

双滤波器平滑估计的递推更新

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摘 要

本文考虑了具有多个测量通道时线性马尔可夫状态固定间隔平滑问题的线性最小二乘解,给出了一种双滤波器递推算法,它随各通道测量数据的依次投入而更新平滑估计。

关键词: 固定间隔平滑,卡尔曼滤波器,双滤波器法,前向估计,后向估计。

1 引言

固定间隔平滑是一类重要的估计问题。对于通常的线性模型,双滤波器(two-filter)法已成为固定间隔平滑的主要方法。在这种方法中,平滑估计被表示为基于过去数据的“前向估计”和基于未来数据的“后向估计”的加权和^[1,2];前向和后向估计可借助于卡尔曼滤波器计算。在分布估计,分层控制以及基于多轨扫描的随机场估计和制图等的应用背景下,Willsky 和 Bello 等首先提出和研究了具有多个线性测量通道时线性-高斯状态平滑估计的组合和更新问题^[3]。组合问题是指如何将基于各个通道测量数据的平滑估计综合成基于所有测量数据的估计;更新问题系指平滑估计随各通道测量数据的相继投入而更新的计算。他们给出了连续参量线性高斯状态双滤波器平滑估计组合和更新的解^[3]。本文对于离散参量线性马尔可夫状态模型推导了双滤波器平滑估计测量更新的递推算法。此算法的关键在于平滑误差也服从线性马尔可夫模型。文中误差模型的推导借重于 Verghese 和 Kailath 推导后向模型所采用的正交分解方法^[4]。

2 双滤波器平滑

考虑线性马尔可夫状态空间模型

$$x_{k+1} = A_k x_k + B_k u_k, \quad (2.1a)$$

$$y_k = H_k x_k + v_k, \quad (2.1b)$$

式中 $\{u_k\}$ 和 $\{v_k\}$ 是互不相关的零均值白噪声序列且

$$E[u_k u_k'] = I, \quad E[v_k v_k'] = R_k; \quad (2.1c)$$

设初始状态 x_0 是与 u_k 和 v_k 不相关的随机向量,

$$E[x_0] = \bar{x}_0, \quad cov\{x_0\} = \pi_0. \quad (2.1d)$$

假设给定测量数据 $Y \triangleq \{y_k: 0 \leq k \leq N-1\}$, 要求估计 x_k , $0 \leq k \leq N-1$. 这就是固定间隔平滑问题.

考虑 LLS (线性最小二乘) 估计. 令 $\hat{x}_{s,k}$ 为 x_k 基于数据 Y 的 LLS 平滑估计,

$$\hat{x}_{s,k} \triangleq E^*[x_k | Y], \quad 0 \leq k \leq N-1. \quad (2.2)$$

这里用 E^* 表示 LLS 估计. 令 $\hat{x}_{fp,k}$, \hat{x}_{fk} , $\hat{x}_{bp,k}$ 和 \hat{x}_{bk} 分别为 x_k 的前、后向预测和滤波估计:

$$\hat{x}_{fp,k} \triangleq E^*[x_k | Y_{0,k-1}], \quad \hat{x}_{fk} \triangleq E^*[x_k | Y_{0,k}], \quad (2.3)$$

$$\hat{x}_{bp,k} \triangleq E^*[x_k | Y_{k+1,N-1}], \quad \hat{x}_{bk} \triangleq E^*[x_k | Y_{k,N-1}]. \quad (2.4)$$

式中 $Y_{i,j} \triangleq \{y_i, y_{i+1}, \dots, y_j\} (j > i)$. 令 P_s, P_{fp}, P_f, P_{bp} 和 P_b 分别是对应于 $\hat{x}_s, \hat{x}_{fp}, x_f, \hat{x}_{bp}$ 和 \hat{x}_b 的误差协方差. 所谓双滤波器平滑就是, $\hat{x}_{s,k}$ 可以表示为 \hat{x}_{fk} 和 $\hat{x}_{bp,k}$ 或 $\hat{x}_{fp,k}$ 和 \hat{x}_{bk} 的加权和. 例如, 一种双滤波器公式如下:

$$\hat{x}_{s,k} = P_{s,k} (P_{fk}^{-1} \hat{x}_{fk} + P_{bp,k}^{-1} \hat{x}_{bp,k} - \pi_k^{-1} \bar{x}_k), \quad (2.5a)$$

$$P_{s,k}^{-1} = P_{fk}^{-1} + P_{bp,k}^{-1} - \pi_k^{-1}. \quad (2.5b)$$

以上 $\bar{x}_k \triangleq E[x_k]$, $\pi_k \triangleq cov\{x_k\}$, 且由(2.1)式

$$\hat{x}_{k+1} = A_k \bar{x}_k, \quad \bar{x}_0 = E[x_0], \quad (2.6)$$

$$\pi_{k+1} = A_k \pi_k A_k' + B_k B_k', \quad \pi_0 = cov\{x_0\}. \quad (2.7)$$

双滤波器公式由一些学者先后用不同的方法导出, 有若干种不同的形式^[1,2].

前向估计 \hat{x}_{fp} 和 \hat{x}_f 可就模型(2.1)用卡尔曼滤波器计算. 后向估计 \hat{x}_{bp} 和 \hat{x}_b 可以利用(2.1)的一种导出模型即所谓后向模型 (backward model) 以及卡尔曼滤波器计算. 为了下面讨论的方便, 将具体算法列出如下.

前向滤波:

$$\hat{x}_{fk} = \hat{x}_{fp,k} + P_{fp,k} H_k' Q_k^{-1} (y_k - H_k \hat{x}_{fp,k}), \quad (2.8a)$$

$$\hat{x}_{fp,k+1} = A_k \hat{x}_{fk}, \quad (2.8b)$$

$$P_{fk} = P_{fp,k} - P_{fp,k} H_k' Q_k^{-1} H_k P_{fp,k}, \quad (2.8c)$$

$$P_{fp,k+1} = A_k P_{fk} A_k' + B_k B_k', \quad (2.8d)$$

$$Q_k = H_k P_{fp,k} H_k' + R_k, \quad (2.8e)$$

$$\hat{x}_{fp,0} = \hat{x}_0, \quad P_{fp,0} = \pi_0, \quad 0 \leq k \leq N-1. \quad (2.8f)$$

后向滤波: 后向模型为

$$x_k = C_{k+1} x_{k+1} + D_{k+1} w_{k+1}, \quad (2.9a)$$

$$C_{k+1} = A_k^{-1} (\| - B_k B_k' \pi_{k+1}^{-1}), \quad D_{k+1} = -A_k^{-1} B_k, \quad (2.9b)$$

这里设 A_k^{-1} 存在¹⁾; $\{w_k\}$ 为零均值白噪序列,

$$S_{k+1} \triangleq E[w_{k+1} w_{k+1}'] = \| - B_k' \pi_{k+1}^{-1} B_k \quad (2.9c)$$

且与 x_N 不相关. 后向模型有一些不同的推导方法, 其中最简单的也许是由 Verghese 和 Kailath 给出的方法^[4]. 后向滤波为

$$\hat{x}_{bk} = \hat{x}_{bp,k} + P_{bp,k} H_k' \phi_k^{-1} (y_k - H_k \hat{x}_{bp,k}), \quad (2.10a)$$

$$\hat{x}_{bp,k-1} = C_k \hat{x}_{bk}, \quad (2.10b)$$

1) A_k^{-1} 不存在的情形参阅文献[4].

$$P_{bk} = P_{bp,k} - P_{bp,k} H_k' \phi_k^{-1} H_k P_{bp,k}, \quad (2.10c)$$

$$P_{bp,k-1} = C_k P_{bk} C_k' + D_k S_k D_k', \quad (2.10d)$$

$$\phi_k = H_k P_{bp,k} H_k' + R_k, \quad (2.10e)$$

$$\hat{x}_{bp,N-1} = \bar{x}_{N-1}, \quad P_{bp,N-1} = \pi_{N-1}, \quad 0 \leq k \leq N-1. \quad (2.10f)$$

式(2.8)和(2.10)都是标准的卡尔曼滤波器方程^[5].

3 平滑估计的更新

设有线性马尔可夫模型

$$x_{k+1} = A_k x_k + B_k u_k, \quad (3.1a)$$

$$y_k^{(i)} = H_k^{(i)} x_k + v_k^{(i)}, \quad i = 1, \dots, r. \quad (3.1b)$$

注意状态 x_k 有 r 路测量; 关于 $\{u_k\}$, $\{v_k^{(i)}\}$ 和 x_0 , 除有和模型(2.1)同样的假设外, 还设诸 $\{v_k^{(i)}\}$ 序列不相关,

$$E[v_k^{(i)}(v_l^{(j)})'] = R_k^{(i)} \delta_{kl} \delta_{ij} \quad (3.1c)$$

($\delta_{kl} = 1$ 若 $k = l$, $\delta_{kl} = 0$ 若 $k \neq l$). 要计算状态 x_k 基于测量数据

$$Y^{(i)} \triangleq \{y_0^{(i)}, \dots, y_{N-1}^{(i)}\}, \quad i = 1, \dots, r \quad (3.2)$$

的平滑估计

$$\hat{x}_{sk}^{(i)} \triangleq E^*[x_k | Y^{(1)}, \dots, Y^{(i)}], \quad i = 1, \dots, r. \quad (3.3)$$

这类平滑问题基本上有两种计算方式^[3]. 一种是将分别基于 $Y^{(1)}, \dots, Y^{(i)}$ 的平滑估计综合成 $\hat{x}_{sk}^{(i)}$; 这是平滑估计的组合问题. 另一种方式是通过 $\hat{x}_{sk}^{(i)}$ (利用数据 $Y^{(j+1)}$) 到 $\hat{x}_{sk}^{(j+1)}$ ($1 \leq j < i$) 的依次更新来获得 $\hat{x}_{sk}^{(i)}$; 这是平滑估计的更新问题. 本文要讨论的就是更新问题.

不失一般性, 可以考虑由 $\hat{x}_{sk}^{(1)}$ 到 $\hat{x}_{sk}^{(2)}$ 的更新; $\hat{x}_{sk}^{(1)}$ 可用上一节中所述双滤波器法计算.

以 $\tilde{x}_{sk}^{(1)}$ 表示 $\hat{x}_{sk}^{(1)}$ 的平滑误差,

$$\tilde{x}_{sk}^{(1)} \triangleq x_k - \hat{x}_{sk}^{(1)} \quad (3.4)$$

则可写

$$\begin{aligned} \hat{x}_{sk}^{(2)} &\triangleq E^*[x_k | Y^{(1)}, Y^{(2)}] = E^*[\hat{x}_{sk}^{(1)} + \tilde{x}_{sk}^{(1)} | Y^{(1)}, Y^{(2)}] \\ &= \hat{x}_{sk}^{(1)} + E^*[\tilde{x}_{sk}^{(1)} | Y^{(1)}, Y^{(2)}]. \end{aligned} \quad (3.5)$$

上式右方第二项可以解释为 $\tilde{x}_{sk}^{(1)}$ 对于向量集合 $Y^{(1)}$ 和 $Y^{(2)}$ 所张子空间 $\{Y^{(1)}\} + \{Y^{(2)}\}$ 的投影. 令 $\{\tilde{Y}^{(2)}\}$ 为 $\{Y^{(2)}\}$ 对于 $\{Y^{(1)}\}$ 的正交部分: $\{Y^{(1)}\} + \{Y^{(2)}\} = \{Y^{(1)}\} \oplus \{\tilde{Y}^{(2)}\}$ (\oplus 表示直和). 依照投影定理, $\tilde{x}_{sk}^{(1)}$ 正交于 $\{Y^{(1)}\}$, 所以

$$\hat{x}_{sk}^{(1)} \triangleq E^*[\tilde{x}_{sk}^{(1)} | Y^{(1)}, Y^{(2)}] = E^*[\tilde{x}_{sk}^{(1)} | Y^{(1)}, \tilde{Y}^{(2)}] = E^*[\tilde{x}_{sk}^{(1)} | \tilde{Y}^{(2)}] \quad (3.6)$$

于是式(3.5)可改写为

$$\hat{x}_{sk}^{(2)} = \hat{x}_{sk}^{(1)} + \hat{\tilde{x}}_{sk}^{(1)} = \hat{x}_{sk}^{(1)} + E^*[\tilde{x}_{sk}^{(1)} | \tilde{Y}^{(2)}]. \quad (3.7)$$

显然 $\tilde{Y}^{(2)} = \{\tilde{y}_0^{(2)}, \dots, \tilde{y}_N^{(2)}\}$,

$$\tilde{y}_k^{(2)} = y_k^{(2)} - E^*[y_k^{(2)} | Y^{(1)}] = y_k^{(2)} - H_k^{(2)} \hat{x}_{sk}^{(1)}. \quad (3.8)$$

图 1 给出式(3.7)分解的几何解释.

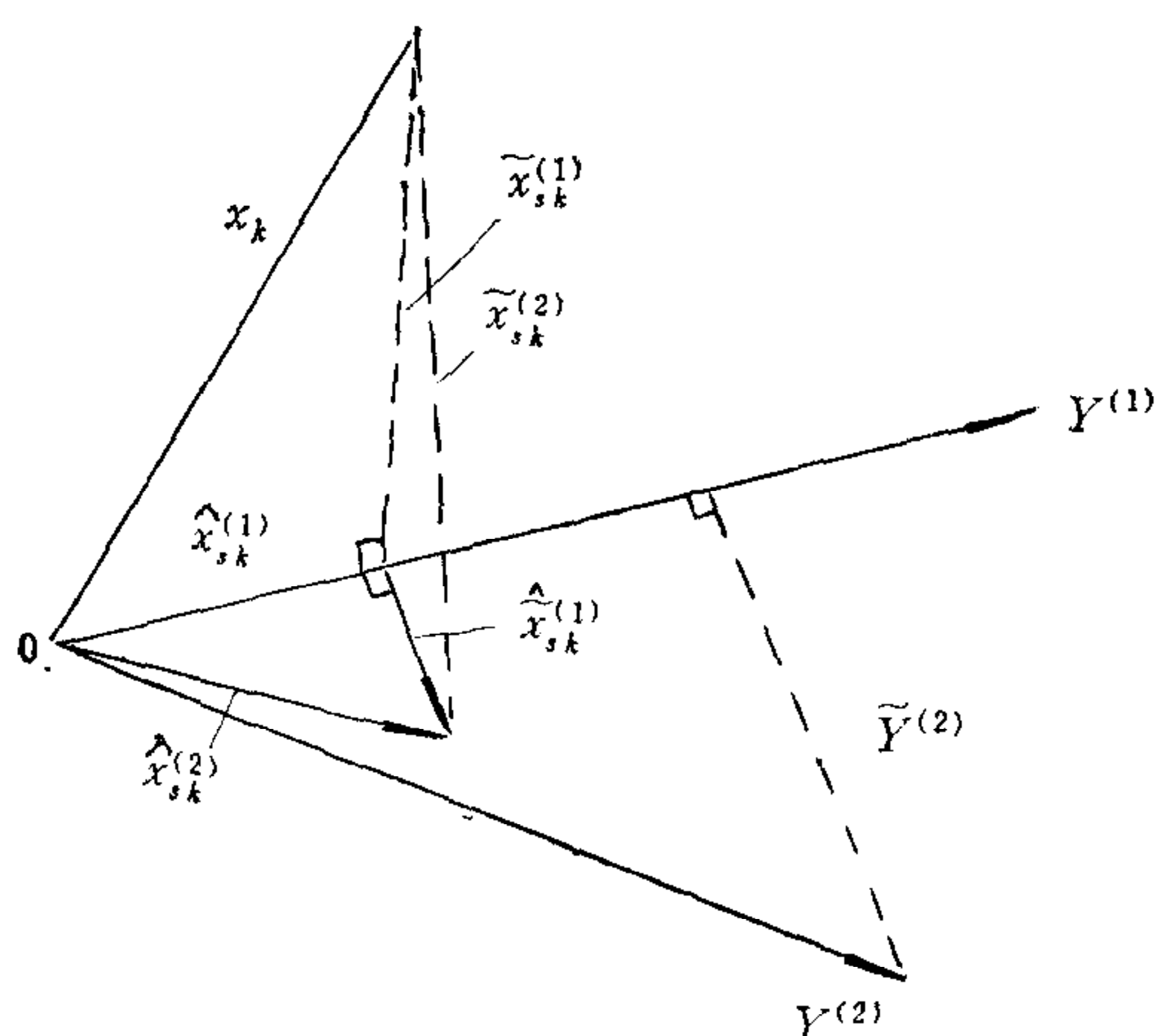


图1 式(3.7)分解的几何解释

只要可由 $Y^{(2)}$ 计算 $\hat{x}_{s,k}^{(1)}$, 式(3.7)便给出由 $\hat{x}_{s,k}^{(1)}$ 到 $\hat{x}_{s,k}^{(2)}$ 的更新. 特别, 如果能够类似于 $\hat{x}_{s,k}^{(1)}$ 那样用双滤波器法计算 $\hat{x}_{s,k}^{(1)}$, 便有 $\hat{x}_{s,k}$ 的双滤波器递推更新.

以 $\tilde{x}_{f_p,k}^{(1)}$ 表示 $\hat{x}_{f_p,k}^{(1)} \triangleq E^*[x_k | Y_{0,k-1}^{(1)}]$ 的预测误差:

$$\tilde{x}_{f_p,k}^{(1)} \triangleq x_k - \hat{x}_{f_p,k}^{(1)} \quad (3.9)$$

则可写

$$\hat{x}_{s,k}^{(1)} \triangleq E^*[x_k | Y^{(1)}] = E^*[\hat{x}_{f_p,k}^{(1)} + \tilde{x}_{f_p,k}^{(1)} | Y^{(1)}] = \hat{x}_{f_p,k}^{(1)} + E^*[\tilde{x}_{f_p,k}^{(1)} | Y^{(1)}] \quad (3.10)$$

由式(3.4), (3.9)和(3.10)得

$$\tilde{x}_{s,k}^{(1)} = \tilde{x}_{f_p,k}^{(1)} - E^*[\tilde{x}_{f_p,k}^{(1)} | Y^{(1)}] \quad (3.11)$$

由式(3.1)以及(2.8a,b)可得

$$\tilde{x}_{f_p,k+1}^{(1)} = A_k \tilde{x}_{f_p,k}^{(1)} + B_k u_k - A_k P_{f_p,k}^{(1)} (H_k^{(1)})' (Q_k^{(1)})^{-1} (y_k^{(1)} - H_k^{(1)} \hat{x}_{f_p,k}^{(1)}) \quad (3.12)$$

取(3.12)式两方对于 $\{Y^{(1)}\}$ 的投影, 用(3.12)式减它, 再利用式(3.11), 得到

$$\tilde{x}_{s,k+1}^{(1)} = A_k \tilde{x}_{s,k}^{(1)} + B_k u_k - B_k E^*[u_k | Y^{(1)}] \quad (3.13)$$

在处理上式右方第三项时, 要注意到 u_k 通过与 x_{k+1} 的相关而与 $y_{k+1}^{(1)}, \dots, y_N^{(1)}$ 相关. 如将 u_k 分解为

$$u_k = \tilde{u}_k + E^*[u_k | \tilde{x}_{f_p,k+1}^{(1)}] \quad (3.14)$$

则可证明 $\{\tilde{u}_k\}$ 是零均值不相关序列且具有下列性质: (1) \tilde{u}_k 正交于 $\{\tilde{x}_{f_p,k+1}^{(1)}, \dots, \tilde{x}_{f_p,N-1}^{(1)}\}$; (2) \tilde{u}_k 正交于 $\{Y^{(1)}\}$ (证明见附录). 由标准的 LLS 公式^[5],

$$E^*[u_k | \tilde{x}_{f_p,k+1}^{(1)}] = E[u_k (\tilde{x}_{f_p,k+1}^{(1)})'] \{E[\tilde{x}_{f_p,k+1}^{(1)} (\tilde{x}_{f_p,k+1}^{(1)})']\}^{-1} \tilde{x}_{f_p,k+1}^{(1)}. \quad (3.15)$$

由式(3.12)有

$$E[u_k (\tilde{x}_{f_p,k+1}^{(1)})'] = E[u_k (\tilde{x}_{f_p,k}^{(1)})'] A_k' + E[u_k u_k'] B_k' = B_k'$$

(u_k 与 $\tilde{x}_{f_p,k}^{(1)}$ 不相关). 所以得

$$\begin{aligned} E^*[u_k | \tilde{x}_{f_p,k+1}^{(1)}] &= B_k' (P_{f_p,k+1}^{(1)})^{-1} \tilde{x}_{f_p,k+1}^{(1)}, \\ \tilde{u}_k &= u_k - B_k' (P_{f_p,k+1}^{(1)})^{-1} \tilde{x}_{f_p,k+1}^{(1)} \end{aligned} \quad (3.16)$$

和

$$\tilde{Q}_k \triangleq E[\tilde{u}_k \tilde{u}_k'] = I - B_k' (P_{f_p,k+1}^{(1)})^{-1} B_k. \quad (3.17)$$

将式(3.16)代入(3.13)并利用(3.11)以及 \tilde{u}_k 的上述性质得

$$[\| - B_k B'_k (P_{fp,k+1}^{(1)})^{-1}] \tilde{x}_{s,k+1}^{(1)} = A_k \tilde{x}_{s,k}^{(1)} + B_k \tilde{u}_k. \quad (3.18)$$

将此式视为 $\tilde{x}_{s,k}^{(1)}$ 的后向模型,

$$\tilde{x}_{s,k}^{(1)} = A_k^{-1} [\| - B_k B'_k (P_{fp,k+1}^{(1)})^{-1}] \tilde{x}_{s,k+1}^{(1)} - A_k^{-1} B_k \tilde{u}_k \quad (3.19a)$$

(设 A_k^{-1} 存在)则容易看出

$$\tilde{x}_{s,N-1}^{(1)} = \tilde{x}_{fp,N-1}^{(1)}. \quad (3.19b)$$

与 $\{\tilde{u}_k\}_{N-2}^0$ 不相关即它具有“初始正交性”即, 所以确系线性马尔可夫模型. 注意模型(3.19)与 x_k 的后向模型(2.9)的相似性: 在式(2.9)中用 $P_{fp,k+1}^{(1)}$ 代 π_{k+1} 和用 \tilde{u}_k 代 u_k 便得式(3.19). 藉助于模型(3.19), 可用卡尔曼滤波器计算 $\tilde{x}_k^{(1)}$ 的后向预测和滤波估计

$$\hat{x}_{bp,k}^{(1)} \triangleq E^*[\tilde{x}_{s,k}^{(1)} | \tilde{Y}_{k+1,N-1}^{(2)}] \text{ 和 } \hat{x}_{b,k}^{(1)} \triangleq E^*[\tilde{x}_{s,k}^{(1)} | \tilde{Y}_{k,N-1}^{(2)}]. \quad (3.20)$$

由于前、后向模型以及前、后向预测和滤波的对称性, 可以用类似的方法导出 $\tilde{x}_{s,k}^{(1)}$ 的前向线性马尔可夫模型. 由式(2.9)和(2.10)得

$$\begin{aligned} \tilde{x}_{bp,k}^{(1)} &= C_{k+1} \tilde{x}_{bp,k+1}^{(1)} + D_{k+1} w_{k+1} \\ &\quad - C_{k+1} P_{bp,k+1}^{(1)} (H_{k+1}^{(1)})' (\phi_{k+1}^{(1)})^{-1} (y_{k+1}^{(1)} - H_{k+1}^{(1)} \hat{x}_{bp,k+1}^{(1)}), \end{aligned} \quad (3.21)$$

式中 $\hat{x}_{bp,k}^{(1)} = x_k - \hat{x}_{bp,k}^{(1)}$. 类似于式(3.11)和(3.13), 有

$$\tilde{x}_{s,k}^{(1)} = \tilde{x}_{bp,k}^{(1)} - E^*[\tilde{x}_{bp,k}^{(1)} | Y^{(1)}], \quad (3.22)$$

$$\tilde{x}_{s,k}^{(1)} = C_{k+1} \tilde{x}_{s,k+1}^{(1)} + D_{k+1} w_{k+1} - D_{k+1} E^*[w_{k+1} | Y^{(1)}]. \quad (3.23)$$

令

$$\tilde{w}_{k+1} = w_{k+1} - E^*[w_{k+1} | \tilde{x}_{bp,k}^{(1)}], \quad (3.24)$$

则可求得 $E^*[w_{k+1} | \tilde{x}_{bp,k}^{(1)}] = S_{k+1} D'_{k+1} (P_{bp,k}^{(1)})^{-1} \tilde{x}_{bp,k}^{(1)}$, 故得

$$\tilde{w}_{k+1} = w_{k+1} - S_{k+1} D'_{k+1} (P_{bp,k}^{(1)})^{-1} \tilde{x}_{bp,k}^{(1)}, \quad (3.25)$$

$$\tilde{S}_{k+1} \triangleq E[\tilde{w}_{k+1}, \tilde{w}'_{k+1}] = S_{k+1} [\| - D'_{k+1} (P_{bp,k}^{(1)})^{-1} D_{k+1} S_{k+1}]. \quad (3.26)$$

类似于证明 $\{\tilde{u}_k\}$ 的性质那样(见附录), 可以证明 $\{\tilde{w}_k\}$ 为零均值不相关序列且 \tilde{w}_{k+1} 正交于 $\{\tilde{x}_{bp,k}^{(1)}, \dots, \tilde{x}_{bp,0}^{(1)}\}$ 和 $\{Y^{(1)}\}$. 利用式(3.23), (3.24)和(3.25)可得

$$\tilde{x}_{s,k+1}^{(1)} = C_{k+1}^{-1} [\| - D_{k+1} S_{k+1} D'_{k+1} (P_{bp,k}^{(1)})^{-1}] \tilde{x}_{s,k}^{(1)} - C_{k+1}^{-1} D_{k+1} \tilde{w}_{k+1} \quad (3.27a)$$

(设 C^{-1} 存在). 式(3.27)以及

$$\tilde{x}_{s,0}^{(1)} = \tilde{x}_{b,0}^{(1)}, \quad (3.27b)$$

构成了 $\tilde{x}_{s,k}^{(1)}$ 的前向模型; 它确为线性马尔可夫模型, 因为它具有初始正交性即 $\tilde{x}_{s,0}^{(1)}$ 与 \tilde{w}_{k+1} 不相关. 藉助于模型(3.27), 可用卡尔曼滤波器计算 $\tilde{x}_{s,k}^{(1)}$ 的前向预测和滤波估计

$$\hat{x}_{fp,k}^{(1)} \triangleq E^*[\tilde{x}_{s,k}^{(1)} | \tilde{Y}_{0,k-1}^{(2)}] \text{ 和 } \hat{x}_{fk}^{(1)} \triangleq E^*[\tilde{x}_{s,k}^{(1)} | \tilde{Y}_{0,k}^{(2)}]. \quad (3.28)$$

利用 $\hat{x}_{fk}^{(1)}$ 和 $\hat{x}_{bp,k}^{(1)}$ 可依式(2.5)构成 $\hat{x}_{s,k}^{(1)}$ 的双滤波器解:

$$\hat{x}_{s,k}^{(1)} = P_{s,k}^{(2)} [(P_{fk}^{(2)})^{-1} \hat{x}_{fk}^{(1)} + (P_{bp,k}^{(2)})^{-1} \hat{x}_{bp,k}^{(1)}], \quad (3.29a)$$

$$(P_{s,k}^{(2)})^{-1} = (P_{fk}^{(2)})^{-1} + (P_{bp,k}^{(2)})^{-1} - (P_{s,k}^{(1)})^{-1}, \quad (3.29b)$$

式中 $P_{fk}^{(2)}$, $P_{bp,k}^{(2)}$ 和 $P_{s,k}^{(2)}$ 分别是误差 $\tilde{x}_{fk}^{(2)} \triangleq \tilde{x}_{s,k}^{(1)} - \hat{x}_{fk}^{(1)}$, $\tilde{x}_{bp,k}^{(2)} \triangleq \tilde{x}_{s,k}^{(1)} - \hat{x}_{bp,k}^{(1)}$ 和 $\tilde{x}_{s,k}^{(2)} = x_k - \hat{x}_{s,k}^{(2)} = \tilde{x}_{s,k}^{(1)} - \hat{x}_{s,k}^{(1)}$ 的协方差.

对 $\tilde{x}_{s,k}^{(2)}$ 又可建立前、后向模型; 它们类似于式(3.27)和(3.19). 如此递推, 可得一种双滤波器平滑更新算法, 它基于误差 $\tilde{x}_{s,k}^{(2)}$ 的下列前、后向模型:

前向模型

$$\begin{aligned}\tilde{x}_{s,k+1}^{(i)} &= A_k^{(i)}\tilde{x}_{s,k}^{(i)} + B_k^{(i)}u_k^{(i)}, \\ A_k^{(i)} &= C_{k+1}^{-1}[\| - D_{k+1}S_{k+1}D'_{k+1}(P_{b,p,k}^{(i)})^{-1}], \\ B_k^{(i)} &= -C_{k+1}^{-1}D_{k+1}, \\ Q_k^{(i)} \triangleq E[u_k^{(i)}(u_k^{(i)})'] &= S_{k+1}[\| - D'_{k+1}(P_{b,p,k}^{(i)})^{-1}D_{k+1}S_{k+1}], \\ \tilde{x}_{s_0}^{(i)} &= \tilde{x}_{b,p,0}^{(i)}.\end{aligned}$$

后向模型

$$\begin{aligned}\tilde{x}_{s,k}^{(i)} &= C_{k+1}^{(i)}\tilde{x}_{s,k+1}^{(i)} + D_{k+1}^{(i)}w_{k+1}^{(i)}, \\ C_{k+1}^{(i)} &= A_k^{-1}[\| - B_kB'_k(P_{f,p,k+1}^{(i)})^{-1}], \\ D_{k+1} &= -A_k^{-1}B_k, \\ S_{k+1}^{(i)} \triangleq E[w_{k+1}^{(i)}(w_{k+1}^{(i)})'] &= \| - B'_k(P_{f,p,k+1}^{(i)})^{-1}B_k, \\ \tilde{x}_{s,N-1}^{(i)} &= \tilde{x}_{f,p,N-1}^{(i)}.\end{aligned}$$

附 录

由 $\{u_k\}$ 的不相关性有

$$E^*[u_k|\tilde{x}_{f,p,k+1}^{(1)}] = E^*[u_k|\tilde{x}_{f,p,k+1}^{(1)}, u_{k+1}, \dots, u_{N-1}].$$

由式(3.12), $\tilde{x}_{f,p,l}^{(1)}$, $k+1 < l \leq N-1$, 可以表示为 $\tilde{x}_{f,p,k+1}^{(1)}$ 和 u_{k+1}, \dots, u_{N-1} 的线性组合, 所以可写

$$E^*[u_k|\tilde{x}_{f,p,k+1}^{(1)}] = E^*[u_k|\tilde{x}_{f,p,k+1}^{(1)}, \dots, \tilde{x}_{f,p,N-1}^{(1)}; u_{k+1}, \dots, u_{N-1}].$$

可见 \tilde{u}_k 正交于 $\{\tilde{x}_{f,p,k+1}^{(1)}, \dots, \tilde{x}_{f,p,N-1}^{(1)}\}$. 其次, 由上式, 对于任意的 $k < l \leq N-1$, \tilde{u}_k 正交于 $\{\tilde{x}_{f,p,k+1}^{(1)}, \dots, \tilde{x}_{f,p,N-1}^{(1)}; u_{k+1}, \dots, u_{N-1}\}$ 而 \tilde{u}_l 在此空间中, 故知 \tilde{u}_k 正交于 \tilde{u}_l . 最后, 因为 u_k 和 $E^*[u_k|\tilde{x}_{f,p,k+1}^{(1)}]$ 正交于 $\{Y_{0,k}^{(1)}\}$, \tilde{u}_k 也正交于 $\{Y_{0,k}^{(1)}\}$, 而且对任意的 $k < l \leq N-1$,

$$\begin{aligned}E[\tilde{u}_k(y_l^{(1)})'] &= E[\tilde{u}_k(H_l^{(1)}x_l + v_l^{(1)})'] \\ &= E[\tilde{u}_k x'_l](H_l^{(1)})' \\ &= E[\tilde{u}_k(\hat{x}_{f,p,l}^{(1)} + \tilde{x}_{f,p,l}^{(1)})'](H_l^{(1)})' = 0,\end{aligned}$$

所以 \tilde{u}_k 正交于 $\{Y^{(1)}\}$.

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A RECURSIVE UPDATE METHOD OF TWO-FILTER SMOOTHED ESTIMATES

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ABSTRACT

Linear Least square solution to the problem of fixed interval smoothing of a linear-Markovian state with multiple measurement channels is considered. An algorithm which recursively updates the two-filter smoothed estimates of the state as measured data from each channel are successively pue foreh is presented.

Key words: Fixed Interval Smoothing; Kalman Filter; Two-Filter Method; Forward Estimation; Backward Estimation.

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