$, $\boldsymbol{u}_t^p(t=1,\cdots,T)$ 和 $\boldsymbol{\xi}^p$, 增加附加约束

$$\boldsymbol{v}_t^p=\boldsymbol{v}_t,\boldsymbol{u}_t^p=\boldsymbol{u}_t,t=1,\cdots,T, \tag{1}$$

$$\boldsymbol{\xi}^p=\boldsymbol{\xi}, \tag{2}$$

以及目标中罚项

$$\frac{1}{2}\|\boldsymbol{v}_t-\boldsymbol{v}_t^p\|_{P_{vt}}^2,\frac{1}{2}\|\boldsymbol{u}_t-\boldsymbol{u}_t^p\|_{P_{ut}}^2,t=1,\cdots,T, \tag{3}$$

$$\frac{1}{2}\|\boldsymbol{\xi}-\boldsymbol{\xi}^p\|_{P_\xi}^2, \tag{4}$$

式中 $\boldsymbol{P}_{vt}$,$\boldsymbol{P}_{ut}$ 和 $\boldsymbol{P}_\xi$ 均为正定对角权矩阵. 再将原问题中某些变量用其预测变量代替,可得到等价问题 EWEP($\boldsymbol{\varepsilon}$)

$$\min\sum_{t=1}^{T}\left(f_t^{l_1}+\frac{1}{2}\|\boldsymbol{v}_t-\boldsymbol{v}_t^p\|_{P_{vt}}^2+\frac{1}{2}\|\boldsymbol{u}_t-\boldsymbol{u}_t^p\|_{P_{ut}}^2\right)+\frac{1}{2}\|\boldsymbol{\xi}-\boldsymbol{\xi}^p\|_{P_\xi}^2, \tag{5}$$

$$s.t.\ \sum_{t=1}^{T}f_t^{l_j}\leqslant\varepsilon_{l_j},j=2,3,4, \tag{6}$$

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1) 高等学校博士学科点专项科研基金资助课题.

$$\left[\sum_{i=1}^{m}(A^i+E^iP^i)\Xi^{pi}-I_n\right]\boldsymbol{x}_t+\sum_{\tau=1}^{k}\sum_{i=1}^{m}(B_\tau^iV_{i\tau}^{pi}+D_\tau^iP^iU_{i\tau}^{pi})\Xi^{pi}(\boldsymbol{x}_{t+\tau}$$

$$-\boldsymbol{x}_{t+\tau-1})+\sum_{i=1}^{m}[\boldsymbol{e}_c^iq_t^i\zeta_t^i+\boldsymbol{d}_c^i(q^i_{t+k_c^i}\zeta^i_{t+k_c^i}-q^i_{t+k_c^i-1}\zeta^i_{t+k_c^i-1})]$$

$$+\boldsymbol{y}_t=0, \tag{7}$$

$$[\Gamma(\boldsymbol{\xi}^p,\boldsymbol{x}_t)\vdots F_0\widetilde{Q}_t][\zeta_t^T\vdots\zeta_t^{sT}]^T=\boldsymbol{\gamma}(\boldsymbol{\xi}^p,\boldsymbol{x}_t), \tag{8}$$

$$F_q\tilde{\boldsymbol{q}}_t=G_q\tilde{\boldsymbol{q}}_t^0+H_qE_t^{\eta0}\boldsymbol{q}_t^r, \tag{9}$$

$$F_0L_t\tilde{\boldsymbol{q}}_t=G_0\widetilde{Q}_t^0\boldsymbol{l}_t^0+H_qE_t^{\zeta0}\boldsymbol{O}_t^r, \tag{10}$$

$$\zeta_t=[\zeta_t^{1T}\vdots\cdots\vdots\zeta_t^{mT}]^T,\zeta_t^i=[\zeta_t^{i1},\cdots,\zeta_t^{im_i}]^T,$$

$$\zeta_t^i=\boldsymbol{\eta}_t^{0iT}\zeta_t^i;\boldsymbol{\eta}_t^{0i}=[\eta_t^{0i1},\cdots,\eta_t^{0im_i}]^T,i=1,\cdots,m, \tag{11}$$

$$q_t^i=\boldsymbol{i}_n^TP^i\Xi^{pi}\boldsymbol{x}_t+w_t^{0i},O_t^i=\boldsymbol{i}_n^TO^i\Xi^{pi}\boldsymbol{x}_t+r_t^{0i},i=1,\cdots,m, \tag{12}$$

$$\Gamma(\boldsymbol{\xi}^p,\boldsymbol{x}_t)=-H_qH_{0t}^0,\boldsymbol{\gamma}(\boldsymbol{\xi}^p,\boldsymbol{x}_t)=F_0\widetilde{Q}_t\boldsymbol{l}_t^g-G_0\widetilde{Q}_t^0\boldsymbol{l}_t^0-H_qE_t^{\eta0}\boldsymbol{O}_t^r,$$

$$\boldsymbol{q}_t^r=[q_t^1,\cdots,q_t^m]^T,\boldsymbol{O}_t^r=[O_t^1,\cdots,O_t^m]^T,$$

$$E_t^{\eta0}=\text{block-diag}\{\boldsymbol{\eta}_t^{01},\cdots,\boldsymbol{\eta}_t^{0m}\},E_t^{\zeta0}=[(1-\mu^{0i}\zeta_t^{ij})\eta_t^{0ij}],H_{0t}^0=[\mu^{0i}\eta_t^{0ij}O_t^i],$$

$$\sum_{i=1}^{m}\boldsymbol{\xi}^i=\boldsymbol{i}_n,\boldsymbol{\xi}^{\min}\leqslant\boldsymbol{\xi}\leqslant\boldsymbol{\xi}^{\max}, \tag{13}$$

$$\left.\begin{aligned}&\sum_{\tau=1}^{k_p^i}\boldsymbol{v}_{t-\tau,\tau}^i-\boldsymbol{i}_n=0,i=1,\cdots,m,\\&\boldsymbol{v}_{t-\tau,\tau}=\boldsymbol{v}_{t-\tau,\tau}^0,t-\tau<1\text{ 时},\end{aligned}\right\}t=2,\cdots,T+1, \tag{14}$$

$$\sum_{\tau=t-T}^{k_p^i}\boldsymbol{v}_{t-\tau,\tau}^i+\boldsymbol{b}_{vt}^{0i}-\boldsymbol{i}_n=0,i=1,\cdots,m;t=T+2,\cdots,T+k_p^i, \tag{15}$$

$$\boldsymbol{v}_t^{\min}\leqslant\boldsymbol{v}_t\leqslant\boldsymbol{v}_t^{\max},t=1,\cdots,T, \tag{16}$$

$$\left.\begin{aligned}&\sum_{\tau=1}^{k_d^i}\boldsymbol{u}_{t-\tau,\tau}^i-\boldsymbol{i}_n=0,i=1,\cdots,m,\\&\boldsymbol{u}_{t-\tau,\tau}=\boldsymbol{u}_{t-\tau,\tau}^0,t-\tau<1\text{ 时},\end{aligned}\right\}t=2,\cdots,T+1, \tag{17}$$

$$\sum_{\tau=t-T}^{k_d^i}\boldsymbol{u}_{t-\tau,\tau}^i+\boldsymbol{b}_{ut}^{0i}-\boldsymbol{i}_n=0,i=1,\cdots,m;t=T+2,\cdots,T+k_d^i, \tag{18}$$

$$\boldsymbol{u}_t^{\min}\leqslant\boldsymbol{u}_t\leqslant\boldsymbol{u}_t^{\max},t=1,\cdots,T, \tag{19}$$

$$\boldsymbol{x}_t^{\min}\leqslant\boldsymbol{x}_t\leqslant\boldsymbol{x}_t^{\max},\boldsymbol{y}_t^{\min}\leqslant\boldsymbol{y}_t\leqslant\boldsymbol{y}_t^{\max},t=1,\cdots,T, \tag{20}$$

$$\boldsymbol{x}_t=D_{xt}^0\boldsymbol{x}_T,t=T+1,\cdots,T+k, \tag{21}$$

$$0\leqslant\boldsymbol{\zeta}_t\leqslant\boldsymbol{i}_m,\zeta_t^i\geqslant0;t=1,\cdots,T, \tag{22}$$

$$\zeta_t^i=d_{\zeta t}^{0i}\zeta_T^i,t=T+1,\cdots,T+k_c^i;i=1,\cdots,m \tag{23}$$

和(1),(2)式，

式中 $\bigcup_{j=1}^{4}l_j=\{1,2,3,4\}$，且 l_1 为优先改进的目标函数标号．此外

$$f_t^1 = f_t^1(\boldsymbol{y}_t) = \frac{1}{2}\|\boldsymbol{y}_t - \boldsymbol{y}_t^d\|_{W_{yt}}^2, \tag{24}$$

$$f_t^2 = f_t^2(\boldsymbol{x}_t) = \frac{1}{2}\|\boldsymbol{x}_t\|_{W_{xt}}^2, \tag{25}$$

$$f_t^3 = f_t^3(\boldsymbol{\xi}^p, \boldsymbol{x}_t) = \frac{1}{2}\left(\sum_{i=1}^m \boldsymbol{c}^{iT}\Xi^{pi}\boldsymbol{x}_t - c_t^d\right)^2 = \frac{1}{2}\left(\sum_{i=1}^m \boldsymbol{c}^{iT}x_t\boldsymbol{\xi}^{pi} - c_t^d\right)^2, \tag{26}$$

$$f_t^4 = f_t^4(\boldsymbol{\xi}^p, \boldsymbol{x}_t, \zeta_t) = \sum_{i=1}^m [\boldsymbol{\alpha}_{f\xi}^{iT}(t)\boldsymbol{x}_t + \beta_f^i(t)]\zeta_t^i = \sum_{i=1}^m [\boldsymbol{\alpha}_{fx}^{iT}(t)\boldsymbol{\xi}^{pi} + \beta_f^i(t)]\zeta_t^i, \tag{27}$$

且

$$\boldsymbol{\alpha}_{f\xi}^{iT}(t) = \begin{cases} \boldsymbol{i}_n^T\tilde{\boldsymbol{e}}_t^i\boldsymbol{i}_n^TP^i\Xi^{pi}, t = 1,\cdots,T-1, \\ \boldsymbol{i}_n^T(\boldsymbol{e}_c^i\boldsymbol{i}_n^T + \boldsymbol{d}_c^i\boldsymbol{i}_n^TD^0_{x,T+k_c^i}d^{0i}_{\zeta,T+k^i})P^i\Xi^{pi}, t = T, \end{cases}$$

$$\boldsymbol{\alpha}_{fx}^{iT}(t) = \begin{cases} \boldsymbol{i}_n^T\tilde{\boldsymbol{e}}_t^i\boldsymbol{i}_n^TP^iX_t, t = 1,\cdots,T-1, \\ \boldsymbol{i}_n^T(\boldsymbol{e}_c^i\boldsymbol{i}_n^T + \boldsymbol{d}_c^i\boldsymbol{i}_n^TD^0_{x,T+k_c^i}d^{0i}_{\zeta,T+k_c^i})P^iX_t, t = T, \end{cases}$$

$$\beta_f^i(t) = \begin{cases} \boldsymbol{i}_n^T\tilde{\boldsymbol{e}}_t^iw_t^{0i}, t = 1,\cdots,T-1, \\ \boldsymbol{i}_n^T(\boldsymbol{e}_c^iw_T^{0i} + \boldsymbol{d}_c^iw^{0i}_{T+k_c^i}d^{0i}_{\zeta,T+k_c^i}), t = T, \end{cases}$$

$$\tilde{\boldsymbol{e}}_t^i = \begin{cases} \boldsymbol{e}_c^i, t = 1,\cdots,T-1 \text{ 但 } t \neq k_c^i, \\ \boldsymbol{e}_c^i - \boldsymbol{d}_c^i, t = k_c^i. \end{cases}$$

以上各式中,(7)式为多子区域水环境经济投入产出方程,(8)—(10)式分别为水系水质约束、水量平衡和 BOD 平衡方程,而其余为变量定义、约束和端点方程[1]. 递阶算法能有效求解该问题.

二、多目标递阶优化算法

采用类似于文献[2,3]的方法建立 EWEP($\boldsymbol{\varepsilon}$) 的拉格朗日函数,并使用分解协调法求解,可得如下三级递阶结构:

主协调级(第三级)采用混合协调法。选择 $\boldsymbol{v}_t^p, \boldsymbol{u}_t^p$ 和 $\boldsymbol{\xi}^p$, 以及(1)式和(2)式的拉格朗日乘子向量 $\boldsymbol{\beta}_{vt}, \boldsymbol{\beta}_{ut}$ 和 $\boldsymbol{\beta}_\xi (t = 1,\cdots,T)$ 为主协调向量,可得第二级的四个独立子问题 $(SP1)$—$(SP4)$:

$(SP1)$
$$\min_{x,y,\zeta} \boldsymbol{\delta}^T \left(\sum_{t=1}^T \boldsymbol{f}_t - \boldsymbol{\varepsilon}\right),$$

s.t. $\begin{cases} (7)—(12)\text{式和}(20)—(23)\text{式}, \\ \boldsymbol{v}^p, \boldsymbol{u}^p \text{ 和 } \boldsymbol{\xi}^p \text{ 由第三级给定}, \end{cases}$

式中 $\boldsymbol{\delta} = [1, \delta_{l_2}, \delta_{l_3}, \delta_{l_4}]^T, \boldsymbol{\varepsilon} = [0, \varepsilon_{l_2}, \varepsilon_{l_3}, \varepsilon_{l_4}]^T$,

$\boldsymbol{f}_t = [f_t^{l_1}, f_t^{l_2}, f_t^{l_3}, f_t^{l_4}]^T$ 且 δ_{l_2}—δ_{l_4} 为 $K-T$ 乘子.

$(SP2)$
$$\min_v \sum_{t=1}^T \left(\frac{1}{2}\|\boldsymbol{v}_t - \boldsymbol{v}_t^p\|_{P_{vt}}^2 + \boldsymbol{\beta}_{vt}^T\boldsymbol{v}_t\right),$$

s.t. $\begin{cases} (14)—(16)\text{式}, \\ \boldsymbol{v}^p \text{ 和 } \boldsymbol{\beta}_v \text{ 由第三级给定}. \end{cases}$

$$(SP3) \qquad \min_{u} \sum_{t=1}^{T}\left(\frac{1}{2}\|\boldsymbol{u}_t-\boldsymbol{u}_t^p\|_{P_{ut}}^2+\boldsymbol{\beta}_{ut}^T\boldsymbol{u}_t\right)$$

s.t. $\begin{cases}(17)\text{—}(19)\text{式},\\ \boldsymbol{u}^p \text{ 和 } \boldsymbol{\beta}_u \text{ 由第三级给定.}\end{cases}$

$$(SP4) \qquad \min_{\xi}\left(\frac{1}{2}\|\boldsymbol{\xi}-\boldsymbol{\xi}^p\|_{P_\xi}^2+\boldsymbol{\beta}_\xi^T\boldsymbol{\xi}\right)$$

s.t $\begin{cases}(13)\text{式},\\ \boldsymbol{\xi}^p \text{ 和 } \boldsymbol{\beta}_\xi \text{由第三级给定,}\end{cases}$

式中没有给出定义的无时间下标向量为各年相应量所合成的向量(下同).

上述子问题中 ($SP2$) 和 ($SP3$) 与文献[2]中的相应子问题完全一样，其解见文献[2]和[4]；此外($SP4$) 已是按部门的分解形式，亦不再讨论.

现讨论 ($SP1$) 的解法. 设(7)式的拉格朗日乘子向量为 $\boldsymbol{\lambda}_t$，使用目标协调法取 $\boldsymbol{\lambda}$ 和 $\boldsymbol{\delta}$ 为 ($SP1$)的协调向量，并使用下式

$$\begin{aligned}
&\sum_{t=1}^{T}\boldsymbol{\lambda}_t^T\Big\{\Big[\sum_{i=1}^{m}(A^i+E^iP^i)\Xi^{pi}-I_n\Big]\boldsymbol{x}_t+\sum_{\tau=1}^{k}\sum_{i=1}^{m}(B_\tau^iV_{t\tau}^{pi}\\
&\qquad+D_\tau^iP^iU_{t\tau}^{pi})\Xi^{pi}(\boldsymbol{x}_{t+\tau}-\boldsymbol{x}_{t+\tau-1})+\sum_{i=1}^{m}[\boldsymbol{e}_c^iq_t^i\zeta_t^i\\
&\qquad+\boldsymbol{d}_c^i(q_{t+k_c}^i\zeta_{t+k_c^i}^i-q_{t+k_c^i-1}^i\zeta_{t+k_c^i-1}^i)]+\boldsymbol{y}_t\Big\}\\
&=\sum_{t=1}^{T}\Big\{\boldsymbol{h}^T(t)\boldsymbol{x}_t+\sum_{i=1}^{m}[\boldsymbol{\alpha}^{iT}(t)\boldsymbol{x}_t+\beta^i(t)]\zeta_t^i+\boldsymbol{\lambda}_t^T\boldsymbol{y}_t\Big\},
\end{aligned} \tag{28}$$

式中

$$\boldsymbol{h}^T(t)=\begin{cases}\displaystyle\sum_{\tau=0}^{k}\boldsymbol{\lambda}_{t-\tau}^TH_{t-\tau,\tau},\ t=1,\cdots,T-1,\\ \displaystyle\sum_{\tau=0}^{k}\Big(\boldsymbol{\lambda}_{t-\tau}^TH_{t-\tau,\tau}+\sum_{\theta=T+1}^{T+\tau}\boldsymbol{\lambda}_{\theta-\tau}^TH_{\theta-\tau,\tau}D_{x\theta}^0\Big),\ t=T,\end{cases}$$

$$\boldsymbol{\alpha}^{iT}(t)=\begin{cases}[\boldsymbol{\lambda}_t^T\boldsymbol{e}_c^i+(\boldsymbol{\lambda}_{t-k_c^i}^T-\boldsymbol{\lambda}_{t-k_c^i+1}^T)\boldsymbol{d}_c^i]\boldsymbol{i}_n^TP^i\Xi^{pi},\ t=1,\cdots,T-1,\\ \Big\{[\boldsymbol{\lambda}_t^T\boldsymbol{e}_c^i+(\boldsymbol{\lambda}_{t-k_c^i}^T-\boldsymbol{\lambda}_{t-k_c^i+1}^T)\boldsymbol{d}_c^i]\boldsymbol{i}_n^T\\ \qquad+\displaystyle\sum_{\theta=T+1}^{T+k_c^i}(\boldsymbol{\lambda}_{\theta-k_c^i}^T-\boldsymbol{\lambda}_{\theta-k_c^i+1}^T)\boldsymbol{d}_c^i\boldsymbol{i}_n^TD_{x\theta}^0d_{\zeta\theta}^{0i}\Big\}P^i\Xi^{pi},\ t=T,\end{cases}$$

$$\beta^i(t)=\begin{cases}[\boldsymbol{\lambda}_t^T\boldsymbol{e}_c^i+(\boldsymbol{\lambda}_{t-k_c^i}^T-\boldsymbol{\lambda}_{t-k_c^i+1}^T)\boldsymbol{d}_c^i]w_t^{0i},\ t=1,\cdots,T-1,\\ [\boldsymbol{\lambda}_t^T\boldsymbol{e}_c^i+(\boldsymbol{\lambda}_{t-k_c^i}^T-\boldsymbol{\lambda}_{t-k_c^i+1}^T)\boldsymbol{d}_c^i]w_t^{0i}\\ \qquad+\displaystyle\sum_{\theta=T+1}^{T+k_c^i}(\boldsymbol{\lambda}_{\theta-k_c^i}^T-\boldsymbol{\lambda}_{\theta-k_c^i+1}^T)\boldsymbol{d}_c^id_{\zeta\theta}^{0i}w_\theta^{0i},\ t=T,\end{cases}$$

且 $H_{t0}=\overline{A}^{\xi}-\overline{B}_{t1}^{\xi}$, $H_{t\tau}=\begin{cases}\overline{B}_{t\tau}^{\xi}-\overline{B}_{t,\tau+1}^{\xi}, \tau=1,\cdots,k-1,\\ \overline{B}_{t\tau}^{\xi}, \tau=k,\end{cases}$

$\boldsymbol{\lambda}_t=0, H_{t\tau}=0, t<1$ 或 $t>T$ 时，

$$\overline{A}^{\xi}=\sum_{i=1}^{m}(A^i+E^iP^i)\Xi^{pi}-I_n, \overline{B}_{t\tau}^{\xi}=\sum_{i=1}^{m}(B_\tau^iV_{t\tau}^{pi}+D_\tau^iP^iU_{t\tau}^{pi})\Xi^{pi}$$

可将 $(SP1)$ 分解成其第一级的T个子问题，其中第一级子问题 $(1-t), t=1,\cdots,T$ 为

$$\min_{x_t,y_t,\zeta_t}\{\mathscr{L}_t^1:(9),(10),(20)\text{ 和}(22)\text{式}\}, \tag{29}$$

且
$$\mathscr{L}_t^1=[\delta_1,\delta_2,\delta_3][f_t^1(\boldsymbol{y}_t),f_t^2(\boldsymbol{x}_t),f_t^3(\boldsymbol{\xi}^p,\boldsymbol{x}_t)]^T+\delta_4\sum_{i=1}^{m}[\boldsymbol{\alpha}_{f\xi}^{iT}(t)\boldsymbol{x}_t$$
$$+\beta_f^i(t)]\zeta_t^i+\boldsymbol{h}^T(t)\boldsymbol{x}_t+\sum_{i=1}^{m}[\boldsymbol{\alpha}^{iT}(t)\boldsymbol{x}_t+\beta^i(t)]\zeta_t^i+\boldsymbol{\lambda}_t^T\boldsymbol{y}_t$$
$$+\boldsymbol{\varphi}_t^T\{[\Gamma(\boldsymbol{\xi}^p,\boldsymbol{x}_t)\vdots F_0\widetilde{Q}_t][\boldsymbol{\zeta}_t^T\vdots\zeta_t^{sT}]^T-\boldsymbol{\gamma}(\boldsymbol{\xi}^p,\boldsymbol{x}_t)\},$$

式中 $\boldsymbol{\varphi}_t$ 为相应的拉格朗日乘子向量. 不难由上式看出，当 $\boldsymbol{x}_t$ 和 $\boldsymbol{y}_t$ 给定时，子问题 $(1-t)$是关于 $\boldsymbol{\zeta}_t$ 的如下线性规划问题：

$$\left.\begin{array}{l}\min\limits_{\xi_t}\sum\limits_{i=1}^{m}\{[\delta_4\boldsymbol{\alpha}_{f\xi}^{iT}(t)+\boldsymbol{\alpha}^{iT}(t)]\boldsymbol{x}_t+\delta_4\beta_f^i(t)+\beta^i(t)\}\zeta_t^i,\\ s.t.\ (8)\text{—}(10)\text{和}(22)\text{式},\end{array}\right\} \tag{30}$$

利用单纯形法求解，可得 $\boldsymbol{\zeta}_t,\boldsymbol{\zeta}_t^s$ 和 $\boldsymbol{\varphi}_t$. 此外，当 $\boldsymbol{\zeta}_t,\boldsymbol{\zeta}_t^s$ 和 $\boldsymbol{\varphi}_t$ 给定时，由 $\mathscr{L}_t^1$ 对于 $\boldsymbol{x}_t$ 和 $\boldsymbol{y}_t$ 的最优性条件并使用(9)和(10)式，得

$$\boldsymbol{x}_t=\boldsymbol{x}_t^{\#}=-\left[\delta_2W_{xt}+\delta_3\left(\sum_{i=1}^{m}\Xi^{pi}\boldsymbol{c}^i\right)\left(\sum_{i=1}^{m}\Xi^{pi}\boldsymbol{c}^i\right)^T\right]^{-1}$$
$$\times\left\{\sum_{i=1}^{m}[\delta_4\boldsymbol{\alpha}_{f\xi}^i(t)+\boldsymbol{\alpha}^i(t)]\zeta_t^i+\boldsymbol{h}(t)-\delta_3c_t^d\sum_{i=1}^{m}\Xi^{pi}\boldsymbol{c}_i\right.$$
$$\left.+[O_\xi^rE_t^{\zeta 0T}H_q^T+P_\xi^rE_t^{\eta 0T}H_q^T](F_q^T)^{-1}(Z_t^s-L_t^g)F_0^T]\boldsymbol{\varphi}_t\right\}, \tag{31}$$

$$\boldsymbol{y}_t=\boldsymbol{y}_t^{\#}=-\delta_1^{-1}W_{yt}^{-1}\boldsymbol{\lambda}_t+\boldsymbol{y}_t^d, \tag{32}$$

式中 Z_t^s 和 L_t^g 分别为 $\boldsymbol{\zeta}_t^s$ 和 $\boldsymbol{l}_t^g$ 元素组成的对角阵(如不加特别定义，文中大写符号与相应的小写黑体符号之间有类似的对应关系)，且

$$O_\xi^r=[O^1\boldsymbol{\xi}^{p1}\vdots\cdots\vdots O^m\boldsymbol{\xi}^{pm}], P_\xi^r=[P^1\boldsymbol{\xi}^{p1}\vdots\cdots\vdots P^m\boldsymbol{\xi}^{pm}]$$

考虑到(20)式，有

$$\boldsymbol{x}_t=\operatorname{sat}(\boldsymbol{x}_t^{\#}), \boldsymbol{y}_t=\operatorname{sat}(\boldsymbol{y}_t^{\#}), \tag{33}$$

而饱和函数 $\operatorname{sat}(\boldsymbol{\theta})$ 的第 j 个元素为

$$\operatorname{sat}(\theta_j)=\begin{cases}\theta_j^{\max}, \theta_j>\theta_j^{\max}\text{ 时},\\ \theta_j, \theta_j^{\min}\leqslant\theta_j\leqslant\theta_j^{\max}\text{ 时},\\ \theta_j^{\min}, \theta_j<\theta_j^{\min}\text{ 时}.\end{cases}$$

根据上述推导可得计算第一级子问题 $(1-t)$ 的如下迭代算法(算法 I)：

(1) 利用(32)式和(33)式计算 $\boldsymbol{y}_t$；

(2) 令 $l=0, \boldsymbol{x}_t^{(l)}=\boldsymbol{x}_t^0, \boldsymbol{x}_t^0$ 为猜测的迭代初值；

(3) 将 $\boldsymbol{x}_t^{(l)}$ 代入问题(30)，解得 $\boldsymbol{\zeta}_t^{(l+1)}, \boldsymbol{\zeta}_t'^{(l+1)}$ 和 $\boldsymbol{\varphi}_t^{(l+1)}$；

(4) 将 $\zeta_t^{(l+1)}, \boldsymbol{\zeta}_t'^{(l+1)}$ 和 $\boldsymbol{\varphi}_t^{(l+1)}$ 代入(31)式和(33)式得 $\boldsymbol{x}_t^{(l+1)}$；

(5) 若 $l=0$ 则转第 6 步，否则若满足 $\|\boldsymbol{x}_t^{(l+1)}-\boldsymbol{x}_t^{(l)}\|+\|\boldsymbol{\zeta}_t^{(l+1)}-\boldsymbol{\zeta}_t^{(l)}\| \leqslant \varepsilon$，则解为 $\boldsymbol{x}_t=\boldsymbol{x}_t^{(l+1)}, \boldsymbol{\zeta}_t=\boldsymbol{\zeta}_t^{(l+1)}$ 和 $\boldsymbol{y}_t$，且 $\boldsymbol{\zeta}_t'^{(l+1)}$ 和 $\boldsymbol{\varphi}_t^{(l+1)}$ 亦提供给上级用以协调，算法终止；

(6) 令 $l=l+1$ 转第 3 步.

子协调级 ($SP1$) 采用梯度型算法，收敛时有

$$\bar{A}^{\xi}\boldsymbol{x}_t+\sum_{\tau=1}^{k}\bar{B}_{t\tau}^{\xi}(\boldsymbol{x}_{t+\tau}-\boldsymbol{x}_{t+\tau-1})+\sum_{i=1}^{m}[\boldsymbol{e}_c^i q_t^i \zeta_t^i+\boldsymbol{d}_c^i(q^i_{t+k_c^i}\zeta^i_{t+k_c^i} - q^i_{t+k_c^i-1}\zeta^i_{t+k_c^i-1})]+\boldsymbol{y}_t=0, t=1,\cdots,T, \tag{34}$$

和

$$\delta_{l_j}\left(\sum_{t=1}^{T} f_t^{l_j}-\varepsilon_{l_j}\right)=0,\ \sum_{t=1}^{T} f_t^{l_j} \leqslant \varepsilon_{l_j}, \delta_{l_j} \geqslant 0, j=2,3,4, \tag{35}$$

式(34)即为式(7)，具体算法类似于文献[3].

使用与文献[2]相似的处理方法，可得到主协调级的直接迭代算法：

$$\boldsymbol{v}_t^{p(L+1)}=\boldsymbol{v}_t^{(L)}, \boldsymbol{\beta}_{vt}^{(L+1)}=\bar{B}_t^T(\boldsymbol{\xi}^{p(L+1)}, \boldsymbol{x}^{(L)})\boldsymbol{\lambda}_t^{(L)}; t=1,\cdots,T, \tag{36}$$

$$\boldsymbol{u}_t^{p(L+1)}=\boldsymbol{u}_t^{(L)}, \boldsymbol{\beta}_{ut}^{(L+1)}=\bar{D}_t^T(\boldsymbol{\xi}^{p(L+1)}, \boldsymbol{x}^{(L)})\boldsymbol{\lambda}_t^{(L)}; t=1,\cdots,T, \tag{37}$$

$$\left.\begin{aligned}&\boldsymbol{\xi}^{p(L+1)}=\boldsymbol{\xi}^{(L)},\\&\boldsymbol{\beta}_{\xi}^{(L+1)}=\sum_{t=1}^{T}[\delta_3\boldsymbol{c}_t(\boldsymbol{\xi}^{p(L+1)}, \boldsymbol{x}_t^{(L)})+\delta_4\boldsymbol{a}_{fx}(\boldsymbol{x}_t^{(L)}, \boldsymbol{\zeta}_t^{(L)})\\&\qquad+C_t^{xT}(\boldsymbol{v}_t^{p(L+1)}, \boldsymbol{u}_t^{p(L+1)}, \boldsymbol{x}^{(L)}, \boldsymbol{\zeta}^{(L)})\boldsymbol{\lambda}_t^{(L)}+R_t^x(\boldsymbol{x}_t^{(L)}, \boldsymbol{\zeta}_t^{(L)})\boldsymbol{\varphi}_t^{(L)}],\end{aligned}\right\} \tag{38}$$

式中

$$\boldsymbol{c}_t=[X_t\boldsymbol{c}^{1T}\vdots\cdots\vdots X_t\boldsymbol{c}^{mT}]^T\left(\sum_{i=1}^{m}\boldsymbol{c}^{iT}\Xi^{pi}\boldsymbol{x}_t-c_t^d\right),$$

$$\boldsymbol{a}_{fx}=[\zeta_t^1\boldsymbol{a}_{fx}^{1T}(t)\vdots\cdots\vdots\zeta_t^m\boldsymbol{a}_{fx}^{mT}(t)]^T,$$

$$\bar{B}_t=[\bar{B}_t^1\vdots\cdots\vdots\bar{B}_t^m], \bar{D}_t=[\bar{D}_t^1\vdots\cdots\vdots\bar{D}_t^m],$$

$$\bar{B}_t^i=[\bar{B}_{t1}^i\vdots\cdots\vdots\bar{B}_{tk_p^i}^i], \bar{D}_t^i=[\bar{D}_{t1}^i\vdots\cdots\vdots\bar{D}_{tk_d^i}^i],$$

$$\bar{B}_{t\tau}^i=B_\tau^i\Xi^{pi}(X_{t+\tau}-X_{t+\tau-1}), \bar{D}_{t\tau}^i=D_\tau^iP^i\Xi^{pi}(X_{t+\tau}-X_{t+\tau-1}),$$

$$C_t^x(\boldsymbol{v}_t^p, \boldsymbol{u}_t^p, \boldsymbol{x}, \boldsymbol{\zeta})=[C_t^{x1}\vdots\cdots\vdots C_t^{xm}],$$

$$C_t^{xi}=(A^i+E^iP^i)X_t+\sum_{\tau=1}^{k}(B_\tau^iV_{t\tau}^{pi}+D_\tau^iP^iU_{t\tau}^{pi})(X_{t+\tau}-X_{t+\tau-1})+\boldsymbol{e}_c^i\boldsymbol{i}_n^TP^iX_t\zeta_t^i+\boldsymbol{d}_c^i\boldsymbol{i}_n^TP^i(X_{t+k_c^i}\zeta^i_{t+k_c^i}-X_{t+k_c-1}\zeta^i_{t+k_c^i-1}),$$

$$R_t^x(\boldsymbol{x}_t, \boldsymbol{\zeta}_t)=O'_{xt}E_t^{\zeta 0T}H_q^T+P'_{xt}E_t^{\eta 0T}H_q^T(F_q^T)^{-1}(Z_t^s-L_t^g)F_0^T,$$

$$O'_{xt}=\text{block-diag}\{O^1\boldsymbol{x}_t,\cdots,O^m\boldsymbol{x}_t\}, P'_{xt}=\text{block-diag}\{P^1\boldsymbol{x}_t,\cdots,P^m\boldsymbol{x}_t\}.$$

当主协调级收敛时可得 EWEP($\boldsymbol{\varepsilon}$)（即原问题 WEP($\boldsymbol{\varepsilon}$)）的解. 求解过程如图 1 所示，其中除使用算法 I 求解子问题(1-1)，…，(1 − T) 以外，其余步骤与文献[2]算法十分类似，不再写出.

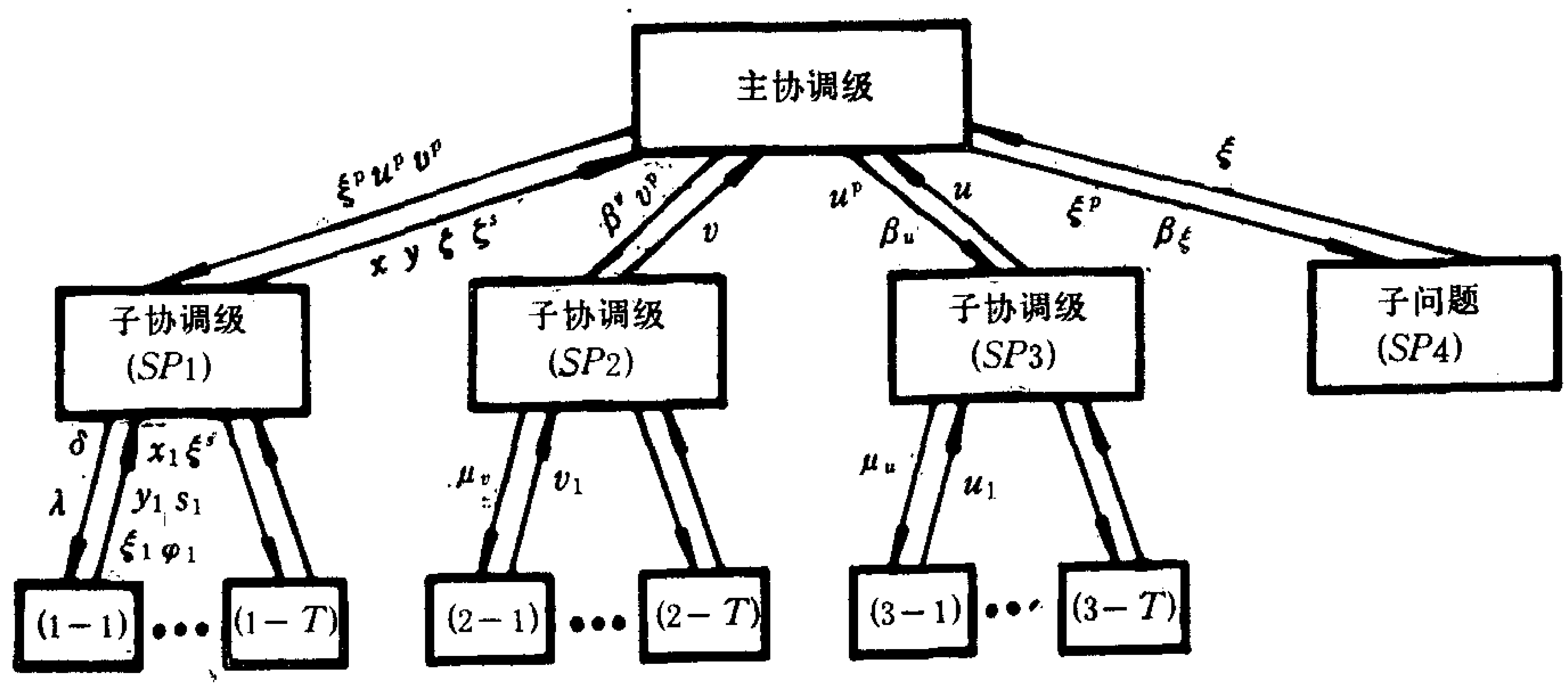

图 1 求解 EWEP(ε) 的三级递阶结构

容易看出该递阶算法有如下特点:

1) 分解-协调结构具有明确的经济和物理意义.子问题 (SP1) 的任务是在主协调级给定 $\boldsymbol{v},\boldsymbol{u}$ 和 $\boldsymbol{\xi}$ 当前值条件下，寻找 $\boldsymbol{x},\boldsymbol{y}$ 和 $\boldsymbol{\zeta}$ 的最优值,而其子问题 $(1-t)$ 实际上是第七年的水环境经济规划，它由该年的经济规划和水系水质规划相互耦合而成；子问题 (SP2)—(SP4) 则是利用主协调级给出的 $\boldsymbol{v},\boldsymbol{\beta}_v,\boldsymbol{u},\boldsymbol{\beta}_u,\boldsymbol{\xi}$ 和 $\boldsymbol{\beta}_\xi$ 的上次迭代值来改进 $\boldsymbol{v},\boldsymbol{u}$ 和 $\boldsymbol{\xi}$.

2) 当子协调级 (SP1) 收敛时，其解显然满足所有约束条件，所以算法为现实法.

在上述算法基础上，直接利用文献 [3] 方法可得求解整个问题的多目标递阶优化算法.

三、仿 真 算 例

研究某一区域水环境经济系统规划问题. 该区域由两个子区域组成，其河流系统如

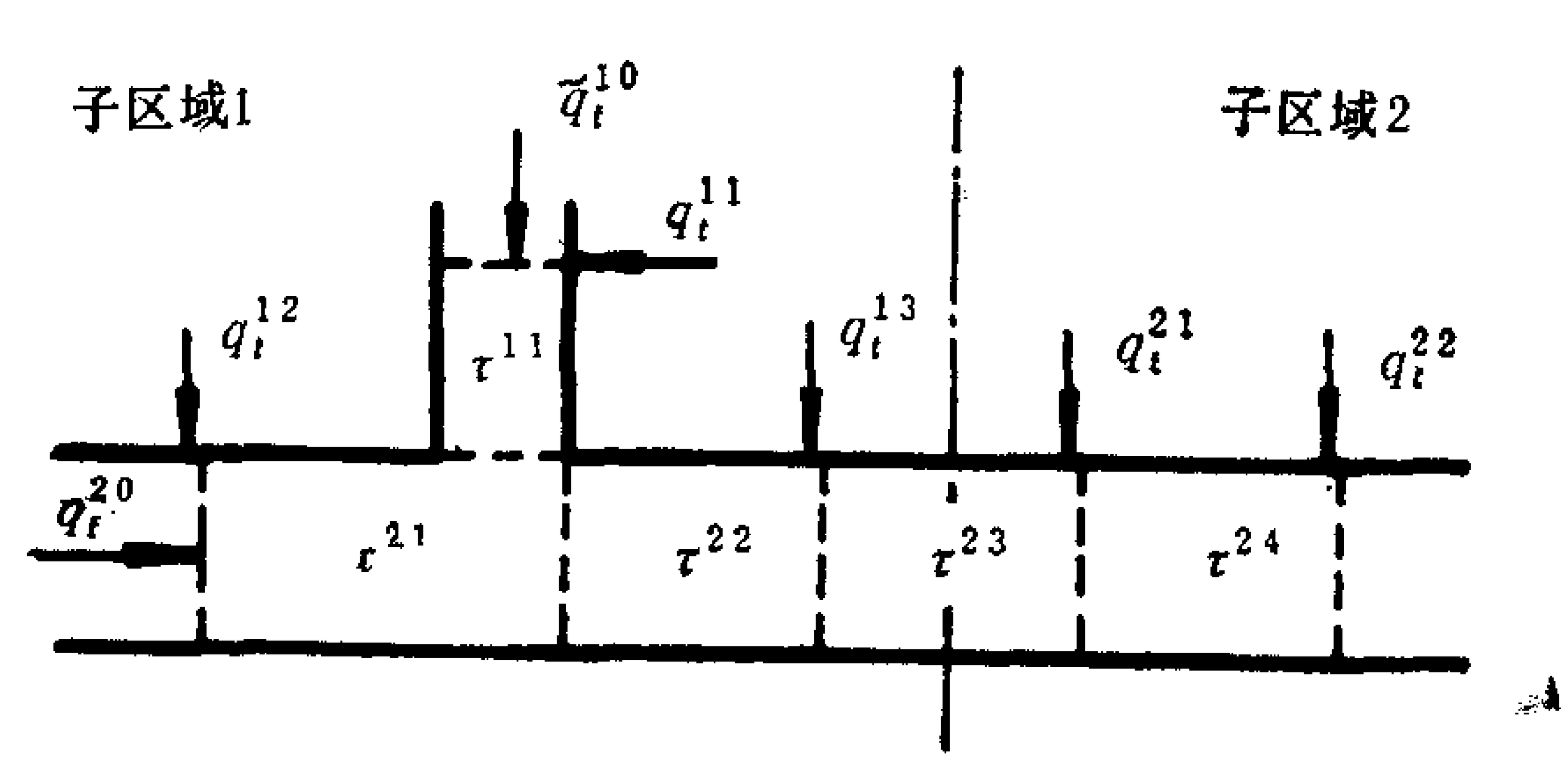

图 2 一个简单的河流系统

图 2 示，它包括两条河流和五个污水集中处理厂. 将它分成图示的五个河段，于是方程(8)—(10)中的向量和矩阵为

$$\tilde{\boldsymbol{q}}_t=[\tilde{q}_t^{11},\tilde{q}_t^{21},\tilde{q}_t^{22},\tilde{q}_t^{23},\tilde{q}_t^{24},\tilde{q}_t^{25}]^T,$$

$$\boldsymbol{l}_t=[l_t^{11},l_t^{21},l_t^{22},l_t^{23},l_t^{24},l_t^{25}]^T,$$

$$\tilde{\boldsymbol{q}}_t^0=[\tilde{q}_t^{10},\tilde{q}_t^{20}]^T=[10,20]^T(\mathrm{m}^3/\mathrm{s}),$$

$$\boldsymbol{l}_t^0=[l_t^{10},l_t^{20}]^T=[2,2]^T(\mathrm{g}/\mathrm{m}^3),$$

$$q_t' = [q_t^1, q_t^2]^T, \zeta_t = [\zeta_t^{11}, \zeta_t^{12}, \zeta_t^{13}, \zeta_t^{21}, \zeta_t^{22}]^T,$$

$$F_q = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ -1 & -1 & 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 & -1 & 1 \end{bmatrix}, \quad H_q = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix},$$

$$G_o = G_q = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, \quad E_t^{\eta 0} = \begin{bmatrix} \eta_t^{011} & 0 \\ \eta_t^{012} & 0 \\ \eta_t^{013} & 0 \\ 0 & \eta^{021} \\ 0 & \eta^{022} \end{bmatrix} = \begin{bmatrix} 0.15 & 0 \\ 0.40 & 0 \\ 0.45 & 0 \\ 0 & 0.6 \\ 0 & 0.4 \end{bmatrix},$$

$$F_o = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ -e^{-\tau^{11}} & -e^{-\tau^{21}} & 1 & 0 & 0 & 0 \\ 0 & 0 & -e^{-\tau^{22}} & 1 & 0 & 0 \\ 0 & 0 & 0 & -e^{-\tau^{23}} & 1 & 0 \\ 0 & 0 & 0 & 0 & e^{-\tau^{24}} & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ -e^{-0.3} & -e^{-0.4} & 1 & 0 & 0 & 0 \\ 0 & 0 & -e^{-0.4} & 1 & 0 & 0 \\ 0 & 0 & 0 & -e^{-0.6} & 1 & 0 \\ 0 & 0 & 0 & 0 & -e^{-0.45} & 1 \end{bmatrix},$$

$$E^{\zeta 0} = \begin{bmatrix} (1-\mu^{01}\zeta_t^{11})\eta_t^{011} & 0 \\ (1-\mu^{01}\zeta_t^{12})\eta_t^{012} & 0 \\ (1-\mu^{01}\zeta_t^{13})\eta_t^{013} & 0 \\ 0 & (1-\mu^{02}\zeta_t^{21})\eta_t^{021} \\ 0 & (1-\mu^{02}\zeta_t^{22})\eta_t^{022} \end{bmatrix} = \begin{bmatrix} (1-0.8\zeta_t^{11})0.15 & 0 \\ (1-0.8\zeta_t^{12})0.40 & 0 \\ (1-0.8\zeta_t^{13})0.45 & 0 \\ 0 & (1-.9\zeta_t^{21})0.6 \\ 0 & (1-0.9\zeta_t^{22})0.4 \end{bmatrix},$$

$$H_{ot}^0 = \mathrm{diag}\{\mu^{01}\eta_t^{011}O_t^1, \mu^{01}\eta^{012}O_t^1, \mu^{01}\eta^{013}O_t^1, \mu^{02}\eta_t^{021}O_t^2, \mu^{02}\eta_t^{022}O_t^2\}$$

$$= \mathrm{diag}\{0.12O_t^1, 0.32O_t^1, 0.36O_t^1, 0.54O_t^2, 0.36O_t^2\},$$

$$l_t^g = [5,5,5,5,6,6]^T(\mathrm{g/m^3}),$$

此外，投入产出平衡方程（7）中生产部门数 $n = 4$，且 $k_p^1 = k_p^2 = k_c^1 = 3, k_d^1 = k_d^2 = k_c^2 = 2, T = 10$；子区域 1 的主要参数为

$$A^1 = \begin{bmatrix} 0.328 & 0.171 & 0.243 & 0.176 \\ 0.075 & 0.120 & 0.006 & 0.043 \\ 0.014 & 0.007 & 0.008 & 0.009 \\ 0.037 & 0.123 & 0.016 & 0.019 \end{bmatrix}, B_1^1 = \begin{bmatrix} 0.550 & 0.122 & 0.315 & 0.212 \\ 0.191 & 0.018 & 0.102 & 0.098 \\ 0.091 & 0.006 & 0.037 & 0.053 \\ 0.073 & 0.004 & 0.063 & 0.054 \end{bmatrix},$$

$$B_2^1=\begin{bmatrix}0.670 & 0.028 & 0.321 & 0.102\\ 0.168 & 0.009 & 0.109 & 0.057\\ 0.068 & 0.003 & 0.039 & 0.027\\ 0.089 & 0.004 & 0.049 & 0.023\end{bmatrix},B_3^1=\begin{bmatrix}0.299 & 0.038 & 0.233 & 0.210\\ 0.090 & 0.026 & 0.071 & 0.089\\ 0.047 & 0.009 & 0.012 & 0.045\\ 0.057 & 0.009 & 0.028 & 0.065\end{bmatrix},$$

$$E^1=\begin{bmatrix}0.118 & 0.203 & 0.143 & 0.165\\ 0.081 & 0.093 & 0.005 & 0.051\\ 0.103 & 0.011 & 0.009 & 0.007\\ 0.017 & 0.019 & 0.017 & 0.017\end{bmatrix}(¥/\mathrm{m}^3),D_1^1=\begin{bmatrix}0.191 & 0.028 & 0.300 & 0.202\\ 0.201 & 0.008 & 0.111 & 0.099\\ 0.068 & 0.003 & 0.027 & 0.045\\ 0.081 & 0.005 & 0.053 & 0.045\end{bmatrix}(¥/\mathrm{m}^3),$$

$$D_2^1=\begin{bmatrix}0.298 & 0.031 & 0.213 & 0.200\\ 0.092 & 0.027 & 0.037 & 0.017\\ 0.043 & 0.009 & 0.005 & 0.051\\ 0.053 & 0.009 & 0.018 & 0.051\end{bmatrix}(¥/\mathrm{m}^3),$$

$$P^1=\mathrm{diag}\{0.056,0.014,0.179,0.028\}(\mathrm{m}^3/¥),$$

$$e_c^1=[0.02,0.028,0.03,0.041]^T(¥/\mathrm{m}^3)\quad d_c^1=[0.015,0.017,0.019,0.037]^T(¥/\mathrm{m}^3)$$

$$D_{xt}^0=D_x^0=\mathrm{diag}\{1.08,1.09,1.08,1.07\},d_{\zeta t}^{01}=d_\zeta^{01}=1.0,$$

$$O^1=\mathrm{diag}\{0.27,1.40,0.98,0.14\}(\mathrm{g}/¥).$$

子区域 2 的参数不再给出.

在 DUAL SYSTEM83/20 计算机上求解上述问题,计算表明:

1) 利用算法 I 求解子问题$(1-t)$一般仅需 3—5 次迭代便可以满意的精度收敛,其余各级收敛特性类似于文献[2]和[3]的相应级,因此本文算法具有良好的收敛性.

2) 规划结果的目标值能够按照决策者意图调整. 优化效果类似于文献[2], 此外结果中的污水集中处理厂规模和处理率的最优值可供设计使用. 因篇幅有限, 详细结果不再列出.

四、结 论

理论分析和计算机仿真表明, 本文算法能够有效地求解以水系水质规划为基础的多子区域多目标水环境经济规划这一复杂大系统问题. 该算法的分解-协调具有明确的经济和物理意义,是现实法并有良好的收敛性能. 这样本文与文献[1]相结合为区域水环境经济系统规划提供了有效的新方法.

最后指出,本算法完全适用于河段水质模型采用 BOD-DO 耦合方程的情况. 此外,进行系统灵敏度分析,进一步分析和证明算法 I 以及主协调级协调算法的收敛性,并利用收敛性证明结果来改进算法,是今后的研究任务.

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HIERARCHICAL MULTIOBJECTIVE PLANNING FOR REGIONAL WATER-ENVIRONMENT-ECONOMY SYSTEM

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Abstract

In this paper, the authors propose an interactive trale-off hierachical optimization algorithm for the multisubregional multiobjective water-environment-economy planning [1], which is based on the water quality planning of river system. The simulation example of a water-environment-economy system containing two subregions and five sewage centralized treatment plants is given to show the effectiveness of this algorithm.

Key words: Environmental economics; multiobjective optimization; hierarchical algorithm.

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