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Passivity Control for Uncertain T-S Fuzzy Descriptor Systems[1)]

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Abstract By means of matrix decomposition method a criterion is presented for the admissibility of T-S fuzzy descriptor system. Then, the problem of passivity control is studied for a kind of T-S fuzzy descriptor system with uncertain parameters, and sufficient conditions which make the closed-loop system admissible and strictly passive are obtained based on linear matrix inequality (LMI). The nonstrict LMIs restricted conditions which characterize the descriptor system are transformed into strict ones, so testing admissibility and passivity of the system can be finished simultaneously. The design scheme of state feedback controller is also obtained. Finally, a numerical example is given to show the validity and feasibility of the proposed approach.

Key words Uncertain T-S fuzzy descriptor systems, admissibility, passivity, linear matrix inequality (LMI)

1 Introduction

The Takagi-Sugeno (T-S for short) fuzzy control method is recognized as an effective one in approximating or describing a complex nonlinear system. Fuzzy control technique has been extensively used in control of nonlinear system for more than two decades[1]. In 2000, Taniguchi T first presented the T-S fuzzy descriptor system model and gave an example to illustrate the advantage of fuzzy descriptor system over the original one[2]. It also demonstrates the importance of fuzzy descriptor system.

Motivated by the dissipation of energy across resistors in an electrical circuit, the concept of passivity of an input-output system has been widely used in stability analysis for nonlinear systems[3]. Willems J C studied passivity for nonlinear system in state space representations in terms of using Lyapunov functions[4,5]. Byrnes C I addressed that finite-dimensional nonlinear system was render passive *via* state feedback and provided a rather complete answer in terms of geometric nonlinear system theory[6]. Sun W Q pointed out that if the nonlinearity or uncertainty could be characterized by a positive real system, then the classical results in stability theory could be used to guarantee robust stability provided that an appropriate closed-loop system was strictly positive real[7]. In recent years, much attention has been given to the study on dissipation and passivity[8,9]. It enriches and develops the Lyapunov stability theory.

However, little attention has been paid to the dissipation and passivity control for the T-S fuzzy descriptor systems. In this paper, we consider the problem of passivity control for a kind of uncertain T-S fuzzy descriptor system. Since many results of the descriptor system cannot be directly used for the system[10~12], we present a necessary and sufficient condition to check admissibility for the T-S descriptor system. The notion of strict passivity is proposed for the system. The sufficient conditions, which make the closed-loop system admissible and strictly passive, are obtained for the system based on linear matrix inequalities (LMIs). The nonstrict LMIs restricted conditions, which characterize the descriptor system, are transformed into the strict ones. It overcomes the limitation of LMI Toolbox. So testing the admissibility and passivity of the system can be finished simultaneously by LMI Toolbox. The design scheme for the state feedback controller is also given. Finally, a numerical example is given to show the validity and feasibility of the proposed approach.

2 Problem formulation and preliminaries

In this section, we consider an uncertain T-S fuzzy descriptor system, and its ith fuzzy rule is of the following form:

$$R_i : \text{if } \xi_1(t) \text{ is } M_{1i} \text{ and } \xi_2(t) \text{ is } M_{2i} \cdots \text{ and } \xi_p(t) \text{ is } M_{pi}$$

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$$\text{then } E\dot{\boldsymbol{x}}(t) = (A_i + \Delta A_i)\boldsymbol{x}(t) + B_{1i}\boldsymbol{w}(t) + (B_{2i} + \Delta B_{2i})\boldsymbol{u}(t) \tag{1a}$$

$$\boldsymbol{z}(t) = (C_i + \Delta C_i)\boldsymbol{x}(t) + D_{1i}\boldsymbol{w}(t) + (D_{2i} + \Delta D_{2i})\boldsymbol{u}(t), \quad i = 1, 2, \cdots, r \tag{1b}$$

where $M_{ji}(j = 1, 2, \cdots, p)$ is the fuzzy set, r is the number of if-then rules, $\boldsymbol{x}(t) \in R^n$ is the state vector, $\boldsymbol{u}(t) \in R^m$ is the control input, $\boldsymbol{w}(t) \in R^l$ is the exogenous input, and $\boldsymbol{w}(t) \in L_2[0, \infty)$. $\boldsymbol{z}(t) \in R^l$ is the controlled output, E and $A_i, B_{1i}, B_{2i}, C_i, D_{1i}, D_{2i}$ are known constant matrices with appropriate dimensions. E is the singular matrix and $\text{rank}E = n_1 < n$. The $\xi_1(t), \xi_2(t), \cdots, \xi_p(t)$ are premise variables which may be functions of the state variables. Let $\boldsymbol{\xi}(t) = [\xi_1(t), \xi_2(t), \cdots, \xi_p(t)]^{\rm T}$. The matrice $\Delta A_i, \Delta B_{2i}, \Delta C_i, \Delta D_{2i}$ represent the time-varying parametric uncertainties with the following structure:

$$[\Delta A_i \quad \Delta B_{2i}] = H_{1i}F_i[E_{1i} \quad E_{2i}], \quad [\Delta C_i \quad \Delta D_{2i}] = H_{2i}F_i[E_{1i} \quad E_{2i}], \ i = 1, 2, \cdots, r$$

where $H_{1i}, H_{2i}, E_{1i}, E_{2i}$ are known constant matrices with appropriate dimensions, and F_i are unknown real matrices satisfying $F_i^{\rm T}F_i \leqslant I$. By using the fuzzy inference methods with singleton fuzzifer and weighted average defuzzifer, the overall fuzzy model for the system can be inferred as follows:

$$E\dot{\boldsymbol{x}}(t) = \sum_{i=1}^{r} h_i(\boldsymbol{\xi}(t))[(A_i + \Delta A_i)\boldsymbol{x}(t) + B_{1i}\boldsymbol{w}(t) + (B_{2i} + \Delta B_{2i})\boldsymbol{u}(t)] \tag{2a}$$

$$\boldsymbol{z}(t) = \sum_{i=1}^{r} h_i(\boldsymbol{\xi}(t))[(C_i + \Delta C_i)\boldsymbol{x}(t) + D_{1i}\boldsymbol{w}(t) + (D_{2i} + \Delta D_{2i})\boldsymbol{u}(t)], \quad i = 1, 2, \cdots, r \tag{2b}$$

$$\beta_i(\boldsymbol{\xi}(t)) = \prod_{j=1}^{P} M_{ji}(\xi_j(t)) \geqslant 0, \quad h_i(\boldsymbol{\xi}(t)) = \frac{\beta_i(\boldsymbol{\xi}(t))}{\sum\limits_{i=1}^{r} \beta_i(\boldsymbol{\xi}(t))} \geqslant 0, \quad \sum_{i=1}^{r} h_i(\boldsymbol{\xi}(t)) = 1$$

where $h_i(\boldsymbol{\xi}(t))$ can be regarded as the normalized weight of each if-then rule.

Consider T-S fuzzy descriptor system of the following form:

$$E\dot{\boldsymbol{x}}(t) = \sum_{i=1}^{r} h_i(\boldsymbol{\xi}(t))A_i\boldsymbol{x}(t) \tag{3}$$

Definition 1. System (3) is said to be regular if $\det(sE - \sum\limits_{i=1}^{r} h_i(\boldsymbol{\xi}(t))A_i)$ is not identically zero for any $t \geqslant 0$. System (3) is said to be impulse-free if it is regular and $\deg_s \det(sE - \sum\limits_{i=1}^{r} h_i(\boldsymbol{\xi}(t))A_i) = \text{rank}E$ for any $t \geqslant 0$. System (3) is said to be stable if it is regular and satisfies $\text{Re}(s) < 0$, $\forall t \geqslant 0$, for $s \in \sigma(E, \sum\limits_{i=1}^{r} h_i(\boldsymbol{\xi}(t))A_i)$, where $\sigma(E, \sum\limits_{i=1}^{r} h_i(\boldsymbol{\xi}(t))A_i) = \left\{ s|\det(sE - \sum\limits_{i=1}^{r} h_i(\boldsymbol{\xi}(t))A_i) = 0 \right\}$. System (3) is said to be admissible if it is regular, impulse-free and stable.

Theorem 1. System (3) is admissible if and only if there exists a common nonsingular matrix $P \in R^{n \times n}$ such that the following inequalities hold.

$$E^{\rm T}P = P^{\rm T}E \geqslant 0 \tag{4}$$

$$(\sum_{i=1}^{r} h_i(\boldsymbol{\xi}(t))A_i)^{\rm T}P + P^{\rm T}(\sum_{i=1}^{r} h_i(\boldsymbol{\xi}(t))A_i) < 0, \quad i = 1, 2, \cdots, r, \ \forall t \geqslant 0 \tag{5}$$

Proof. Sufficiency. Suppose $s \in \sigma(E, \sum\limits_{i=1}^{r} h_i(\boldsymbol{\xi}(t))A_i)$, and that the generalized eigenvector $\boldsymbol{x} \in C^n$ is consistent with it. We can obtain that $2\text{Re}(s)\boldsymbol{x}^*E^{\rm T}P\boldsymbol{x} < 0$. Thus, system (3) is regular and stable. So there exist invertible matrices M_1 and N_1 such that

$$M_1EN_1 = \text{diag}(I_{n_1}, N) \tag{6}$$

$$M_1(\sum_{i=1}^{r} h_i(\boldsymbol{\xi}(t))A_i)N_1 = \mathrm{diag}(\tilde{A}_1, I_{n_2}) \tag{7}$$

where $\tilde{A}_1 \in R^{n_1\times n_1}$, $N \in R^{n_2\times n_2}$, N is a nilpotent matrix, $n_1+n_2=n$, $\tilde{A}_1$ and N are related to membership functions.

Suppose $N^{h-1}\neq 0$, $N^h=0$, $h>1$. Partition $M_1^{-\mathrm{T}}PN_1$ to conform with (6) and (7), that is,

$$M_1^{-\mathrm{T}}PN_1 = \begin{bmatrix} P_1 & P_2 \\ P_3 & P_4 \end{bmatrix} \tag{8}$$

From (4), (5) and (8), it can be shown that $P_4+P_4^{\mathrm{T}}<0$, and $N^{\mathrm{T}}P_4 = P_4^{\mathrm{T}}N \geqslant 0$. Then it follows that

$$0=(N^{\mathrm{T}})^{h-2}P_4^{\mathrm{T}}N^h + (N^{\mathrm{T}})^h P_4 N^{h-2} < 0$$

It is a contradiction. So that $N=0$. Namely, system (3) is impulse-free. Therefore, it is admissible.

Necessity. Suppose that system (3) is regular and impulse-free. Then there exist invertible matrices M_2 and N_2 such that

$$M_2EN_2 = \mathrm{diag}(I_{n_1}, 0) \tag{9}$$

$$M_2(\sum_{i=1}^{r} h_i(\boldsymbol{\xi}(t))A_i)N_2 = \mathrm{diag}(\bar{A}_1, I_{n_2}) \tag{10}$$

where $\bar{A}_1 \in R^{n_1\times n_1}$, $N \in R^{n_2\times n_2}$, N is a nilpotent matrix, $n_1+n_2=n$, $\bar{A}_1$ and N are related to membership functions. Since system (3) is stable , there exists a positive definite symmetric matrix $P_{11}\in R^{n_1\times n_1}$ such that $\bar{A}_1^{\mathrm{T}}P_{11}+P_{11}\bar{A}_1<0$. Now define a matrix P as

$$P = M_2^{\mathrm{T}}\begin{bmatrix} P_{11} & 0 \\ 0 & -I_{n_2} \end{bmatrix} N_2^{-1}$$

It is easy to verify that P in the above form is a nonsingular matrix and satisfies (4) and (5). □

The following lemma will be used later.

Lemma 1[13]**.** For matrices D and G with appropriate dimensions and symmetric Y,

$$Y + DFG + G^{\mathrm{T}}F^{\mathrm{T}}D^{\mathrm{T}} < 0$$

for all F satisfying $F^{\mathrm{T}}F\leqslant I$, if and only if there exists some scalar $\varepsilon>0$ such that

$$Y + \varepsilon DD^{\mathrm{T}} + \varepsilon^{-1}G^{\mathrm{T}}G < 0$$

3 Passivity control

Definition 2. The system of the form (2) is said to be strictly passive if there exists a positive constant δ, for each solution with initial condition $\boldsymbol{x}(0)=0$, such that the following inequality

$$V(\boldsymbol{x}(\tau)) \leqslant \int_0^{\tau} \boldsymbol{z}^{\mathrm{T}}(t)\boldsymbol{w}(t)\mathrm{d}t - \delta\int_0^{\tau}\boldsymbol{w}^{\mathrm{T}}(t)\boldsymbol{w}(t)\mathrm{d}t$$

holds for all positive constants τ and $\boldsymbol{w}(t)\in L_2[0,\tau]$, where V is a nonnegative continuous real function satisfying $V(0)=0$.

For the fuzzy model (2), we construct the following fuzzy controller *via* the PDC:

$$\boldsymbol{u}(t) = \sum_{i=1}^{r} h_i(\boldsymbol{\xi}(t))K_i\boldsymbol{x}(t) \tag{11}$$

where K_i is the local feedback gain matrix. By substituting (11) into system (2), we obtain the following closed-loop system

$$E\dot{\boldsymbol{x}}(t) = \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(\boldsymbol{\xi}(t))h_j(\boldsymbol{\xi}(t))\{[(A_i+\Delta A_i)+(B_{2i}+\Delta B_{2i})K_j]\boldsymbol{x}(t)+B_{1i}\boldsymbol{w}(t)\} \tag{12a}$$

$$\boldsymbol{z}(t)=\sum_{i=1}^{r}\sum_{j=1}^{r}h_i(\boldsymbol{\xi}(t))h_j(\boldsymbol{\xi}(t))\{[(C_i+\Delta C_i)+(D_{2i}+\Delta D_{2i})K_j]\boldsymbol{x}(t)+D_{1i}\boldsymbol{w}(t)\} \tag{12b}$$

Theorem 2. For system (12) with the zero initial condition, if there exist a common nonsingular matrix $P\in R^{n\times n}$ and a positive constant δ such that the following LMIs

$$E^{\mathrm{T}}P=P^{\mathrm{T}}E\geqslant 0 \tag{13a}$$

$$\sum_{i=1}^{r}h_i(\boldsymbol{\xi}(t))(-D_{1i}-D_{1i}^{\mathrm{T}}+\delta I)<0 \tag{13b}$$

$$\sum_{i=1}^{r}\sum_{j=1}^{r}h_i(\boldsymbol{\xi}(t))h_j(\boldsymbol{\xi}(t))\begin{bmatrix}\Delta & \varOmega\\ * & (-D_{1i}-D_{1i}^{\mathrm{T}}+\delta I)\end{bmatrix}<0,\quad i,j=1,2,\cdots,r,\ \forall r\geqslant 0 \tag{13c}$$

hold then system (12) is admissible and strictly passive, where the asterisk denotes the transposed elements of symmetric positions in the matrix, and

$$\Delta=[(A_i+\Delta A_i)+(B_{2i}+\Delta B_{2i})K_j]^{\mathrm{T}}P+P^{\mathrm{T}}[(A_i+\Delta A_i)+(B_{2i}+\Delta B_{2i})K_j]$$
$$\varOmega=P^{\mathrm{T}}B_{1i}-[(C_i+\Delta C_i)+(D_{2i}+\Delta D_{2i})K_j]^{\mathrm{T}}$$

Proof. Consider the nonnegative functional $V(\boldsymbol{x}(t))$ of the form $V(\boldsymbol{x}(t))=\dfrac{1}{2}\boldsymbol{x}^{\mathrm{T}}(t)E^{\mathrm{T}}P\boldsymbol{x}(t)$. From Theorem 1 and Definition 2, we can obtain that system (12) is admissible when $\boldsymbol{w}(t)=0$ and

$$V(\boldsymbol{x}(\tau))\leqslant\int_0^{\tau}\boldsymbol{z}^{\mathrm{T}}(t)\boldsymbol{w}(t)\mathrm{d}t-\frac{1}{2}\delta\int_0^{\tau}\boldsymbol{w}^{\mathrm{T}}(t)\boldsymbol{w}(t)\mathrm{d}t$$

That is, system (12) is admissible and strictly passive. □

Theorem 3. For system (12) with the zero initial condition, if there exist a common nonsingular matrix $X\in R^{n\times n}$, matrix N_i and positive constants $\delta,\varepsilon_i,\varepsilon_{ij}$ such that the following LMIs

$$XE^{\mathrm{T}}=EX^{\mathrm{T}}\geqslant 0 \tag{14a}$$

$$-D_{1i}-D_{1i}^{\mathrm{T}}+\delta I<0,\quad i=1,2,\cdots,r \tag{14b}$$

$$\begin{bmatrix}\Delta_1 & \varOmega_1 & \varepsilon_iH_{1i} & \varXi_1\\ * & -D_{1i}-D_{1i}^{\mathrm{T}}+\delta I & -\varepsilon_iH_{2i} & E_{2i}^{\mathrm{T}}\\ * & * & -\varepsilon_iI & 0\\ * & * & * & -\varepsilon_iI\end{bmatrix}<0,\quad i=1,2,\cdots,r \tag{14c}$$

$$\begin{bmatrix}\Delta_2 & \varOmega_2 & \varepsilon_{ij}H_{1i} & \varepsilon_{ij}H_{1j} & \varXi_2 & \varXi_3\\ * & -D_{1i}-D_{1i}^{\mathrm{T}}-D_{1j}-D_{1j}^{\mathrm{T}}+2\delta I & -\varepsilon_{ij}H_{2i} & -\varepsilon_{ij}H_{2j} & 0 & 0\\ * & * & -\varepsilon_{ij}I & 0 & 0 & 0\\ * & * & * & -\varepsilon_{ij}I & 0 & 0\\ * & * & * & * & -\varepsilon_{ij}I & 0\\ * & * & * & * & * & -\varepsilon_{ij}I\end{bmatrix}<0,\ i<j\leqslant r \tag{14d}$$

hold then system (12) is admissible and strictly passive, the state feedback controller $\boldsymbol{u}(t)=\sum_{i=1}^{r}h_i(\boldsymbol{\xi}(t))N_iX^{-\mathrm{T}}\boldsymbol{x}(t)$, where the asterisk denotes the transposed elements of symmetric positions, and

$$\Delta_1=XA_i^{\mathrm{T}}+A_iX^{\mathrm{T}}+N_i^{\mathrm{T}}B_{2i}^{\mathrm{T}}+B_{2i}N_i$$
$$\Delta_2=XA_i^{\mathrm{T}}+A_iX^{\mathrm{T}}+N_j^{\mathrm{T}}B_{2i}^{\mathrm{T}}+B_{2i}N_j+XA_j^{\mathrm{T}}+A_jX^{\mathrm{T}}+N_i^{\mathrm{T}}B_{2j}^{\mathrm{T}}+B_{2j}N_i$$
$$\varOmega_1=B_{1i}-XC_i^{\mathrm{T}}-N_i^{\mathrm{T}}D_{2i}^{\mathrm{T}}$$
$$\varOmega_2=B_{1i}-XC_i^{\mathrm{T}}-N_j^{\mathrm{T}}D_{2i}^{\mathrm{T}}+B_{1j}-XC_j^{\mathrm{T}}-N_i^{\mathrm{T}}D_{2j}^{\mathrm{T}}$$
$$\varXi_1=XE_{1i}^{\mathrm{T}}+N_i^{\mathrm{T}}E_{2i}^{\mathrm{T}},\quad \varXi_2=XE_{1i}^{\mathrm{T}}+N_j^{\mathrm{T}}E_{2i}^{\mathrm{T}},\quad \varXi_3=XE_{1j}^{\mathrm{T}}+N_i^{\mathrm{T}}E_{2j}^{\mathrm{T}}$$
$$X=P^{-\mathrm{T}},\quad N_i=K_iX^{\mathrm{T}}$$

Proof. By using Theorem 2, Lemma 1, the Schur complement Lemma and the elementary transformation of matrix, we can infer the theorem. □

Remark 1. When normalized membership functions are $h_i(\boldsymbol{\xi}(t)) = 1$, $h_j(\boldsymbol{\xi}(t)) = 0$, $\forall j \neq i$, $\delta = 2$, rank$E = n$, and the system is without uncertain parameters, Theorm 2 is the sufficient condition to check the strictly positive real character for original linear system[7]. When normal membership functions are $h_i(\boldsymbol{\xi}(t)) = 1$, $h_j(\boldsymbol{\xi}(t)) = 0$, $\forall j \neq i$, Theorem 2 is the sufficient condition to test strict passivity and admissibility for descriptor system with uncertain parameters.

Now, we deal with the nonstrict matrix inequality constraint in (13a). Without loss of generality, we assume $E = \begin{pmatrix} I_{n_1} & 0 \\ 0 & 0 \end{pmatrix}$. Partition $P = \begin{bmatrix} P_1 & P_2 \\ P_3 & P_4 \end{bmatrix}$ to conform with E. We can infer that the constraint in (13a) is equivalent to $P = \begin{bmatrix} P_1 & 0 \\ P_3 & P_4 \end{bmatrix}$, $P_1 > 0$, det$P_4 \neq 0$. Thus, we obtain $XE^{\mathrm{T}} = EX^{\mathrm{T}} = \begin{bmatrix} P_1^{-1} & 0 \\ 0 & 0 \end{bmatrix} \geqslant 0$. Therefore, the following theorem holds.

Theorem 4. For system (12) with the zero initial condition, if there exist a common nonsingular matrix $X \in R^{n\times n}$, matrix N_i and positive constants $\delta, \varepsilon_i, \varepsilon_{ij}$ such that the LMIs (14b), (14c) and (14d) hold, then system (12) is admissible and strictly passive, where $X = P^{-\mathrm{T}} = \begin{bmatrix} P_1^{-1} & -P_1^{-1}P_3^{\mathrm{T}}P_4^{-\mathrm{T}} \\ 0 & P_4^{-\mathrm{T}} \end{bmatrix} := \begin{bmatrix} X_1 & X_2 \\ 0 & X_3 \end{bmatrix}$, $X_1 = P_1^{-1} > 0$, $N_i = K_i X^{\mathrm{T}}$. The state feedback controller $\boldsymbol{u}(t) = \sum_{i=1}^{r} h_i(\boldsymbol{\xi}(t)) N_i X^{-\mathrm{T}} \boldsymbol{x}(t)$.

Remark 2. By Theorem 4, the nonstrict LMIs restricted conditions in (13a), which characterize the descriptor system, are transformed into the strict ones. So it overcomes the limitation of LMI Toolbox. Thus we can use the LMI Toolbox to check admissibility and strict passivity only once.

4 Numerical example

Consider the following uncertain T-S fuzzy system

$$\begin{aligned} &R_i : \text{if } x_1(t) \text{ is } M_i \\ &\quad \text{then } E\dot{\boldsymbol{x}}(t) = (A_i + \Delta A_i)\boldsymbol{x}(t) + B_{1i}\boldsymbol{w}(t) + (B_{2i} + \Delta B_{2i})\boldsymbol{u}(t) \\ &\qquad \boldsymbol{z}(t) = (C_i + \Delta C_i)\boldsymbol{x}(t) + D_{1i}\boldsymbol{w}(t) + (D_{2i} + \Delta D_{2i})\boldsymbol{u}(t), \quad i = 1, 2 \end{aligned} \tag{15}$$

The membership functions are chosen as $h_1(x_1(t)) = \frac{x_1^2(t)}{2}$, $h_2(x_1(t)) = 1 - \frac{x_1^2(t)}{2}$, where state vector $\boldsymbol{x}(t) = [x_1^{\mathrm{T}}(t), x_2^{\mathrm{T}}(t), x_3^{\mathrm{T}}(t)]^{\mathrm{T}}$, $x_i(t) \in R^1$, $i = 1, 2, 3$. The control parameters are defined as follows

$$E = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix},\ A_1 = \begin{bmatrix} 1 & 8 & 1 \\ 1 & 1 & 7 \\ 1 & 3 & 5 \end{bmatrix},\ A_2 = \begin{bmatrix} 2 & -7 & 1 \\ 2 & 0 & 7 \\ 9 & 1 & 1 \end{bmatrix},\ B_{11} = \begin{bmatrix} 1 & 0 \\ 1 & 2 \\ 0 & 1 \end{bmatrix},\ B_{12} = \begin{bmatrix} 1 & 1 \\ 1 & 0 \\ 1 & 1 \end{bmatrix}$$

$$B_{21} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 1 \end{bmatrix},\ B_{22} = \begin{bmatrix} 1 & 3 \\ 0 & 1 \\ 2 & 1 \end{bmatrix},\ C_1 = \begin{bmatrix} 1 & 0 & 1 \\ 0 & -1 & -1 \end{bmatrix},\ C_2 = \begin{bmatrix} 0.1 & 4 & -1 \\ 2 & -1 & 1 \end{bmatrix}$$

$$D_{11} = \begin{bmatrix} 1 & -0.2 \\ 0.1 & 0.1 \end{bmatrix},\ D_{12} = \begin{bmatrix} 2 & 0 \\ 2 & 1 \end{bmatrix},\ D_{21} = \begin{bmatrix} 1 & -1 \\ 1 & 0 \end{bmatrix},\ D_{22} = \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix},\ H_{11} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \\ 0 & -1 \end{bmatrix}$$

$$H_{12} = \begin{bmatrix} 0.1 & 0 \\ 0 & -1 \\ 0 & -0.2 \end{bmatrix},\ H_{21} = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix},\ H_{22} = \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix},\ E_{11} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & -0.3 & -0.2 \end{bmatrix}$$

$$E_{12} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & -1 & -0.2 \end{bmatrix},\ E_{21} = \begin{bmatrix} 0 & 0.3 \\ 0 & 0.1 \end{bmatrix},\ E_{22} = \begin{bmatrix} 0.2 & 0.1 \\ 0 & 1 \end{bmatrix}$$

Solving the matrix inequalities in Theorem 4, we can obtain the state feedback controller as

$$\boldsymbol{u}(t) = \left\{ \frac{x_1^2(t)}{2} \begin{bmatrix} 247.7694 & 325.4279 & -4.3742 \\ 221.5945 & 300.4592 & 5.8559 \end{bmatrix} + (1 - \frac{x_1^2(t)}{2}) \begin{bmatrix} 442.2767 & 593.3450 & 4.6406 \\ 338.9410 & 461.4361 & 9.8363 \end{bmatrix} \right\} \boldsymbol{x}(t)$$

It makes system (12) admissible and strictly passive.

5 Conclusions

In this paper, we have addressed the problem of passivity control for a kind of uncertain T-S fuzzy descriptor system. We have given a method to check the admissibility of the system. The controller which makes the closed-loop system admissible and strictly passive has been obtained in term of LMI Toolbox. The proposed approach can overcome the limitation of the LMI Toolbox. An example has been given to show the validity and feasibility of the approach.

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