Fault-Tolerant Control Research for Networked Control System under Communication Constraints[1)]

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Abstract Implementing a control system over a communication network induces inevitable time delays that may degrade performance and even cause instability. One of the most effective ways to reduce the negative effect of delays on the performance of networked control system (NCS) is to reduce network traffic. In this paper, adjustable deadbands are explored as a solution to reduce network traffic in NCS. A method of fault-tolerant control of networked control system is presented, which takes into account system response as well as network traffic. The integrity design for a kind of NCS with sensor failures and actuator failures is analyzed based on robust fault-tolerant control theory and information scheduling. After detailed theoretical analysis, the paper also provides the simulation results, which further validate the proposed scheme.

Key words Networked control system (NCS), deadbands, fault-tolerant control (FTC), integrity design, time delay

1 Introduction

Networked control system is one type of distributed control systems. Serial communication network is used to exchange system information and control signals between various physical components of the system that may be physically distributed[1,2]. Fig. 1 illustrates a typical setup of an NCS. Compared with conventional point-to-point interconnected control systems, the primary advantages of NCS are modular and flexible system design (*e.g.*, distributed processing and interoperability), simple and fast implementation (*e.g.*, reduced system wiring and powerful configuration tools), ease of system maintenance, and increased system agility. However, implementing a control system over a communication network induces inevitable time delays that may degrade performance and even cause instability. Time delays are a function of device processing time and the sharing of the communication medium. Thus system performance is not only dependent on the performance of its components but also on their network interaction. Consequently, there are growing theoretical interests in the field of networked control system. Modeling, analysis, control, optimization of networked control system are main research topics now[3~5].

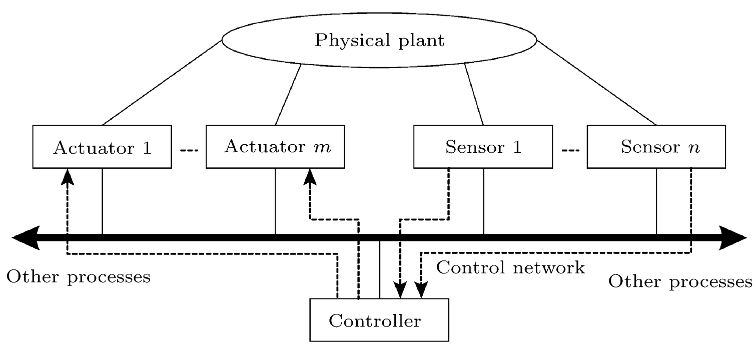

Fig. 1 Typical NCS setup and information flows

With the development of network technology, NCS is more and more popular in most engineering practices. Because of the complexity induced by network, this kind of automated control system is more vulnerable to faults. Defects in sensors and actuators, in the process itself, or within the controller can

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be amplified by the networked control system. The closed-loop may hide a fault from being observed until a failure is inevitable. The consequence could do damage to technical parts of the plant, to personnel or the environment. A cost effective way to obtain increased availability and reliability in networked control system is to introduce fault-tolerant control. Therefore, research on fault-tolerant control of networked control system has great theoretical and applied significance, and it is now a new research field in which only time delay is considered for NCS at present[6]. In this paper, adjustable deadbands are explored as a solution to reduce network traffic in NCS, and the integrity design of a kind of NCS with sensor failures and actuator failures is analyzed based on robust fault-tolerant control theory and information scheduling. The new idea about NCS robust fault-tolerant control that links the quality of control performance (QoP) and quality of network service (QoS) by synthesizing both control and scheduling schemes is put forward.

2 NCS model under communication constraints

The process to be controlled is assumed to be of the form:

$$\dot{x}(t) = Ax(t) + Bu(t), \quad y(t) = Cx(t)$$

and a discrete controller is

$$u(k) = Kx(k), \ k = 0, 1, 2, \cdots$$

where $x \in R^n$, $u \in R^m$, $y \in R^p$, and A, B, C, K are of compatible dimensions, h is the sampling period. Assume that the network-induced delay τ is bounded and the packet is transmitted in sequence. The network time-varying delays can be obtained online[6]. The closed-loop networked control system with time-varying delays can be described as[1,6]

$$x(k+1) = (\hat{A} + \hat{B}_{0,k}K)x(k) + \sum_{i=1}^{n} \hat{B}_{i,k}Kx(k-i) \tag{1}$$

where $\hat{A} = \Phi^{Ah}$, $\hat{B}_{i,k} = \int_{t_i^k}^{t_{i-1}^k} e^{A(h-s)}B\mathrm{d}s$, where t_i^k denotes the ith data package arriving time[6].

The implementation of deadbands results in network traffic reduction while maintaining acceptable system performance[7]. A node with a deadband compares the value prepared to send to the network, x, to either the last value sent, x_{sent}, or a constant threshold, δ_t. If the absolute value of the difference between x and x_{sent} is within the deadband, δx_{sent} or δ_t, then no update is sent to the network.

IF $|x - x_{sent}| \geqslant \delta x_{sent}$ (or δ_t)
THEN
  Broadcast x
  $x_{sent} = x$
ELSE
No message broadcast

When the term δx_{sent} is used the deadband becomes a function of the node state and be viewed as a relative deadband. When δ_t is used, the deadband remains constant independent of the state.

As the size of the deadband coefficient increases, δ or δ_t, the node broadcasts less message. Implementing deadbands to reduce network traffic can produce uncertainty in the state of the system. Since the controller relies on the broadcast state x_b to compute the control signal, it is important to determine whether this uncertainty could drive the system to instability. At any given time, the true state of the system x is

$$x = x_b \pm \delta x_b$$

In case of the controller node, the control signal it sends to the plant is the following

$$u = Kx_b$$

The discrete-time version of the control signal is the following

$$u(k) = K(I + \delta)^{-1}x(k) \tag{2}$$

Substituting equation (2) into equation (1), the closed-loop networked control system with time-varying delays can be described as

$$x(k+1)=(\hat{A}+\hat{B}_{0,k}K(I+\delta)^{-1})x(k)+\sum_{i=1}^{n}(\hat{B}_{i,k}K(I+\delta)^{-1}x(k-i)) \tag{3}$$

Lemma 1[8]**.** For any compatible dimension vector y and z, $z^{\rm T}y+y^{\rm T}z\leqslant y^{\rm T}y+z^{\rm T}z$.
Lemma 2[9]**.** For any $\varepsilon>0$, and $\varepsilon E^{\rm T}E\leqslant I$, $F^{\rm T}F\leqslant I$, then

$$[G+DF(t)E][G+DF(t)E]^{\rm T}\leqslant G(I-\varepsilon E^{\rm T}E)^{-1}G^{\rm T}+\frac{1}{\varepsilon}DD^{\rm T}$$

3 NCS fault-tolerant control under communication constraints

Fault tolerant control (FTC) strives to make the system stable and retain acceptable performance under system faults. Integrity design is an important means for the fault tolerant control. Integrity design is that the system can keep asymptotically stable under some sensor failures or actuator failures.

If system (3) is under sensor failures, then we introduce a switching matrix M, which is between the feedback gain matrix K and the system state $x(kh)$, where $M=\mathrm{diag}\{m_1,m_2,\cdots,m_n\}$.

Let

$$m_j=\begin{cases}1, & \text{no sensor fault}\\ 0, & j\text{th sensor fault}\end{cases},\ j=1,2,\cdots,n,\ M\neq 0$$

"1" denotes that the sensor is in the normal situation, "0" denotes that the sensor is in the full fault situation, and then the closed-loop networked control system can be described as

$$x(k+1)=(\hat{A}+\hat{B}_{0,k}K(I+\delta)^{-1}M)x(k)+\sum_{i=1}^{n}(\hat{B}_{i,k}K(I+\delta)^{-1}Mx(k-i)) \tag{4}$$

For convenience, let $\bar{K}=K(I+\delta)^{-1}$. Then $u(k)=\bar{K}x(k)$.

$$x(k+1)=(\hat{A}+\hat{B}_{0,k}\bar{K}M)x(k)+\sum_{i=1}^{n}(\hat{B}_{i,k}\bar{K}Mx(k-i))$$

The corresponding integrity design is to find the state feedback gain matrix $\bar{K}$ such that the system can keep stable under any sensors failures $M\in\Omega$, where Ω denotes the set of all possible sensor failures switching the matrix.

Theorem 1. Let $\|\hat{B}_k\|_2=\max\limits_{\substack{0\leqslant i\leqslant n\\ i,k\in N}}(\|\hat{B}_{i,k}\|_2)$. If there exists positive define matrix, $P=P^{\rm T}$, such that

$$\lambda_{\substack{\min\\ M\in\Omega}}(S)>0$$

where

$$S=P-2\hat{A}^{\rm T}P\hat{A}-2M\bar{K}^{\rm T}\bar{B}_0^{\rm T}-2\hat{A}^{\rm T}P\hat{B}_0\bar{K}M-2M\bar{K}^{\rm T}\hat{B}_0^{\rm T}P\hat{B}_0\bar{K}M-(n+1)\sum_{i=1}^{n}(M\bar{K}^{\rm T}\hat{B}_k^{\rm T}P\hat{B}_k\bar{K}M)$$

then system (4) has the ability of fault tolerant against sensor failures $M\in\Omega$.

Note that $\lambda_m(\cdot)$ denotes the minimum eigenvalue of matrix S.

Proof. Consider the Lyapunov function

$$V(k)=x^{\rm T}(k)Px(k)+(n+1)\sum_{i=1}^{n}\sum_{j=k-i}^{k-1}[x^{\rm T}(k)M\bar{K}^{\rm T}\hat{B}_{j,k}^{\rm T}P\hat{B}_{j,k}\bar{K}Mx(k)]$$

$$\begin{aligned}\Delta V=&V(k+1)-V(k)=\\ &x^{\rm T}(k+1)Px(k+1)-x^{\rm T}(k)Px(k)+\\ &(n+1)\sum_{i=1}^{n}[x^{\rm T}(k)M\bar{K}^{\rm T}\hat{B}_{i,k}^{\rm T}P\hat{B}_{i,k}\bar{K}Mx(k)-x^{\rm T}(k-i)M\bar{K}^{\rm T}\hat{B}_{i,k}^{\rm T}P\hat{B}_{i,k}\bar{K}Mx(k-i)]=\end{aligned}$$

$$x^{\rm T}(k)[(\hat{A}+\hat{B}_0\bar{K}M)^{\rm T}P(\hat{A}+\hat{B}_0\bar{K}M)-P+(n+1)\sum_{i=1}^{n}(M\bar{K}^{\rm T}\hat{B}_{i,k}^{\rm T}P\hat{B}_{i,k}\bar{K}M)]x(k)+$$
$$\sum_{i=1}^{n}[x^{\rm T}(k-i)M\bar{K}^{\rm T}\hat{B}_{i,k}^{\rm T}]P(\hat{A}+\hat{B}_0\bar{K}M)x(k)+x^{\rm T}(k)(\hat{A}+\hat{B}_0\bar{K}M)^{\rm T}P\sum_{i=1}^{n}[\hat{B}_{i,k}\bar{K}Mx(k-i)]+$$
$$\sum_{i=1}^{n}[x^{\rm T}(k-i)M\bar{K}^{\rm T}\hat{B}_{i,k}^{\rm T}]P\sum_{i=1}^{n}[\hat{B}_{i,k}\bar{K}Mx(k-i)-(n+1)\sum_{i=1}^{n}[x^{\rm T}(k-i)M\bar{K}^{\rm T}\hat{B}_{i,k}^{\rm T}P\hat{B}_{i,k}\bar{K}Mx(k-i)]]$$

By lemma 1, we have

$$\sum_{i=1}^{n}[x^{\rm T}(k-i)M\bar{K}^{\rm T}\hat{B}_{i,k}^{\rm T}]P(\hat{A}+\hat{B}_0\bar{K}M)x(k)+x^{\rm T}(k)(\hat{A}+\hat{B}_0\bar{K}M)^{\rm T}P\sum_{i=1}^{n}[\hat{B}_{i,k}\bar{K}Mx(k-i)]\leqslant$$
$$x^{\rm T}(k)(\hat{A}+\hat{B}_0\bar{K}M)^{\rm T}P(\hat{A}+\hat{B}_0\bar{K}M)x(k)+\sum_{i=1}^{n}[x^{\rm T}(k-i)M\bar{K}^{\rm T}\hat{B}_{i,k}^{\rm T}P\hat{B}_{i,k}\bar{K}Mx(k-i)]$$

and

$$\sum_{i=1}^{n}[x^{\rm T}(k-i)M\bar{K}^{\rm T}\hat{B}_{i,k}^{\rm T}]P(\hat{A}+\hat{B}_0\bar{K}M)x(k)=\sum_{i=1}^{n}\sum_{j=1}^{n}[x^{\rm T}(k-j)M\bar{K}^{\rm T}\hat{B}_{j,k}^{\rm T}P\hat{B}_{i,k}\bar{K}Mx(k-i)]\leqslant$$
$$n\sum_{i=1}^{n}[x^{\rm T}(k-i)M\bar{K}^{\rm T}\hat{B}_{i,k}^{\rm T}P\hat{B}_{i,k}\bar{K}Mx(k-i)]$$

Thus,

$$\Delta V\leqslant x^{\rm T}(k)[(\hat{A}+\hat{B}_0\bar{K}M)^{\rm T}P(\hat{A}+\hat{B}_0\bar{K}M)-P+(n+1)\sum_{i=1}^{n}(M\bar{K}^{\rm T}\hat{B}_{i,k}^{\rm T}P\hat{B}_{i,k}\bar{K}M)]x(k)+$$
$$x^{\rm T}(k)(\hat{A}+\hat{B}_0\bar{K}M)^{\rm T}P(\hat{A}+\hat{B}_0\bar{K}M)x(k)+\sum_{i=1}^{n}[x^{\rm T}(k-i)M\bar{K}^{\rm T}\hat{B}_{i,k}^{\rm T}P\hat{B}_{i,k}\bar{K}Mx(k-i)]+$$
$$n\sum_{i=1}^{n}[x^{\rm T}(k-i)M\bar{K}^{\rm T}\hat{B}_{i,k}^{\rm T}P\hat{B}_{i,k}\bar{K}Mx(k-i)]+(n+1)\sum_{i=1}^{n}[x^{\rm T}(k-i)M\bar{K}^{\rm T}\hat{B}_{i,k}^{\rm T}P\hat{B}_{i,k}\bar{K}Mx(k-i)]=$$
$$x^{\rm T}(k)[2(\hat{A}+\hat{B}_0\bar{K}M)^{\rm T}P(\hat{A}+\hat{B}_0\bar{K}M)-P+(n+1)\sum_{i=1}^{n}(M\bar{K}^{\rm T}\hat{B}_{i,k}^{\rm T}P\hat{B}_{i,k}\bar{K}M)]x(k)\leqslant$$
$$-x^{\rm T}(k)[P-2\hat{A}^{\rm T}P\hat{A}-2M\bar{K}^{\rm T}\hat{B}_0^{\rm T}P\hat{A}-2\hat{A}^{\rm T}P\hat{B}_0\bar{K}M-2M\bar{K}^{\rm T}\hat{B}_0^{\rm T}P\hat{B}_0\bar{K}M]-$$
$$(n+1)\sum_{i=1}^{n}[M\bar{K}^{\rm T}\hat{B}_k^{\rm T}P\hat{B}_k\bar{K}Mx(k)]x(k)=-x^{\rm T}(k)Sx(k)\leqslant-\lambda_{\min}(S)x^{\rm T}(k)x(k)$$

If $\lambda_{\min\limits_{M\in\Omega}}(S)>0$, then $\Delta V<0$. From Lyapunov stability theory, system (4) is asymptotically stable. The system has the ability of fault tolerant against sensor failures. □

If system (3) is under actuator failures, then we introduce switching matrix L, which is between the system matrix B and feedback gain matrix K, where $L={\rm diag}\{l_1,l_2,\cdots,l_n\}$.

Let

$$l_j=\begin{cases}1, & \text{no actuator fault}\\ 0, & j\text{th actuator fault}\end{cases},\ j=1,2,\cdots,n,\ L\neq 0$$

Suppose that the system is controllable, and consider the state feedback control law $u(k)=Kx(k)$. Then the close-loop control system can be written as

$$x(k+1)=(\hat{A}+\hat{B}_{0,k}LK(I+\delta)^{-1})x(k)+\sum_{i=1}^{n}(\hat{B}_{i,k}LK(I+\delta)^{-1}x(k-i))\tag{5}$$

The corresponding integrity design is to find the state feedback gain matrix K such that the system can keep stable under any actuator failures $L\in\Omega'$, where Ω' denotes the set of all possible actuator failures switching matrix.

Theorem 2. Let $\|\hat{B}_k\|_2 = \max\limits_{\substack{0\leqslant i\leqslant n\\ i,k\in N}}(\|\hat{B}_{i,k}\|_2)$. If there exists positive define matrix $P = P^{\mathrm{T}}$, such that

$$\lambda_{\min\limits_{L\in\Omega'}}(S') > 0$$

where

$$S' = P - 2\hat{A}^{\mathrm{T}}P\hat{A} - 2\bar{K}^{\mathrm{T}}L\hat{B}_0^{\mathrm{T}}P\hat{A} - 2\hat{A}^{\mathrm{T}}P\hat{B}_0L\bar{K} - 2\bar{K}^{\mathrm{T}}L\hat{B}_0^{\mathrm{T}}P\hat{B}_0L\bar{K} - (n+1)\sum_{i=1}^{n}(\bar{K}^{\mathrm{T}}L\hat{B}_k^{\mathrm{T}}P\hat{B}_kL\bar{K})$$

then system (5) has the ability of fault tolerant against actuator failures $L \in \Omega'$.

The proof of the Theorem 2 is similar to Theorem 1.

The design step of a robust fault-tolerant controller with sensor failures or actuator failure is given as follows. Choose state feedback control matrix as

$$\bar{K} = -r\hat{B}_0^{\mathrm{T}}P \tag{6}$$

where r is constant.

Step 1. Choose an appropriate P. From (6) we can get state feedback matrix $\bar{K}$, and substitute it into Theorem 1 or Theorem 2.

Step 2. If P satisfies these inequality, then stop and have a feasible controller.

Step 3. Otherwise, go back to Step 1.

It is difficult to describe a real system with an accurate mathematical model. So ΔA, ΔB are employed to describe the uncertainty with appropriate dimension, indicating the parameter uncertainties of the system.

Consider the uncertain networked control system model

$$\dot{x}(t) = (A + \Delta A)x(t) + (B + \Delta B)u(t)$$

Consider the state feedback control law $u(k) = Kx(k)$. Then the close-loop control system can be written as

$$x(k+1) = (\hat{A} + \Delta\hat{A})x(k) + \sum_{i=0}^{q}(\hat{B}_{i,k} + \Delta\hat{B}_{i,k})Kx(k-i)$$

where

$$\Delta\hat{A} = \mathrm{e}^{(A+\Delta A)h} - \mathrm{e}^{Ah} = \mathrm{e}^{Ah}(\mathrm{e}^{\Delta A\cdot h} - I)$$

$$\Delta\hat{B}_{i,k} = \int_{t_i^k}^{t_{i-1}^k}[\mathrm{e}^{(A+\Delta A)(h-s)}(B+\Delta B) - \mathrm{e}^{A(h-s)}B]\mathrm{d}s = \int_{t_i^k}^{t_{i-1}^k}[\mathrm{e}^{A(h-s)}((\mathrm{e}^{\Delta A(h-s)} - I)B + \mathrm{e}^{\Delta A(h-s)}\Delta B)]\mathrm{d}s$$

$\Delta\hat{A}$, $\Delta\hat{B}_{i,k}$ are admissible parameter uncertainties of the system that can be described as $\Delta\hat{A} = DF(k)E$, $\Delta\hat{B}_{i,k} = D_iF_i(k)E_i$ $(i = 0, 1, \cdots, n)$, where D_i and E_i are known constant matrices. $F_i(k)$ are time-varying uncertainty matrices satisfying $F(k)^{\mathrm{T}}F(k) \leqslant I$ and $F_i(k)^{\mathrm{T}}F_i(k) \leqslant I$ $(i = 0, 1, \cdots, n)$.

The close-loop networked control system model with sensor failures or actuator failures can be written respectively as

$$x(k+1) = (\hat{A} + \Delta\hat{A} + (\hat{B}_{0,k} + \Delta\hat{B}_{0,k})K(I+\delta)^{-1}M)x(k) + \sum_{i=1}^{n}((\hat{B}_{i,k} + \Delta\hat{B}_{i,k})K(I+\delta)^{-1}Mx(k-i)) \tag{7}$$

$$x(k+1) = (\hat{A} + \Delta\hat{A} + (\hat{B}_{0,k} + \Delta\hat{B}_{0,k})LK(I+\delta)^{-1})x(k) + \sum_{i=1}^{n}((\hat{B}_{i,k} + \Delta\hat{B}_{i,k})LK(I+\delta)^{-1}x(k-i)) \tag{8}$$

Theorem 3. Let $\|\hat{B}_k\|_2 = \max\limits_{\substack{0\leqslant i\leqslant n\\ i,k\in N}}(\|\hat{B}_{i,k}\|_2)$, ε, ε_i $(i = 0, 1, \cdots, n) > 0$ and $P^{-1} - \varepsilon DD^{\mathrm{T}} > 0$, $P^{-1} - \varepsilon_iD_iD_i^{\mathrm{T}} > 0$. If there exists positive define matrix $P = P^{\mathrm{T}}$, such that

$$\lambda_{\min\limits_{M\in\Omega}}(T) > 0$$

where

$$T = P - 4\hat{A}^{\mathrm{T}}(P^{-1} - \varepsilon DD^{\mathrm{T}})^{-1}\hat{A} - 4\varepsilon^{-1}E^{\mathrm{T}}E - 4M\bar{K}^{\mathrm{T}}\hat{B}_0^{\mathrm{T}}(P^{-1} - \varepsilon_0 D_0 D_0^{\mathrm{T}})^{-1}\hat{B}_0\bar{K}M - 4\varepsilon_0^{-1}M\bar{K}^{\mathrm{T}}E_0^{\mathrm{T}}E_0\bar{K}M - (n+1)\sum_{i=1}^{n} G_i$$

$$G_i = M\bar{K}^{\mathrm{T}}\hat{B}_k^{\mathrm{T}}(P^{-1} - \varepsilon_i D_i D_i^{\mathrm{T}})^{-1}\hat{B}_k\bar{K}M + \varepsilon_i^{-1}M\bar{K}^{\mathrm{T}}E_i^{\mathrm{T}}E_i\bar{K}M$$

then system (7) has the ability of fault tolerant against sensor failures.

Proof. Consider Lyapunov function

$$V(k) = x^{\mathrm{T}}(k)Px(k) + (n+1)\sum_{i=1}^{n}\sum_{j=k-i}^{k-1}[x^{\mathrm{T}}(k)G_j x(k)]$$

Let $\bar{A} = \hat{A} + \Delta\hat{A} + \hat{B}_{0,k}\bar{K}M + \Delta\hat{B}_{0,k}\bar{K}M$, $\bar{B}_{i,k} = \hat{B}_{i,k} + \Delta\hat{B}_{i,k}$. From Theorem 1, we have

$$\Delta V = V(k+1) - V(k) = x^{\mathrm{T}}(k)[2\bar{A}^{\mathrm{T}}P\bar{A} - P + (n+1)\sum_{i=1}^{n} G_i]x(k) + (n+1)\sum_{i=1}^{n}[x^{\mathrm{T}}(k-i)M\bar{K}^{\mathrm{T}}\bar{B}_{i,k}^{\mathrm{T}}P\bar{B}_{i,k}\bar{K}Mx(k-i)] - (n+1)\sum_{i=1}^{n}[x^{\mathrm{T}}(k-i)G_i x(k-i)]$$

By lemma 2 and $P^{-1} - \varepsilon_i D_i D_i^{\mathrm{T}} > 0$, we have

$$[\hat{B}_{i,k} + D_i F_i(k)E_i]^{\mathrm{T}}P[\hat{B}_{i,k} + D_i F_i(k)E_i] \leqslant \hat{B}_{i,k}^{\mathrm{T}}(P^{-1} - \varepsilon_i D_i D_i^{\mathrm{T}})^{-1}\hat{B}_{i,k} + \varepsilon_i^{-1}E_i^{\mathrm{T}}E_i$$

Then $M\bar{K}^{\mathrm{T}}\bar{B}_{i,k}^{\mathrm{T}}P\bar{B}_{i,K}\bar{K}M \leqslant G_i$.

From the above and lemma1 and lemma2, when $P^{-1} - \varepsilon DD^{\mathrm{T}} > 0$, $P^{-1} - \varepsilon_0 D_0 D_0^{\mathrm{T}} > 0$, we have

$$\begin{aligned}\bar{A}^{\mathrm{T}}P\bar{A} =& (\hat{A} + \Delta\hat{A} + \hat{B}_{0,k}\bar{K}M + \Delta\hat{B}_{0,k}\bar{K}M)^{\mathrm{T}}P(\hat{A} + \Delta\hat{A} + \hat{B}_{0,k}\bar{K}M + \Delta\hat{B}_{0,k}\bar{K}M) = \\ & (\hat{A} + \Delta\hat{A})^{\mathrm{T}}P(\hat{A} + \Delta\hat{A}) + (\hat{B}_{0,k}\bar{K}M + \Delta\hat{B}_{0,k}\bar{K}M)^{\mathrm{T}}P(\hat{B}_{0,k}\bar{K}M + \Delta\hat{B}_{0,k}\bar{K}M) + \\ & (\hat{A} + \Delta\hat{A})^{\mathrm{T}}P(\hat{B}_{0,k}\bar{K}M + \Delta\hat{B}_{0,k}\bar{K}M) + (\hat{B}_{0,k}\bar{K}M + \Delta\hat{B}_{0,k}\bar{K}M)^{\mathrm{T}}P(\hat{A} + \Delta\hat{A}) \leqslant \\ & 2(\hat{A} + \Delta\hat{A})^{\mathrm{T}}P(\hat{A} + \Delta\hat{A}) + 2(\hat{B}_{0,k}\bar{K}M + \Delta\hat{B}_{0,k}\bar{K}M)^{\mathrm{T}}P(\hat{B}_{0,k}\bar{K}M + \Delta\hat{B}_{0,k}\bar{K}M) \leqslant \\ & 2\hat{A}^{\mathrm{T}}(P^{-1} - \varepsilon DD^{\mathrm{T}})^{-1}\hat{A} + 2\varepsilon^{-1}E^{\mathrm{T}}E + 2M\bar{K}^{\mathrm{T}}\hat{B}_0^{\mathrm{T}}(P^{-1} - \varepsilon_0 D_0 D_0^{\mathrm{T}})^{-1}\hat{B}_0\bar{K}M + \\ & 2\varepsilon_0^{-1}M\bar{K}E_0^{\mathrm{T}}E_0\bar{K}M\end{aligned}$$

Thus

$$\begin{aligned}\Delta V \leqslant & x^{\mathrm{T}}(k)[2\bar{A}^{\mathrm{T}}P\bar{A} - P + (n+1)\sum_{i=1}^{n} G_i]x(k) \leqslant \\ & x^{\mathrm{T}}(k)[4\hat{A}^{\mathrm{T}}(P^{-1} - \varepsilon DD^{\mathrm{T}})^{-1} + 4\varepsilon^{-1}E^{\mathrm{T}}E + 4M\bar{K}^{\mathrm{T}}\hat{B}_0^{\mathrm{T}}(P^{-1} - \varepsilon_0 D_0 D_0^{\mathrm{T}})^{-1}\hat{B}_0\bar{K}M + \\ & 4\varepsilon_0^{-1}M\bar{K}^{\mathrm{T}}E_0^{\mathrm{T}}E_0\bar{K}M - P + (n+1)\sum_{i=1}^{n} G_i]x(k) = \\ & -x^{\mathrm{T}}(k)Px(k) \leqslant -\lambda_{\min}(T)x^{\mathrm{T}}(k)x(k)\end{aligned}$$

If $\lambda_{\min\limits_{M\in\Omega}}(T) > 0$, then $\Delta V < 0$. From the Lyapunov stability theory, system (7) is robustly asymptotically stable. The system has the ability of fault tolerant against sensor failures. □

Theorem 4. Let $\|\hat{B}_k\|_2 = \max\limits_{\substack{0\leqslant i\leqslant n\\ i,k\in N}}(\|\hat{B}_{i,k}\|_2)$, ε, ε_i $(i = 0, 1, \cdots, n) > 0$ and $P^{-1} - \varepsilon DD^{\mathrm{T}} > 0$, $P^{-1} - \varepsilon_i D_i D_i^{\mathrm{T}} > 0$. If there exists positive define matrix $P = P^{\mathrm{T}}$, such that

$$\lambda_{\min\limits_{L\in\Omega'}}(T') > 0$$

where

$$T' = P - 4\hat{A}^{\mathrm{T}}(P^{-1} - \varepsilon DD^{\mathrm{T}})^{-1}\hat{A} - 4\varepsilon^{-1}E^{\mathrm{T}}E - 4\bar{K}^{\mathrm{T}}L\hat{B}_0^{\mathrm{T}}(P^{-1} - \varepsilon_0 D_0 D_0^{\mathrm{T}})^{-1}\hat{B}_0 L\bar{K} - 4\varepsilon_0^{-1}\bar{K}^{\mathrm{T}}LE_0^{\mathrm{T}}E_0L\bar{K} - (n+1)\sum_{i=1}^{n} G_i$$

$$G_i = \bar{K}^{\mathrm{T}}L\hat{B}_k^{\mathrm{T}}(P^{-1} - \varepsilon_i D_i D_i^{\mathrm{T}})^{-1}\hat{B}_k LK + \varepsilon_i^{-1}K^{\mathrm{T}}LE_i^{\mathrm{T}}E_iLK$$

then system (8) has the ability of fault tolerant against actuator failures.

The proof of the Theorem 4 is similar to Theorem 3.

4 Simulation analysis

Consider the simulation example in [6]: $\hat{A} = \begin{bmatrix} 0.2759 & 0.1839 \\ 0.1379 & 0.0921 \end{bmatrix}$, $\hat{B}_0 = \begin{bmatrix} 0.0778 \\ 0.1425 \end{bmatrix}$, $\hat{B}_1 = \begin{bmatrix} 0.1185 \\ 0.0665 \end{bmatrix}$, $\hat{B}_2 = \begin{bmatrix} 0.0157 \\ 0.0079 \end{bmatrix}$ and assume $\Delta\hat{A} = \begin{bmatrix} 0 & 0.01\sin(k) \\ 0 & 0 \end{bmatrix}$, $\Delta\hat{B}_0 = \begin{bmatrix} 0 \\ 0.02\cos(k) \end{bmatrix}$, $\Delta\hat{B}_1 = \begin{bmatrix} 0.02\cos(k) \\ 0 \end{bmatrix}$, $\Delta\hat{B}_2 = \begin{bmatrix} 0 \\ 0.02\cos(k) \end{bmatrix}$, $F(k) = F_i(k) = \begin{bmatrix} \sin(k) & 0 \\ 0 & \cos(k) \end{bmatrix}$, $(i = 0, 1, \cdots, n)$.

We can get $D = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}$, $E = \begin{bmatrix} 0 & 0.1 \\ 0 & 0 \end{bmatrix}$, $D_0 = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.1 \end{bmatrix}$, $E_0 = \begin{bmatrix} 0 \\ 0.2 \end{bmatrix}$, $D_1 = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.1 \end{bmatrix}$, $E_1 = \begin{bmatrix} 0.1 \\ 0 \end{bmatrix}$, $D_2 = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.2 \end{bmatrix}$, $E_2 = \begin{bmatrix} 0 \\ 0.1 \end{bmatrix}$, $D_1 = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.1 \end{bmatrix}$.

Consider $\bar{K} = -r\hat{B}_0^{\mathrm{T}}P$, and let $r = 5$, $\varepsilon = \varepsilon_i$ $(i = 0, 1, \cdots, n) = 1$. By Theorem 4, letting $P = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, we have the feedback matrix $\bar{K} = [-0.3890 \quad -0.7125]$, and $\lambda_{\min\limits_{L\in\Omega'}}(T') > 0.2623 > 0$.

The CAN connects the plant to the controller. A deadband is defined on each node, so that it remembers the last value it sent to the network. The magnitude of the deadband coefficient for the two sensor nodes and the controller node are $\delta_1 = 0.05$, $\delta_2 = 0.01$, $\delta_u = 0.02$, respectively. If a node does not broadcast a new value, then all other nodes assume that the previous value is still valid and act accordingly.

The simulation result is shown in Fig. 2. The solid lines denote the state in a normal situation, and the dash lines denote the state in actuator failures. In view of simulation results, the system can keep steady when actuator failures happen. Simulation results further validate the proposed scheme.

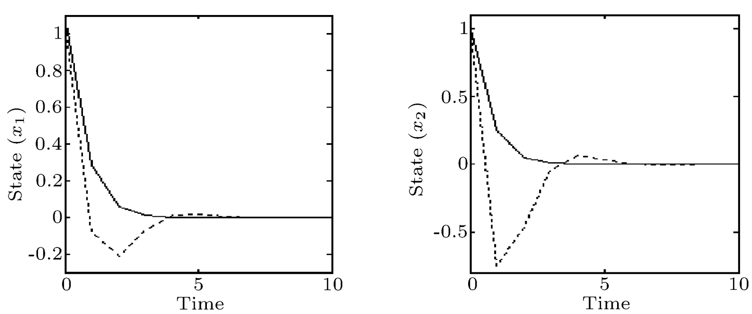

Fig. 2 Zero input response of state in actuator failures

5 Conclusion and prospect

Networked control system could degrade the system dynamic performance and is a source of potential instability in complex dynamical processes like advanced aircraft, spacecraft, and autonomous manufacturing plants. Fault-tolerant controller design for a kind of NCS is investigated in this paper, which takes into account system response as well as network traffic. The implementing of deadbands not only results in network traffic reduction but also maintains acceptable system performance. The integrity design of an NCS with sensor failures and actuator failures is analyzed based on robust fault-tolerant control theory and information scheduling, and the simulation results demonstrated satisfying performance.

The research presented in this paper provides a foundation for future research in NCS fault-tolerant control. In future work, more complex networked control system, in which network effects on system performance should be further considered, is to be investigated in terms of modeling algorithm and fault-tolerant controller design. In addition, an intelligent fault-tolerant controller design that incorporates both network and control parameters to further improve both network and control performance should be further studied. This intelligent networked controller can identify/estimate network performance and operate among multiple nodes of control action, by means of network traffic or control performance.

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