Model-Based Fault Tolerant Control for Hybrid Dynamic Systems with Sensor Faults

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Abstract A model-based fault tolerant control approach for hybrid linear dynamic systems is proposed in this paper. The proposed method, taking advantage of reliable control, can maintain the performance of the faulty system during the time delay of fault detection and diagnosis (FDD) and fault accommodation (FA), which can be regarded as the first line of defence against sensor faults. Simulation results of a three-tank system with sensor fault are given to show the efficiency of the method.

Key words Hybrid system, bond graph, reliable control, fault tolerant control

1 Introduction

The study of hybrid systems (HS) in control is motivated by the fundamentally hybrid nature of many modern control systems[1]. However, many results about HS only consider the fault-free case.

Fault-tolerant control (FTC), which rests on fault detection and diagnosis (FDD) and fault accommodation (FA), has received considerable attention in the past two decades[2,3]. But the hybrid nature of systems requires new forms of FDD and FTC analysis. Only a few existing papers are devoted to FDD and FTC of hybrid systems[4~7]. Those techniques are mainly based on hidden Markov model (HMM), statistical method and global Petri nets and so on. In [7], a switching-based FTC method was proposed, whose availability largely depends on the value of states when faults occur. Little attention has been paid to the time delay between the FDD and FA due to physical limitations[8]. During this time delay, the faulty systems still work under the original control law, and control inadmissibility may cause high risk of the system. Therefore, some measures have to be taken to avoid the rapid deterioration of the system performance.

The main contribution of this paper is: 1) to model the HS and detect the current mode by bond graph method. 2) to propose an observer-based state feedback strategy to guarantee the stability of each mode of HS. 3) to design a set of observers according to different modes, which can maintain the performance of the faulty system during the time delay of FDD and FA. The proposed method can be regarded as the first line of defence when sensor faults occur.

2 System Description

2.1 Characteristics of hybrid linear system and its stability

Let us start from a general description of controlled hybrid systems in the form

$$\begin{aligned} &\dot{\boldsymbol{x}}(t) = A_{s(t)}\boldsymbol{x}(t) + B_{s(t)}\boldsymbol{u}(t), \quad \boldsymbol{y}(t) = C_{s(t)}\boldsymbol{x}(t) \\ &s(t) = \varphi(\boldsymbol{x}(t), s(t^-), \boldsymbol{d}(t), t) \end{aligned} \tag{1}$$

where $\boldsymbol{x}(t)$ is the state vector, $\boldsymbol{u}(t)$ is thye control vector, $s(t) \in \{1, 2, \dots, M\}$ represents a discrete state at instant t that indexes the vector fields determining the states , $\boldsymbol{d}(t)$ is a “discrete event” (possibly control) input, and $\varphi(t)$ is a transition function.

The existence of a common Lyapunov function leads to the stability of system (1). However, this condition is restrictive in many real situations. We have the following theorem[1]:

Theorem 1. If some A_i have eigenvalues in the closed right-half complex plane, then a sufficient condition for the existence of $s(t)$ and $\boldsymbol{u}(t)$ to produce a stable x satisfying system (1) is that there exist $\alpha_i > 0$, $\sum\limits_i \alpha_i = 1$, such that $A_{eq} = \sum\limits_{i=1}^{M} \alpha_i A_i$ is stable matrix.

2.2 Description of three tanks system

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Consider a typical example of hybrid system: the three-tank system illustrated in Fig. 1, which has been investigated intensively recently in [4,6,9]. This system consists of tanks, flow sources, outlet pipes, and connecting pipes. Two types of discrete events may occur in the system: 1) (External jumps) Opening and closing valves change the system configuration; 2) (Autonomous jumps) The levels of fluid in the tanks exceeding a predefined value cause a flow in the pipes. The proposed method in this paper is based on the three-tank system described above, while the theory below can be applied to a more general class of hybrid systems, especially for those HS whose mode transitions are autonomous or whose states are required to be in a stability region all the time.

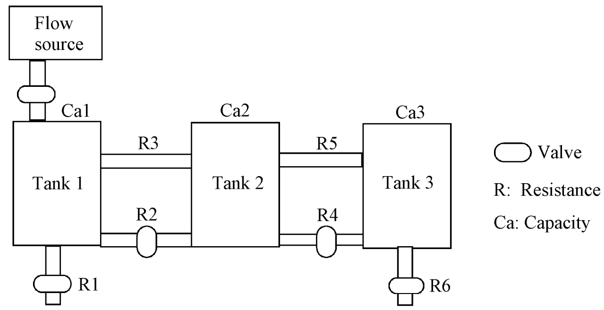

Fig. 1 The three-tank system

The fault tolerant control strategy in this paper involves three steps:

1) Determine the current mode by bond graph;

2) Switch to the relevant observer;

3) Apply the reliable state feedback control.

The whole framework of the proposed method is shown in Fig. 2. The part represented by dashed line is FDD and FA (see [2,3] and reference therein for some related research), which is not the focus of this paper. What we focus on is the part "Reliable control". We will discuss bond graphs method and the state feedback control respectively in the following sections.

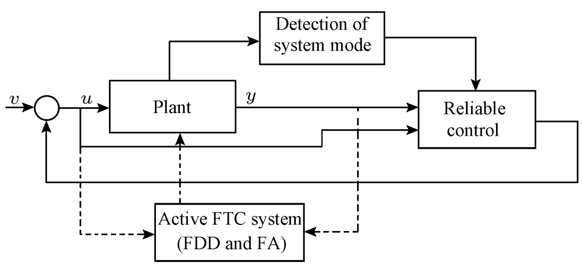

Fig. 2 The whole framework of FTC for hybrid system

3 Bond graphs

Bond graphs method developed in [9,10] can be used for representation of the system and detection of the mode change. In [4] and [6], the bond graph method was applied to the three-tank system. The bond graph is a graph in which nodes represent conservation of energy equations, and terminal nodes represent either system elements such as resistance, capacitance, inertia or sources. A bond is a power interconnection between elements that exchange energy. Power variables can be divided into two parts: 1) effort, which represents either force, torque, pressure, or voltage; 2) flow, which represents either velocity, rotational frequency, volume flow rate, or current. The numbers of the bonds correspond to the power variables.

The nodes of a bond graph are of two types: 1-junction and 0-junction. For a junction, the power of all bonds adds to zero at each moment. For the 1-junction, the flow is common to all the bonds and the algebraic sum of all efforts is zero. For the 0-junction, the effort is common to all the bonds and the sum of flows equals zero.

There exist standard techniques to build bond graph models of systems based on physical principles[4,9,10]. Fig. 3 shows the switched bond graph model of the three tank system, which depicts the sequential automata for the switched junctions $1_1, 1_2, 1_4, 1_6$ (externally controlled) and $1_3, 1_5$ (autonomously controlled). We enumerate states of the hybrid automata dynamically as system behavior evolves. For any given state we identify all its neighboring states by changing the status of each switch in the switched bond graph one at a time. We can determine the transition and reset conditions for these transitions from the local sequential automata of the corresponding switched junction. The states equations can be given from the bond graph as:

$$\begin{aligned} \dot{\boldsymbol{h}} &= A_i(Ca_1, \ldots, Ca_3, R_1, \ldots, R_6)\boldsymbol{h} + B_i(Ca_1, \ldots, Ca_3, R_1, \ldots, R_6)f \\ \boldsymbol{y} &= C_i\boldsymbol{h} \end{aligned} \tag{2}$$

When the current mode is determined by bond graph, the reliable state feedback control will be applied, which will be discussed in Section 4.

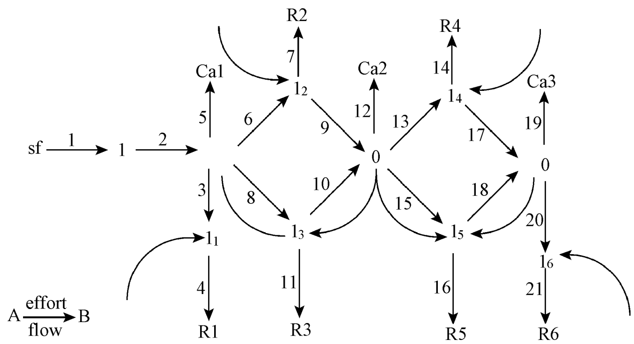

Fig. 3 Bond graph of three-tank system

4 Observer-based state feedback control

4.1 State feedback and vector-valued decision

The observer designed is based on [11] and [12]. Consider the system of (1) at the ith mode :

$$\dot{\boldsymbol{x}}(t) = A_i\boldsymbol{x}(t) + B_i\boldsymbol{u}(t), \quad \boldsymbol{y}(t) = C_i\boldsymbol{x}(t) \tag{3}$$

where $y(t)$ is the output vector, which is assumed to be divided into three parts: $\boldsymbol{y}(t) = [\boldsymbol{y}_1(t) \;\; \boldsymbol{y}_2(t) \;\; \boldsymbol{y}_3(t)]^T$, such that $\boldsymbol{x}(t)$ is detectable by using only $\boldsymbol{y}_i(t) (i = 1, 2, 3)$ and $\boldsymbol{u}(t)$. Under this assumption, C_i can be divided into three matrices as $C_i = [C_{i1} \;\; C_{i2} \;\; C_{i3}]^T$, where (C_{ij}, A_i) is a detectable pair, $j = 1, 2, 3$. C_{ij} represents the sensor set (ij). Consider the following observers for mode i:

$$Observer_{ij}: \quad \dot{\hat{\boldsymbol{x}}}_j(t) = A_i\hat{\boldsymbol{x}}_j(t) + B_i\boldsymbol{u}(t) + L_{ij}\Big(\hat{\boldsymbol{y}}(t) - (\boldsymbol{y}_j(t) + \boldsymbol{\omega}_j(t))\Big), \quad \hat{\boldsymbol{y}}_j(t) = C_{ij}\hat{\boldsymbol{x}}_j(t)$$

where $\hat{\boldsymbol{x}_j}(t)$ is the jth estimated value of $\boldsymbol{x}(t)$, $\boldsymbol{\omega}_j(t)$ is bounded measurement noise, L_{ij} is such that $(A_i + L_{ij}C_{ij})$ is a stable matrix. The reliable state feedback adopts one out of the three estimated states, $\hat{\boldsymbol{x}}_1(t)$, $\hat{\boldsymbol{x}}_2(t)$, $\hat{\boldsymbol{x}}_3(t)$, as the value $\hat{\boldsymbol{x}}(t)$, which is used in the state-feedback $\boldsymbol{u}(t) = F_i\hat{\boldsymbol{x}}(t) + \boldsymbol{v}(t)$, where F_i is such that $A_i + B_iF_i$ is a stable matrix, $\boldsymbol{v}(t)$ is an additional control input. Define that $\| A \| = \max\limits_i \sum\limits_j | a_{ij} |$, where a_{ij} is the element at the ith row and the jth column of A. If

$$\| \hat{\boldsymbol{x}}_p(t) - \hat{\boldsymbol{x}}_q(t) \| \quad = \quad \max\Big\{ \| \hat{\boldsymbol{x}}_p(t) - \hat{\boldsymbol{x}}_q(t) \|, \| \hat{\boldsymbol{x}}_q(t) - \hat{\boldsymbol{x}}_r(t) \|, \| \hat{\boldsymbol{x}}_p(t) - \hat{\boldsymbol{x}}_r(t) \| \Big\} \tag{4}$$

where $p, q, r \in \{1, 2, 3\}$, $p \neq q \neq r$, then the adopted value is $\hat{\boldsymbol{x}}(t) = \hat{\boldsymbol{x}}_r(t)$.

4.2 Augmented system for faulty case

Denotes the estimation error of $Observer_{ij}$ by $\boldsymbol{e}_j(t) = \hat{\boldsymbol{x}}_j(t) - \boldsymbol{x}(t)$, and $\bar{\boldsymbol{x}}(t) \triangleq [\boldsymbol{x}(t) \;\; \boldsymbol{e}_1(t) \;\; \boldsymbol{e}_2(t) \;\; \boldsymbol{e}_3(t)]^T$, $\bar{\boldsymbol{u}}(t) \triangleq [\boldsymbol{v}(t) \;\; \boldsymbol{\omega}_1(t) \;\; \boldsymbol{\omega}_2(t) \;\; \boldsymbol{\omega}_3(t)]^T$, $\bar{\boldsymbol{y}}(t) \triangleq [\boldsymbol{y}(t) \;\; \hat{\boldsymbol{x}}_1(t) \;\; \hat{\boldsymbol{x}}_2(t) \;\; \hat{\boldsymbol{x}}_3(t)]^T$. Then the augmented system can be written as

$$\dot{\bar{\boldsymbol{x}}}(t) = \bar{A}_i\bar{\boldsymbol{x}}(t) + \bar{B}_i\bar{\boldsymbol{u}}(t), \quad \bar{\boldsymbol{y}}(t) = \bar{C}_i\bar{\boldsymbol{x}}(t) \tag{5}$$

where

$$\bar{A}_i = \begin{bmatrix} A_i + B_iF_i & B_iF_i & 0 & 0 \\ 0 & \Lambda_{i1} & 0 & 0 \\ 0 & 0 & \Lambda_{i2} & 0 \\ 0 & 0 & 0 & \Lambda_{i3} \end{bmatrix}, \quad \bar{B}_i = \begin{bmatrix} B_i & 0 & 0 & 0 \\ 0 & -L_1 & 0 & 0 \\ 0 & 0 & -L_2 & 0 \\ 0 & 0 & 0 & -L_3 \end{bmatrix}, \quad \bar{C}_i = \begin{bmatrix} C_i & 0 & 0 & 0 \\ I & I & 0 & 0 \\ I & 0 & I & 0 \\ I & 0 & 0 & I \end{bmatrix}$$

where $\Lambda_{ij} = A_i + L_{ij}C_{ij}$. When one of sensors in sensor set (ij) is out of order, the faulty situation can be formulated by $C'_{ij} = diag[1, \ldots, 1, \beta, 1, \ldots, 1]C_{ij}$, where $0 < \beta < 1$. Without loss of generality, we assume that if a sensor in sensor $(i3)$ is faulty, then

$$\bar{A}_i = \begin{bmatrix} A_i + B_iF_i & a_1B_iF_i & a_2B_iF_i & a_3B_iF_i \\ 0 & A_i + L_{i1}C_{i1} & 0 & 0 \\ 0 & 0 & A_i + L_{i2}C_{i2} & 0 \\ L_3\bar{C}_3 & 0 & 0 & A_i + L_{i3}C_{i3} \end{bmatrix}$$

where a_1, a_2, a_3 are setting 0 or 1, and $a_i = 1$ only when $\hat{\boldsymbol{x}}_i(t)$ is adopted. Note that $\bar{A}_i$ is stable when $\hat{\boldsymbol{x}}_1(t)$ or $\hat{\boldsymbol{x}}_2(t)$ is adopted, and $\bar{A}_i$ is not always stable when $\hat{\boldsymbol{x}}_3(t)$ is adopted. The following theorem will show that even if an incorrect estimated state is adopted and the system is in an unstable structural state, the observers connected to healthy sensor sets are still making steady progress.

Theorem 2. The system is BIBO stable, *i.e.*, $\| \bar{\boldsymbol{x}}(t) \| < \infty$, $\| \bar{\boldsymbol{y}}(t) \| < \infty$ for any $\bar{\boldsymbol{u}}(t)$ against any sensor fault only in one sensor set.

The proof of Theorem 2 is based on two lemmas in [11] and some related work in [12]. We can obtain that there exist $\bar{M} > 0$ and $\bar{\eta} > 0$ satisfying $\| \Phi(t, t_{i0}) \| \leqslant \bar{M}e^{-\bar{\eta}t}$, where t_{i0} denotes the initial time when mode i is activated. Then the transition of (5) is given by

$$\bar{\boldsymbol{x}}(t) = \Phi(t, t_{i0})\bar{\boldsymbol{x}}(t_{i0}) + \int_{t_0}^{t} \Phi(t, \tau)\bar{B}\bar{\boldsymbol{u}}(\tau)d\tau \tag{6}$$

We can conclude that $\| \bar{\boldsymbol{x}}(t) \| < \infty$, $\| \bar{\boldsymbol{y}}(t) \| < \infty$ for any $\bar{\boldsymbol{u}}(t)$ in this domain. Considering the domain $\| \boldsymbol{e}_3(t) \| \leqslant \psi(t)$, where $\psi(t)$ is defined in [11], we have

$$\dot{\boldsymbol{x}}(t) = (A_i + B_iF_i)\boldsymbol{x}(t) + B_i\boldsymbol{v}(t) + B_iF_i\bar{\boldsymbol{e}}(t) \tag{7}$$

where $\bar{\boldsymbol{e}}(t)$ is one of $\boldsymbol{e}_1(t)$, $\boldsymbol{e}_2(t)$ and $\boldsymbol{e}_3(t)$. $\boldsymbol{e}_1(t)$, $\boldsymbol{e}_2(t)$ are bounded. Therefore, $\boldsymbol{x}(t)$ is bounded against any $\boldsymbol{v}(t)$. This completes proof of the theorem.

The block diagram of the reliable state feedback control is represented in Fig.4.

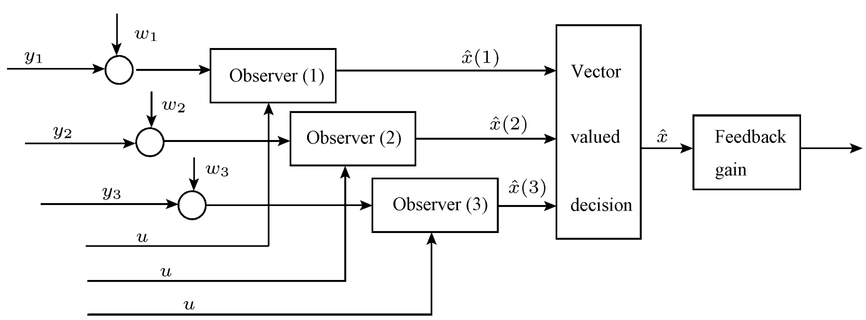

Fig. 4 The block diagram of reliable state feedback control

4.3 The stability of overall HS

For the overall HS, the method in sections 4.1 and 4.2 needs to be modified. The system will switch to the relevant observer at each transition time, and the initial states of the current observer are chosen the same as the final states in the previous mode under both controlled jumps and autonomous jumps.

Considering the stability of the overall systems, several tools (*e.g.* Multiple Lyapunov functions[1,7], *etc.*) can be used, which should be considered in the design of FDD and FA methods for hybrid systems (see [7] for example). The proposed method in this paper, together with these FDD and FA methods, can provide an integral FTC strategy for hybrid systems.

5 Implementation and simulation results

Consider the three tank system described above. Two modes are designed. Mode 1: all the valves are open except R_1 and R_6 , and the fluid levels in the tanks are higher than pipes R_3 and R_5; Mode 2: open R_1 and R_6, and the fluid levels in the tanks are still higher than pipes R_3 and R_5. The state equations of Modes 1, 2 can be obtained as[4]

$$\text{Mode } 1: \quad \begin{bmatrix} \dot{h}_1 \\ \dot{h}_2 \\ \dot{h}_3 \end{bmatrix} = \begin{bmatrix} \Xi_1 & \Xi_2 & \Xi_3 \end{bmatrix} \begin{bmatrix} h_1 \\ h_2 \\ h_3 \end{bmatrix} + \begin{bmatrix} \dfrac{1}{Ca_1} \\ 0 \\ 0 \end{bmatrix} f$$

where

$$\Xi_1 = \begin{bmatrix} -\dfrac{1}{Ca_1R_2} - \dfrac{1}{Ca_1R_3} \\ \dfrac{1}{Ca_2R_2} + \dfrac{1}{Ca_2R_3} \\ 0 \end{bmatrix}, \quad \Xi_2 = \begin{bmatrix} \dfrac{1}{Ca_1R_2} + \dfrac{1}{Ca_1R_3} \\ -\dfrac{1}{Ca_2R_2} - \dfrac{1}{Ca_2R_3} - \dfrac{1}{Ca_2R_4} - \dfrac{1}{Ca_2R_5} \\ \dfrac{1}{Ca_3R_4} + \dfrac{1}{Ca_3R_5} \end{bmatrix}$$

$$\Xi_3 = \begin{bmatrix} 0 \\ \dfrac{1}{Ca_2R_4} + \dfrac{1}{Ca_2R_5} \\ -\dfrac{1}{Ca_3R_4} - \dfrac{1}{Ca_3R_5} \end{bmatrix}, \quad \text{Mode } 2: \quad \begin{bmatrix} \dot{h}_1 \\ \dot{h}_2 \\ \dot{h}_3 \end{bmatrix} = \begin{bmatrix} \Xi'_1 & \Xi'_2 & \Xi'_3 \end{bmatrix} \begin{bmatrix} h_1 \\ h_2 \\ h_3 \end{bmatrix} + \begin{bmatrix} \dfrac{1}{Ca_1} \\ 0 \\ 0 \end{bmatrix} f$$

where

$$\Xi'_1 = \begin{bmatrix} -\dfrac{1}{Ca_1R_1} - \dfrac{1}{Ca_1R_2} - \dfrac{1}{Ca_1R_3} \\ \dfrac{1}{Ca_2R_2} + \dfrac{1}{Ca_2R_3} \\ 0 \end{bmatrix}, \quad \Xi'_2 = \begin{bmatrix} \dfrac{1}{Ca_1R_2} + \dfrac{1}{Ca_1R_3} \\ -\dfrac{1}{Ca_2R_2} - \dfrac{1}{Ca_2R_3} - \dfrac{1}{Ca_2R_4} - \dfrac{1}{Ca_2R_5} \\ \dfrac{1}{Ca_3R_4} + \dfrac{1}{Ca_3R_5} \end{bmatrix}$$

$$\Xi'_3 = \begin{bmatrix} 0 \\ \dfrac{1}{Ca_2R_4} + \dfrac{1}{Ca_2R_5} \\ -\dfrac{1}{Ca_3R_4} - \dfrac{1}{Ca_3R_5} - \dfrac{1}{Ca_3R_6} \end{bmatrix}$$

h_1, h_2, and h_3 correspond to the heights in the three tanks, and f is the flow into tank 1. These state equations are modelled based on the circuit equivalence relationship of three tanks[4,9]. Ca_i is equivalent to Capacity, and R_i is equivalent to Resistance. Ca_1=Ca_2=Ca_3. The values of R_i and Ca_i are chosen appropriately such that A_i satisfies Theorem 1.

Assume that a sensor fault occurs at mode 2 and height of R_3 (R_5) is chosen as $0.5m$, and assume the third sensor channel of C_3 is faulty and the degradation percentage is 40 percent. Fig. 5 shows the heights of fluid in three tanks. From $t = 0s$ to $t = 2s$, the system is working at Mode 1 and after $t = 2s$, the system switches to Mode 2. When fault occurs at $t = 4s$, trajectory of the height of fluid in each tank changes unexpectedly. It can be seen that system's performance can be maintained under the reliable state feedback control.

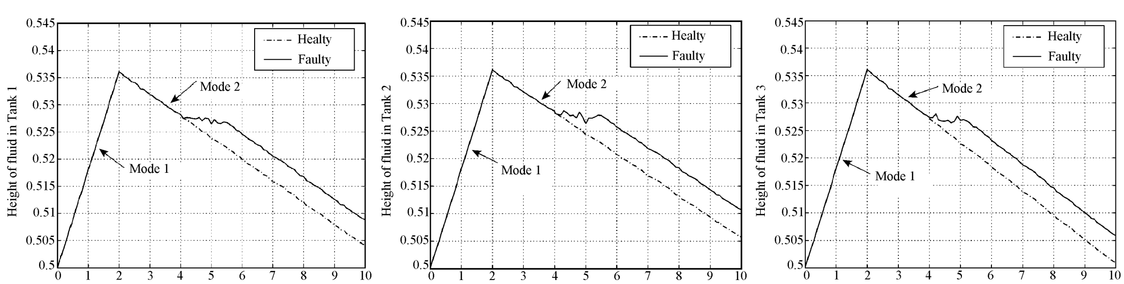

Fig. 5 Heights of fluid in the tank system

6 Conclusion

In this paper, a model-based fault tolerant control approach for hybrid linear dynamic system is proposed. The proposed method, based on bond graph modelling and reliable state feedback control, effectively maintains the system's performance when sensor faults occur. This method can be regarded as the first line of defence of system against faults during the time delay of FDD and FA, which can be extended to more general hybrid systems.

Note that a complex system can not just depend on the reliable control strategy, the combination of active FTC and reliable control is important. Future work will be focused on the design of control strategy which can take both the advantages of active FTC and reliable control.

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