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Optimal Control of Fixed Boundary Transpiration Cooling System1)

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Abstract The fixed boundary control of transpiration cooling system is discussed. The non-linear optimal control problem is replaced by one in which a linear form is minimized over a set of positive measures satisfying linear constraints. The method to calculate the minimized weight of coolant loaded in transpiration control system is given. The numerical simulation shows that the approach is valid in developing numerical technique.

Key words Transpiration cooling system, optimal control, measure theory, linear program

1 Introduction and question

Transpiration cooling control is a kind of important means of thermal protection. Applying the principle of transpiration to temperature control field has prospects for both military and commercial application such as in the thermal protection of the cabin of supersonic aircraft, in dealing with the ablation problem of heat protection shield of missile nose, and in the design of thermal protection for the throat of rocket jets or the gas rudder.

The research of transpiration cooling control problem in China has made remarkable progress, the main work include system model, numerical method and simulation, mathematical theory, control theory, and experiment research. For details one can see the survey [1]. On the study of control method, Yang proposed a feedback control scheme for water transpiration cooling system[2].

In this paper, the optimal control problem of fixed boundary transpiration cooling system as following is discussed. In this case, no ablation occurs, the shield temperature $T < T_a$, where T_a is the material melting temperature. We choose the coordinate system as in Fig. 1.

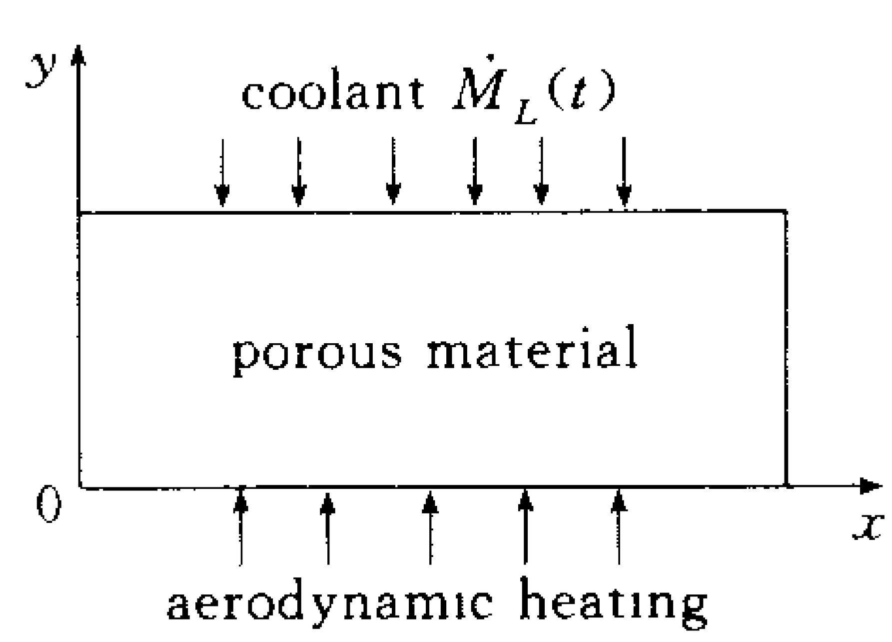

Fig. 1 Diagram of the heating shield

$$c\rho \frac{\partial T}{\partial t} = k \frac{\partial^2 T}{\partial y^2} + \dot{M}_L(t) c_{PL} \frac{\partial T}{\partial y}, \ (y,t) \in (0,l) \times (0,h) \tag{1}$$

$$T|_{t=0} = T_c \tag{2}$$

$$-k \frac{\partial T}{\partial y}\bigg|_{y=l} = \dot{M}_L(t) c_{PL} (T - T_c) \tag{3}$$

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$$-k\left.\frac{\partial T}{\partial y}\right|_{y=0}=\Psi(\dot{M}_L(t))q(t)\tag{4}$$

where $T(y,t)$ is the temperature distribution of porous material with thickness l. c, ρ and k represent the specific heat, the density and thermal conductivity, respectively. The transpiration mass flux $\dot{M}_L(t)$ acts as control parameter. h is terminal time. c_{PL} represents the coolant specific heat. $\Psi(\dot{M}_L(t))$ is the heat blockage function, which is a nonlinear continuous function of control variable $\dot{M}_L(t)$. $q(t)$ is the surface thermal flux of aerodynamic heating which we assume is continuous. In practice, $\dot{M}_L(t)$ must be non-negative, bounded and piecewise continuous function[1], as

$$\dot{M}_L(t)\in U_{ad}=\{\dot{M}_L(t)\mid 0\leqslant\dot{M}_L(t)\leqslant M,\ \text{piecewise continuous}\}.$$

Since the coolant carried by rocket affects the delivery cost. The carried coolant should be minimum under cooling prerequisite, so we choose the following criterion functional:

$$J(\dot{M}_L)=\int_0^h\dot{M}_L(t)\mathrm{d}t\tag{5}$$

Then the problem to calculate the minimized weight of coolant loaded of transpiration cooling system is the same as the nonlinear optimal control problem that minimizes functional (5) under the constraint condition equations (1)～(4) and $0\leqslant\dot{M}_L(t)\leqslant M$, where M is a positive constant, and $T(y,t)\leqslant T_b<T_a$.

Make non-dimensional transformations to simplify equations:

$$u=\frac{T-T_c}{T_b-T_c},\ v(t)=\frac{l\dot{M}_L(t)c_{PL}}{k},\ \tau=\frac{k}{\rho c l^2}t,\ \xi=\frac{l-y}{l}.$$

The original system is translated into:

$$\frac{\partial u}{\partial\tau}=\frac{\partial^2u}{\partial\xi^2}-v(\tau)\frac{\partial u}{\partial\xi},(\xi,\ \tau)\in(0,1)\times(0,H)\tag{6}$$

$$u\mid_{\tau=0}=0\tag{7}$$

$$\left.\frac{\partial u}{\partial\xi}\right|_{\xi=0}=v(\tau)u\tag{8}$$

$$\left.\frac{\partial u}{\partial\xi}\right|_{\xi=1}=\frac{l\Psi(v(\tau))}{k(T_b-T_c)}q(\tau)\tag{9}$$

the criterion functional is translated into:

$$J(v)=\int_0^H v(\tau)\mathrm{d}\tau\tag{10}$$

and satisfies the constraint condition $0\leqslant v(\tau)\leqslant A$ and $0\leqslant u(\xi,\tau)<1$, where $\xi\in[0,1]$ and $\tau\in[0,H]$, H is non-dimensional time and A is constant, i. e., $A=\frac{lMc_{PL}}{k}$.

In this problem, the control parameter appears both in equation and boundary, and the criterion functional is not a quadratics and not convex. [1] analyses the difficulty to minimize (10). [4] gives the result concerning existence, uniqueness and approximation of minimum-norm control problem with flow velocity as its control parameter. In this paper, the main approach is based on measure theory, consisting of the replacement of classical variation problems by problems in measure spaces using the tools of linear analysis. The advantages of new formulation are that the new problem is linear and there is always a minimizer for our measure problem. The method has resolved many linear and non-linear optimal control problems[5～7]. [8] improved the method on the basis of main principle, the method is extended to be applicable to more general problems.

If $v(\tau)$ is piecewise continuous, there exists unique $u\in C^2(0,1)\times C^1(0,H)$ for linear equations (6)～(9)[9]. Then the week solution of equation (11) exists and equals to classical solution. For every $\varphi\in C^2(0,1)\times C^1(0,H)$

$$\int_0^H\int_0^1 u\left(\frac{\partial\varphi}{\partial\tau}+v(\tau)\frac{\partial\varphi}{\partial\xi}+\frac{\partial^2\varphi}{\partial\xi^2}\right)\mathrm{d}\xi\mathrm{d}\tau-\int_0^1 u(\xi,H)\varphi(\xi,H)\mathrm{d}\xi+\int_0^H\varphi_\xi(0,\tau)u(0,\tau)\mathrm{d}\tau-$$

$$\int_0^H [u(1,\tau)(\varphi_\xi(1,\tau)+v(\tau)\varphi(1,\tau))-\frac{l\psi(v(\tau))}{k(T_b-T_c)}q_0(\tau)\varphi(1,\tau)]\mathrm{d}\tau=0 \quad (11)$$

We call the solution pair (u, v) satisfying equation (11) an admissible control pair.

2 Metamorphosis

The minimization of the functional (10) may not be possible, since the criterion functional is not strictly convex. But, according to Ekeland variation principle[10], the nearly-optimal control exists, that is, for every $\lambda>0$, there exists $v_\lambda \in V$ such that $J(v_\lambda)\leqslant J(v_0)-\varepsilon\lambda d(v_0,v_\lambda)$, where $\varepsilon>0, v_0\in V$ and $J(v_0)\leqslant \inf_{v\in V} J(v)+\varepsilon$. In the following, we translate the original problem to find nearly-optimal control.

For every fixed admissible pair (u,v), define the following linear bounded positive functional in the real-value continuous functions space $C(\Lambda)$

$$[u(\xi,\tau),v(\tau)]:\ F\rightarrow\int_{Q_H}F(u(\xi,\tau),v(\tau),\xi,\tau)\mathrm{d}\xi\mathrm{d}\tau \quad (12)$$

where $\Lambda=(0,1)\times(0,A)\times Q_H$; $Q_H=(0,1)\times(0,H)$.

By Riesz. A-Markov. S-Kakutani theorem[11], an admissible pair (u, v) defines the unique Randon measure μ:

$$\int_{Q_H}F(u(\xi,\tau),v(\tau),\xi,\tau)\mathrm{d}\xi\mathrm{d}\tau=\int_\Lambda F\mathrm{d}\mu=:\mu(F) \quad (13)$$

Notice that the formula $\int_0^H\varphi_\xi(0,\tau)u(0,\tau)\mathrm{d}\tau$ can be translated into $\lim_{\Delta\xi\to 0}\frac{1}{\Delta\xi}\int_0^H\int_0^1\varphi_\xi(\xi,\tau)u(\xi,\tau)\chi_{\xi_0}(\xi)\mathrm{d}\xi\mathrm{d}\tau$, where $\chi_{\xi_0}(\xi)$ is continuous function equal to one on $[0,\Delta\xi]$, and rapidly decreasing to zero on $(\Delta\xi, 1)$. Similarly,

$$\int_0^H[u(1,\tau)(\varphi_\xi(1,\tau)+v(\tau)\varphi(1,\tau))-\frac{l\psi(v(\tau))}{k(T_a-T_c)}q(\tau)\varphi(1,\tau)]\mathrm{d}\tau=$$

$$\lim_{\Delta\xi\to 0}\frac{1}{\Delta\xi}\int_0^H\int_0^1[u(\xi,\tau)(\varphi_\xi(\xi,\tau)+v(\tau)\varphi(\xi,\tau))-\frac{l\psi(v(\tau))}{k(T_a-T_c)}q(\tau)\varphi(\xi,\tau)]\chi_{\xi_1}(\xi)\mathrm{d}\xi\mathrm{d}\tau,$$

$\chi_{\xi_1}(\xi)$ is continuous function equal to one on $(1-\Delta\xi,1)$, and rapidly decreasing to zero on $[0,1-\Delta\xi]$. In the same way,

$$\int_0^1 u(\xi,H)\varphi(\xi,H)\mathrm{d}\xi=\lim_{\Delta\tau\to 0}\frac{1}{\Delta\tau}\int_0^H\int_0^1 u(\xi,\tau)\varphi(\xi,\tau)\chi_\tau(\tau)\mathrm{d}\xi\mathrm{d}\tau,$$

where $\chi_\tau(\tau)$ is continuous function equal to one on $(H-\Delta\tau,H)$ and rapidly decreasing to zero on $[0,H-\Delta\tau]$. Then equation (13) can be translated into

$$\mu(F\varphi)-\lim_{\Delta\tau\to 0}\mu(G\varphi)-\lim_{\Delta\xi\to 0}\mu(P\varphi)+\lim_{\Delta\xi\to 0}\mu(D\varphi)=0 \quad (14)$$

where $F_\varphi=u(\varphi_\tau+v\varphi_\xi+\varphi_{\xi\xi})$, $G_\varphi=u\varphi\chi_\tau/\Delta\tau$; $P_\varphi=\left(u\varphi_\xi+uv\varphi-\frac{l\psi(v)}{k(T_b-T_c)}q\varphi\right)\chi_{\xi_1}/\Delta\xi$, and $D_\varphi=\varphi_\xi u\chi_{\xi_0}/\Delta\xi$.

Due to $\int_0^H v(\tau)\mathrm{d}\tau=\int_0^H\int_0^1 v(\tau)\mathrm{d}\xi\mathrm{d}\tau$, the criterion functional (10) can be translated into:

$$I(\mu)=\mu(v) \quad (15)$$

Then the original problem has been modified into one of finding an optional measure over one measure space.

We denote by Q the set of measure μ satisfying (14)

$$Q=\{\mu:\ \mu\in M^+(\Lambda)\text{ and satisfying (14)}\},$$

where $M^+(\Lambda)$ is positive measure on Λ. Obviously, the measure has some extra properties which can be deduced from the definition of measure μ: If $\eta:\Lambda\rightarrow R$, depends only on (ξ,τ), then $\mu(\eta)$ is the Lebesgue integral of η over Q_H

$$\mu(\eta)=\int_{Q_T}\eta(\xi,\tau)\mathrm{d}\xi\mathrm{d}\tau=a_\eta \quad (16)$$

3 Approximation

Now the original problem has been modified into one of finding an optional measure over the measure space Q, that is, to find a measure μ^* satisfying (14) and (16) such that $I(\mu)=\mu(v)$ is a minimum. In the following, we will discuss the existence of the minimum and how to realize the finite-dimensional approximation.

We put week* topology on the measure space $M^+(\Lambda)$. We note that this is a linear space which will become a local convex topological vector space when given the week* topology. Then the space Q is compact and $\mu \to I(\mu)$ is continuous. We denote the set S of measure μ defined in $M^+(\Lambda)$ and satisfying (14) and (16), and it is compact.

Proposition 1. There exists an optimal $\mu^* \in S$ that minimizes the functional $I(\mu)$.

It is easy to conclude the result by Theorem2. 1 in [6]. In the same way as the proof of Theorem 7. 1 in [5], we have

Proposition 2. The set $S_1 \subset S$ of measures μ, which is piecewise-constant function on Λ and satisfies equation (11) and constraint condition (16), is weak*-dense in S.

The original problem has been transformed into an infinite dimensional minimization problem over set S. But we can only compute in finite dimensional spaces in practice. In the following, we will discuss now how to realize the finite-dimensional approximation. At first, we consider the minimization of $I(\mu)$ not over the set Q, but over a subset of Q defined by requiring that only a finite number of constraints in (14) and (16) be satisfied. This can be achieved by choosing countable sets of functions $\{\phi_i\}$ that are in $C^2(0,l)\times C(0,H)$, that is , for $\phi \in \phi \in C^2(0,1)\times C(0,H)$ and $\varepsilon>0$, there exists integer $N>0$ and scalars $\sigma_i, i=1,2,\cdots,N$, such that

$$\max_{Q_H}\left\{\left|\phi-\sum_{i=1}^{N}\sigma_i\phi_i\right|,\left|\phi_t-\sum_{i=1}^{N}\sigma_i\phi_{it}\right|,\left|\phi_\xi-\sum_{i=1}^{N}\sigma_i\phi_{i\xi}\right|,\left|\phi_{\xi\xi}-\sum_{i=1}^{N}\sigma_i\phi_{i\xi\xi}\right|\right\}<\varepsilon .$$

Proposition 3. Let M_1 and M_2 be positive integers. Consider the problem of minimizing $I(\mu)$ over the set $S(M_1, M_2)\subset M^+(\Lambda)$, and

$$\mu(F\varphi_i)-\lim_{\Delta\tau\to 0}\mu(G\varphi_i)-\lim_{\Delta\xi\to 0}\mu(P\varphi_i)+\lim_{\Delta\xi\to 0}\mu(D\varphi_i)=0,\ i=1,2\cdots,M_1 \tag{17a}$$

$$\mu(\eta_i)=\alpha_i \quad i=1,2,\cdots,M_2 \tag{17b}$$

Then as $M_i\to\infty(i=1,2)$, $\inf\limits_{S(M_1,M_2)}[I(\mu)]\to\inf\limits_{S}[I(\mu)]$.

Due to proposition 3, we have limited the number of constraints in the original problem. But the underlying space $S(M_1, M_2)$ is still infinite-dimensional. In what follows, we derive a finite programming by use of piecewise constant functions. Let the measure pair μ_v be the measure corresponding to piecewise constant control u_v. Since S_1 is density in S, then there exists measure $\mu_v\in S_1$ by piecewise constant control function, such that $|I(\mu_v)-\inf\limits_{S}I(\mu)|<\varepsilon$. The final approximation must be finite-dimensional and finite-constraint conditions. We shall prove below that if the numbers $M_i\to\infty$, $(i=1,2)$ and the set of two piecewise constant function (u, v) is sufficiently good, then $J(u_v, v)=\mu_{u_v}(v)\to\inf\limits_{S}I(\mu)$.

Theorem 1. Let (u_v, v) be the pair constructed as explained above. Then, under the appropriate conditions on the approximation, we have

1) (u_v, v) satisfies the weak-solution equation (11) for every $\phi\in C^2(0,1)\times C^1(0,H)$,

2) As $M_i\to\infty(i=1,2)$, $J(u_v, v)=\mu_{u_v}(v)\to\inf\limits_{S}I(\mu)$.

Proof. For every $\varepsilon>0$, let μ^* be the minimizer for the functional $I(\mu)$ over the S. Because of the density of S_1 in S, we can find a measure μ_v in S_1 which corresponds to piecewise constant trajectory-control function (u, v) such that $|I(\mu_v)-I(\mu^*)|<\varepsilon/8$; and we have

$$|\mu_v(F\varphi)-\lim_{\Delta\tau\to 0}\mu_v(G\varphi)-\lim_{\Delta\xi\to 0}\mu_v(P\varphi)+\lim_{\Delta\xi\to 0}\mu_v(D\varphi)|<\varepsilon/8 \tag{18}$$

1) First, we shall prove that u_v is the week solution of equation (11). Due to its construction, u_v satisfies equation (11) for every $\varphi_i \in C^2(0,1)\times C^1(0,H), i=1,2,\cdots M_1$. Notice that the set $\{\varphi_i\}$ serves as a basis in space $C^2(0,1)\times C^1(0,H)$ in the sense: for every $\varphi\in C^2(0,1)\times C^1(0,H)$, there exists an integer N and $\sigma_i \in R$ such that $\varphi=\sum_{i=1}^{N}\sigma_i\varphi_i$. Thus, if we choose $M_1>N$, u_v satisfies the week solution equation (11).

2) We will prove the second part of the theorem, that is, for every $\varepsilon>0$, as $M_i\to\infty$, $i=1,2$, $|\mu_{u_v}(v)-\inf_S I(\mu)|<\varepsilon$.

$$|\mu_{u_v}(v)-\inf_S I(\mu)|<|\mu_{u_v}(v)-\mu_v|+|\mu_v-\mu^*|<|\mu_{u_v}(v)-\mu_v(v)|+\varepsilon/8 \quad (19)$$

We shall prove $|\mu_{u_v}(v)-\mu_v(v)|<\varepsilon/2$ below. By (18), we have

$$|(\mu_{u_v}-\mu_v)F_{\varphi_i}-\lim_{\Delta\tau\to0}((\mu_{u_v}-\mu_v)G_{\varphi_i})-\lim_{\Delta\xi\to0}((\mu_{u_v}-\mu_v)P_{\varphi_i})+\lim_{\Delta\xi\to0}((\mu_{u_\tau}-\mu_v)D_{\varphi_i})|<\varepsilon/4 \quad (20)$$

Decompose $\varphi_i=\phi_i+\chi_i$, where ϕ_i satisfies $\phi_i|_{\xi=0}=\phi_i|_{\xi=1}=\frac{\partial\phi_i}{\partial\xi}\Big|_{\xi=1}=\phi_i|_{\tau=H}=0$. The function $\chi_i(\xi,\tau)$ is chosen so that the Lebesgue measure of the support of $\chi_i(\xi,\tau)$ in Q_H is not higher than a positive number ε_1, and $\frac{\partial\chi_i}{\partial\tau},\frac{\partial\chi_i}{\partial\xi},\frac{\partial^2\chi_i}{\partial\xi^2}$ are not larger than a positive number ρ. Notice that $F_{\varphi_i}=F_{\phi_i}+F_{\chi_i}, G_{\varphi_i}, P_{\varphi_i}, D_{\varphi_i}$ have the same properties. By (20), if we choose ρ and ε_1 properly, the following formula holds.

$$|(\mu_{u_\tau}-\mu_v)F_{\varphi_i}|<\varepsilon/4.$$

Choose φ, such that $\varphi=\sum_{i=1}^{K}\beta_i\varphi_i$, $\sum_{i=1}^{K}|\beta_i|\leqslant\lambda$, $K\leqslant M_1$. Let $\lambda<1$. Then we have

$$|(\mu_{u_v}-\mu_v)F_{\varphi}|<\lambda\varepsilon/4<\varepsilon/4.$$

Further, choose β_i, $i=1,2,\cdots,K$. such that $|u\varphi_\tau|<\varepsilon/16$, $v|u\varphi_\xi-1|<\varepsilon/16$, $u\|\varphi_{\xi\xi}\|<\varepsilon/16$. We have

$$|(\mu_{u_v}-\mu_v)v|<|(\mu_{u_v}-\mu_v)(u\varphi_\tau+uv\varphi_\xi+u\varphi_{\xi\xi})|+|(\mu_{u_v}-\mu_v)(u\varphi_\tau+v(u\varphi_\xi-1)+u\varphi_{\xi\xi})|<\varepsilon/2.$$

Then by (19) we have $|\mu_{u_v}(v)-\inf_S I(\mu)|<\varepsilon$. End of the proof. □

4 Numerical results

Choose aluminum material as an example. The test data is as below: $l=1.5$cm, $T_a=933.3$(K), $T_c=288$(K), $q(t)=425$(cal/cm^2 • s), $k=0.57$(cal/cm • K • s), $c_{PL}=1.24$ (cal/g • K). According to $v(t)=\frac{l\dot{M}_L(t)c_{PL}}{k}$, and using the experience formula of heating blockage function in [12], we have $\Psi(v(\tau))=1-[0.5848v(\tau)-0.084v(\tau)^2]$; then the boundary condition: $\frac{\partial u}{\partial\xi}\Big|_{\xi=1}=1.5(1-0.5848v(\tau)+0.084v(\tau)^2)$. Set control variables $0\leqslant v\leqslant1$, and $0\leqslant u\leqslant1$, $\tau\in[0,1.4]$. The algorithm is given as below.

1) To decompose Λ with $v\in[0,1]=\bigcup_{i=1}^{5}[v_{i-1},v_i)$, $u\in[0,1]=\bigcup_{i=1}^{5}[u_{j-1},u_j)$, $\xi\in[0,1]=\bigcup_{i=1}^{5}[\xi_{k-1},\xi_k)$, $\tau\in[0,1.4]=\bigcup_{i=1}^{7}[\tau_{l-1},\tau_l)$. The space Λ is divided into 875 equal sub-domains $\Lambda_n=[v_{i-1},v_i]\times[u_{j-1},u_j]\times[\xi_{k-1},\xi_k]\times[\tau_{l-1},\tau_l]$ with the corresponding Randon measure β_n. We take a constant value $\lambda_n=(v_n,u_n,\xi_n,\tau_n)$ in Λ_n, where $v_n\in[v_{i-1},v_i]$, $u_n\in[u_{j-1},u_j]$, $\xi_n\in[\xi_{k-1},\xi_k]$, $\tau_n\in[\tau_{l-1},\tau_l]$, and $\Delta\xi=\Delta\tau=0.2$. We put the sub-domains in the order:

$$\Lambda_n=\Lambda_{175(i-1)+35(j-1)+7(l-1)+k-1},\ i=1,2,\cdots,5,\ j=1,2,\cdots,5,\ l=1,2,\cdots,7,\ k=1,2,\cdots,5.$$

2) The original problem has been translated into one of finding $\{\beta_0,\beta_2,\cdots,\beta_{874}\}$ so as to minimize $\sum_{n=0}^{874}\beta_n v_n$ with the following constraint conditions:

$$\sum_{n=0}^{874}\beta_n F_{\varphi_i}(\lambda_n) - \sum_{n=0}^{874}\beta_n G_{\varphi_i}(\lambda_n) - \sum_{n=0}^{874}\beta_n P_{\varphi_i}(\lambda_n) + \sum_{n=0}^{874}\beta_n D_{\varphi_i}(\lambda_n) = 0, i = 1,2,\cdots,10.$$

$$\sum_{n=0}^{874}\beta_n \eta_i(\lambda_n) = \alpha_{\eta_i}, \ i = 1,2,\cdots,35.$$

This is a linear optimization problem with 875 variables and 45 constraint conditions, where $\beta n \geqslant 0$. The test functions $\varphi_i \in C^2(0,1)\times C^1(0,1.4)$, $i=1,2\cdots 10$, where $\varphi_i = \tau^p \sin(q\pi\xi)+(1/8+\tau)^2$. Setting $p=0, q=1$, $p=1, q=0.5$, $p=1, q=1$, $p=2, q=0.5$, $p=2, q=1$, $p=2, q=1.5$, $p=2, q=2$, $p=3, q=0.5$, we get φ_i, $i=1,2,\cdots,8$. Letting $\varphi_i = \tau^p\sin(q\pi\xi)+(1/8+\tau)^2$ and $p=1.5, q=3$, $p=2, q=0.525$, we have φ_9 and φ_{10}. The 35 functions $\eta_i (i=1,2,\cdots,35)$ are chosen as the characteristic function on $[\xi_{k-1},\xi_k]\times[\tau_{l-1},\tau_l]$. Those functions only depend on (ξ,τ). Resolving the problem, we get the solution of linear programming β_n as Fig. 2, and $J_{\min}=0.6834$.

3) How to choose control function. If some one β_n is not zero, then β_n corresponds to the specified time and control v. For example, the first element $\beta_0=0.04$ corresponds to time $[0,0.2]$ and control $v\in[0,0.2]$, its average is 0.1 on the region. Then we get a control component $(0.1\times 0.04)/0.2=0.02$ on the first time region. Also, the element $\beta_{351}=0.02$ still corresponds to the time region $[0,0.2]$, but to control $v\in[0.2,0.4]$. Then we get another control component $(0.3\times 0.02)/0.2=0.03$ on the first time region. Adding up all this kind of components, we can get the control on the first time region $[0,0.2]$. In the same way, by nonzero β_n, we can get the control on other time regions. We get the control on every time region as below: $v_1=0.1$, $v_2=0.2945$, $v_3=0.3151$, $v_4=0.578$, $v_5=0.74$, $v_6=0.74$, $v_7=0.7036$. With the control input, we get the temperature distribution on heating surface $\xi=1$ which is shown in Fig. 3.

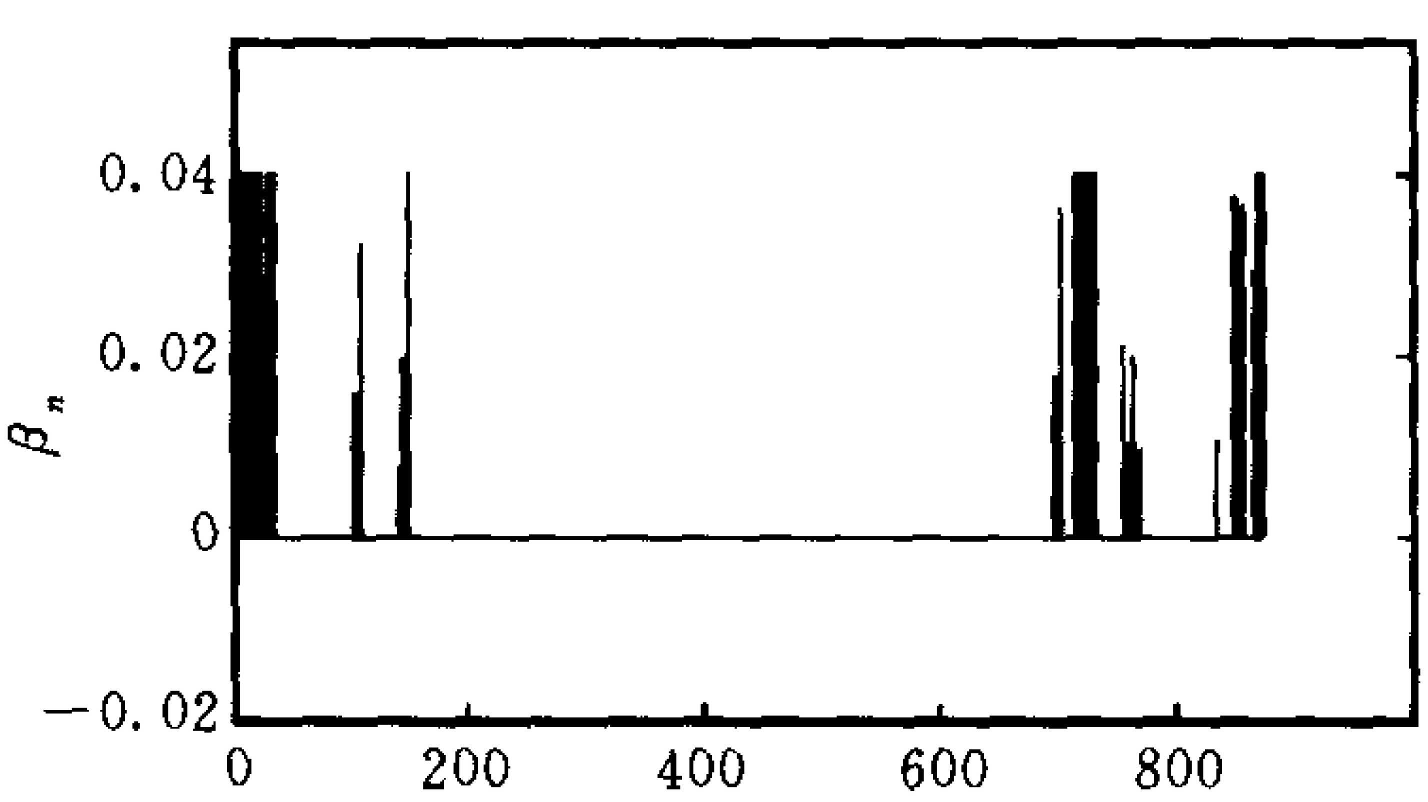

Fig. 2 The solution of linear programming

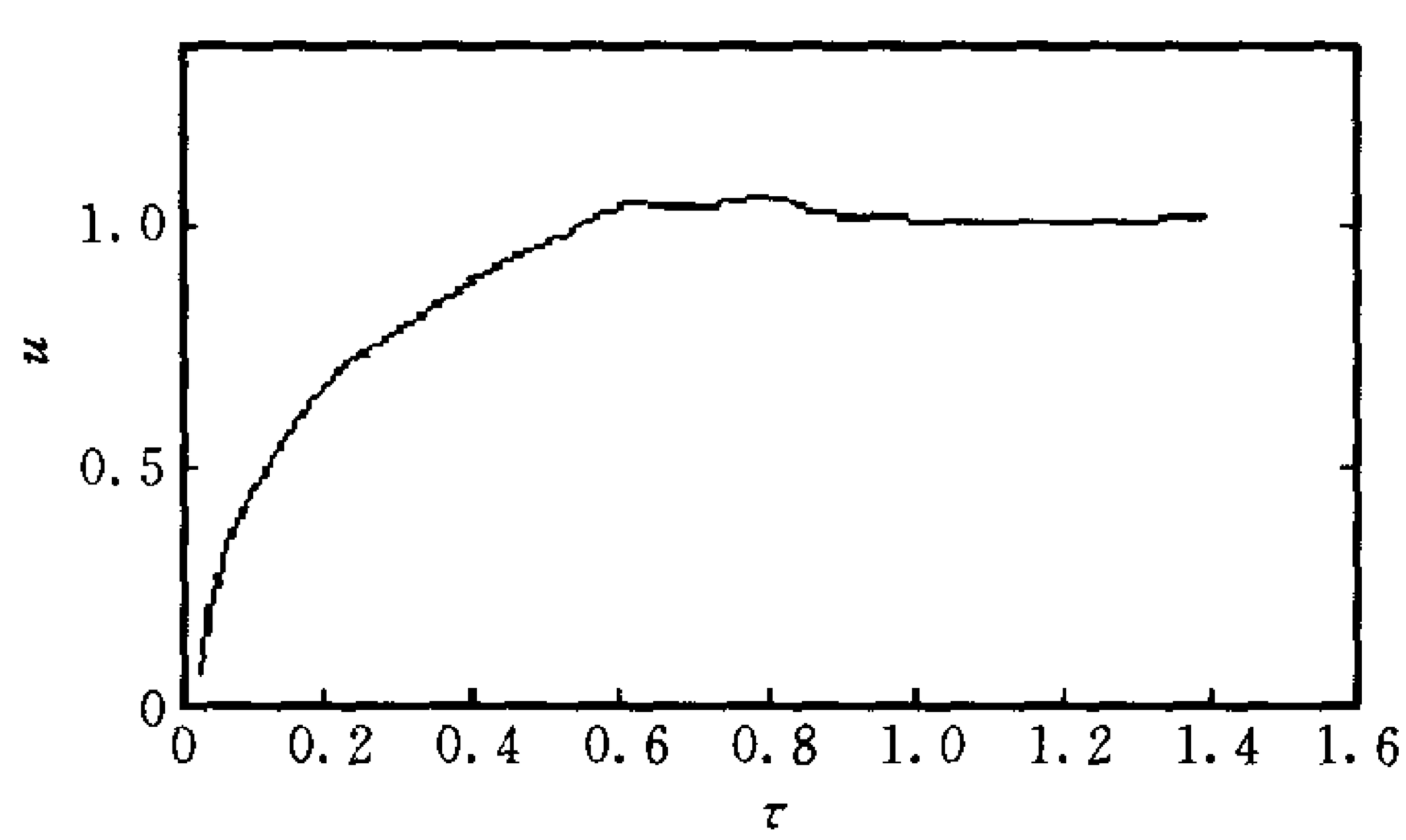

Fig. 3 The temperature distribution of heating surface

5 Conclusion

In this paper, the optimal control of transpiration cooling system has been discussed. The optimal control problem is replaced by one in which a linear form is minimized over a set of positive measures satisfying linear constraints. The method to calculate the minimized weight of coolant loaded in transpiration control system is given. The numerical simulation shows that the approach is valid in developing numerical technique. Moreover, the distributed parameter system with bilinear term $v\frac{\partial u}{\partial \xi}$ is treated by measure method for the first time. We mention, finally, that the approach can be applied to other nonlinear system.

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固定边界发汗冷却系统的最优控制问题

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摘 要 对固定边界发汗冷却控制系统的最优控制问题进行了讨论；利用测度理论，将最优控制问题转化为在一定的正测度空间上求解带线性约束条件的线性优化问题. 从而给出了控制律及冷却剂携带量最优值的计算方法，数据试验显示此方法是有效的.

关键词 发汗冷却控制系统，最优控制，测度理论，线性优化

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