

非线性时滞系统的高阶迭代学习控制

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摘 要

针对非线性时滞系统,讨论了输出跟踪控制的高阶迭代学习算法,并给出了算法的收敛性证明。当由于重复定位等原因造成初态偏差时,提出一种反复学习方案,完成初态和轨迹跟踪,它对初态偏差有较强的鲁棒性。仿真结果表明了该算法的有效性。

关键词: 迭代算法,跟踪控制,时滞系统

1、高阶迭代学习算法

近年来,迭代学习控制已得到人们的注意^[1-4]。这里针对非线性时滞系统,讨论 N 阶 PID 型迭代学习算法。

考虑 r 个输入和 m 个输出 (n 维状态)的非线性时滞系统

$$\left. \begin{aligned} \dot{\mathbf{x}}_i(t) &= \mathbf{f}(\mathbf{x}_i(t), \mathbf{x}_i(t - t_{d_1}), t) \\ &\quad + \mathbf{B}(\mathbf{x}_i(t), \mathbf{x}_i(t - t_{d_2}), t) \mathbf{u}_i(t) + \mathbf{w}_i(t), \\ \mathbf{y}_i(t) &= \mathbf{g}(\mathbf{x}_i(t), t) + \mathbf{v}_i(t). \end{aligned} \right\} \quad (1)$$

其中 $t \in [0, T]$ 。当 $t \in [-t_d, 0]$ 时, $\mathbf{x}_i(t) = \boldsymbol{\phi}(t)$, $t_d = \max\{t_{d_1}, t_{d_2}\}$ 。假定 $\mathbf{f}, \mathbf{g}, \mathbf{g}_x, \mathbf{g}_i, \mathbf{B}$ 关于 $\mathbf{x}(t), \mathbf{x}(t - t_{d_1})$ 和 $\mathbf{x}(t - t_{d_2})$ 满足 Lipschitz 条件,且 \mathbf{g}_x, \mathbf{B} 有界;干扰 $\mathbf{w}_i(t), \mathbf{v}_i(t)$ 和 $\dot{\mathbf{v}}_i(t)$ 有界。

N 阶 PID 型迭代学习算法

$$\begin{aligned} \mathbf{u}_{i+1}(t) &= (1 - \gamma) \sum_{k=1}^N P_k \mathbf{u}_{i-k+1}(t) + \gamma \mathbf{u}_0(t) \\ &\quad + \sum_{k=1}^N \left\{ Q_k \mathbf{e}_{i-k+1}(t) + R_k \dot{\mathbf{e}}_{i-k+1}(t) + S_k \int_0^t \mathbf{e}_{i-k+1}(\tau) d\tau \right\}. \end{aligned} \quad (2)$$

式中 $\mathbf{e}_i(t) \triangleq \mathbf{y}_d(t) - \mathbf{y}_i(t)$, $\mathbf{y}_d(t)$ 为期望轨迹; N 为学习阶次; P_k, Q_k, R_k, S_k 为学习系数矩阵; $0 \leq \gamma < 1$, 用来限制迭代开始时控制波动太大。

引理 1. 设正实数列 $\{a_n\}_1^\infty$, 满足

$$a_n \leq \rho_1 a_{n-1} + \rho_2 a_{n-2} + \cdots + \rho_N a_{n-N} + \varepsilon, \quad (n = N + 1, N + 2, \cdots).$$

其中 $\rho_i \geq 0$, $\varepsilon \geq 0$. 则当 $\rho \triangleq \sum_{i=1}^N \rho_i < 1$ 时, 有

$$\limsup_{n \rightarrow \infty} a_n \leq \frac{\varepsilon}{1 - \rho}.$$

证明. 略.

定理 1. 由式(1)描述的系统满足假设, 若

$$(c1) \quad \sum_{k=1}^N P_k = I,$$

(c2) 存在 $\rho_k > 0$, $k = 1, 2, \cdots, N$, 满足

$$\|(1 - \gamma)P_k - R_k \mathbf{g}_x B\| \leq \rho_k, \quad \text{且} \quad \rho \triangleq \sum_{k=1}^N \rho_k < 1,$$

以及初态偏差有界, 则由算法(2)得到的 $\|\mathbf{u}_d(t) - \mathbf{u}_i(t)\|_\lambda$, $\|\mathbf{x}_d(t) - \mathbf{x}_i(t)\|_\lambda$ 和 $\|\mathbf{y}_d(t) - \mathbf{y}_i(t)\|_\lambda$ 有界; 如果状态和输出无扰动及初态无偏差, 那么所有界为零.

证明. 记 $l = i - k + 1$. $(\cdot)_d$ 表示期望轨迹, $(\cdot)_l$ 表示第 $i - k + 1$ 次学习轨迹, $(\cdot)_x$ 和 $(\cdot)_t$ 分别表示对 \mathbf{x} 和 t 求偏导. 由 (c1) 可得

$$\begin{aligned} \mathbf{u}_d - \mathbf{u}_{i+1} = & \sum_{k=1}^N [(1 - \gamma)P_k - R_k \mathbf{g}_{xl} B_l] (\mathbf{u}_d - \mathbf{u}_l) + \gamma (\mathbf{u}_d - \mathbf{u}_0) \\ & - \sum_{k=1}^N \left\{ Q_k (\mathbf{g}_d - \mathbf{g}_l - \mathbf{v}_l) + S_k \int_0^t (\mathbf{g}_d - \mathbf{g}_l - \mathbf{v}_l) dt \right. \\ & + R_k \left[(\mathbf{g}_{xd} - \mathbf{g}_{xl}) (\mathbf{f}_d + B_d \mathbf{u}_d) + \mathbf{g}_{xl} (\mathbf{f}_d - \mathbf{f}_l) \right. \\ & \left. \left. + \mathbf{g}_{xl} (B_d - B_l) \mathbf{u}_d + \mathbf{g}_{ld} - \mathbf{g}_{ll} - \mathbf{g}_{xl} \mathbf{w}_l - \dot{\mathbf{v}}_l \right] \right\}. \end{aligned} \quad (3)$$

估计上式的模, 有

$$\begin{aligned} \|\mathbf{u}_d - \mathbf{u}_{i+1}\| \leq & \sum_{k=1}^N \|(1 - \gamma)P_k - R_k \mathbf{g}_{xl} B_l\| \|\mathbf{u}_d - \mathbf{u}_l\| + \gamma \|\mathbf{u}_d - \mathbf{u}_0\| \\ & + \sum_{k=1}^N \left\{ \|Q_k\| (\|\mathbf{g}_d - \mathbf{g}_l\| + \|\mathbf{v}_l\|) + \|S_k\| \int_0^t (\|\mathbf{g}_d - \mathbf{g}_l\| + \|\mathbf{v}_l\|) dt \right. \\ & + \|R_k\| \left[\|\mathbf{g}_{xd} - \mathbf{g}_{xl}\| \|\mathbf{f}_d + B_d \mathbf{u}_d\| + \|\mathbf{g}_{xl}\| \|\mathbf{f}_d - \mathbf{f}_l\| \right. \\ & \left. \left. + \|\mathbf{g}_{xl}\| \|B_d - B_l\| \|\mathbf{u}_d\| + \|\mathbf{g}_{ld} - \mathbf{g}_{ll}\| + \|\mathbf{g}_{xl}\| \|\mathbf{w}_l\| + \|\dot{\mathbf{v}}_l\| \right] \right\}. \end{aligned} \quad (4)$$

由假设, 以 k_* 表示相应(*)的 Lipschitz 常数, b_* 表示相应(*)的界, 若记

$$\begin{aligned} b_d & \triangleq \sup_{t \in [0, T]} \|\mathbf{f}_d + B_d \mathbf{u}_d\|, \quad b_{u_d} \triangleq \sup_{t \in [0, T]} \|\mathbf{u}_d\|, \\ b_w & \triangleq \sup_{t \in [0, T]} \|\mathbf{w}\|, \quad b_v \triangleq \sup_{t \in [0, T]} \|\mathbf{v}\|, \quad b_{\dot{v}} \triangleq \sup_{t \in [0, T]} \|\dot{\mathbf{v}}\| \end{aligned}$$

和

$$\begin{aligned} b_{1k} &\triangleq \|Q_k\|k_g + \|R_k\|(b_d k_{gx} + b_{gx} k_f + b_{gx} k_B b_{ud} + k_{gt}), \\ b_{2k} &\triangleq \|R_k\|b_{gx} k_f, \quad b_{3k} \triangleq \|R_k\|b_{gx} k_B b_{ud}, \quad b_{4k} \triangleq \|S_k\|k_g, \\ b_{rdk} &\triangleq \|Q_k\|b_w + \|R_k\|(b_{gx} b_w + b_i) + \|S_k\|T b_w. \end{aligned}$$

则由 (c2) 整理得

$$\begin{aligned} \|\mathbf{u}_d(t) - \mathbf{u}_{i+1}(t)\| &\leq \sum_{k=1}^N \rho_k \|\mathbf{u}_d(t) - \mathbf{u}_i(t)\| + \gamma \|\mathbf{u}_d(t) - \mathbf{u}_0(t)\| \\ &+ \sum_{k=1}^N \left(b_{1k} \|\mathbf{x}_d(t) - \mathbf{x}_i(t)\| + b_{2k} \|\mathbf{x}_d(t - t_{d_1}) - \mathbf{x}_i(t - t_{d_1})\| \right. \\ &+ b_{3k} \|\mathbf{x}_d(t - t_{d_2}) - \mathbf{x}_i(t - t_{d_2})\| \\ &\left. + b_{4k} \int_0^t \|\mathbf{x}_d(\tau) - \mathbf{x}_i(\tau)\| d\tau + b_{rdk} \right). \end{aligned} \quad (5)$$

应用 Bellman-Gronwall 引理[3], 有

$$\begin{aligned} \|\mathbf{x}_d(t) - \mathbf{x}_i(t)\| &= \|\mathbf{x}_d(0) - \mathbf{x}_i(0) \\ &+ \int_0^t [\mathbf{f}_d + B_d \mathbf{u}_d - (\mathbf{f}_i + B_i \mathbf{u}_i + \mathbf{w}_i)] d\tau \| \\ &\leq \|\mathbf{x}_d(0) - \mathbf{x}_i(0)\| e^{b_5 t} \\ &+ \int_0^t e^{b_5(t-\tau)} (b_B \|\mathbf{u}_d(\tau) - \mathbf{u}_i(\tau)\| + b_w) d\tau. \end{aligned} \quad (6)$$

这里, $b_5 \triangleq 2(k_f + b_{ud} k_B)$, $b_B \triangleq \sup_{t \in [0, T]} \|B\|$. 将(6)式代入(5)式, 得到

$$\begin{aligned} \|\delta \mathbf{u}_{i+1}\| &\leq \sum_{k=1}^N \rho_k \|\delta \mathbf{u}_i\| + \gamma \|\delta \mathbf{u}_0\| \\ &+ \sum_{k=1}^N \left\{ b_k \left[\|\delta \mathbf{x}_i(0)\| e^{b_5 t} + \int_0^t e^{b_5(t-\tau)} (b_B \|\delta \mathbf{u}_i\| + b_w) d\tau \right] + b_{rdk} \right\}. \end{aligned} \quad (7)$$

上式中, $\delta(\cdot)_i \triangleq (\cdot)_d - (\cdot)_i$, $b_k \triangleq b_{1k} + b_{2k} + b_{3k} + b_{4k} T$.

定义函数 $h(t)$ 的 λ -模 $\|h(t)\|_\lambda \triangleq \sup_{t \in [0, T]} e^{-\lambda t} \|h(t)\|$, 将(7)式两端同乘 $e^{-\lambda t}$, 有

$$\begin{aligned} \|\delta \mathbf{u}_{i+1}\|_\lambda &\leq \sum_{k=1}^N \rho_k \|\delta \mathbf{u}_i\|_\lambda + \gamma \|\delta \mathbf{u}_0\|_\lambda + \sum_{k=1}^N \left\{ b_k \|\delta \mathbf{x}_i(0)\| e^{(b_5 - \lambda)t} \right. \\ &+ b_{rdk} e^{-\lambda t} + b_k b_w \int_0^t e^{-\lambda \tau} e^{(b_5 - \lambda)(t-\tau)} d\tau \\ &\left. + b_k b_B \int_0^t \|\delta \mathbf{u}_i\|_\lambda e^{(b_5 - \lambda)(t-\tau)} d\tau \right\}. \end{aligned} \quad (8)$$

当 $\lambda > b_5$ 时可得

$$\|\delta \mathbf{u}_{i+1}\|_\lambda \leq \sum_{k=1}^N \bar{\rho}_k \|\delta \mathbf{u}_i\|_\lambda + \varepsilon. \quad (9)$$

这里, $\bar{\rho}_k \triangleq \rho_k + \frac{b_k b_B}{\lambda - b_s} (1 - e^{(b_s - \lambda)T})$;

$$\varepsilon \triangleq \gamma \|\delta \mathbf{u}_0\|_\lambda + \sum_{k=1}^N \left\{ b_k \|\delta \mathbf{x}_i(0)\| + b_{rdk} + \frac{b_k b_\omega}{\lambda - b_s} (1 - e^{(b_s - \lambda)T}) \right\}.$$

由假设条件 (c2) 知, 当 λ 足够大时, 有 $\sum_{k=1}^N \bar{\rho}_k < 1$, 且 $\bar{\rho}_k > 0$, 根据引理 1, 存在 $\bar{\rho}$, 使得

$$\limsup_{i \rightarrow \infty} \|\delta \mathbf{u}_i\|_\lambda \leq \frac{1}{1 - \bar{\rho}} \varepsilon. \quad (10)$$

同时, 易证 $\|\delta \mathbf{x}_i\|_\lambda$ 和 $\|\delta \mathbf{y}_i\|_\lambda$ 有界. 当所有扰动、偏差和 γ 相除时, 由 ε 的定义知 $\varepsilon = 0$, 即所有界将为零.

2、反复学习方案

实际中当 $\mathbf{x}_i(0)$ 偏离 $\mathbf{x}_d(0)$ 时, 上述算法本身并不能使其偏差消除. 对此我们给出一种反复学习方案 (F, B 分别表示正, 反向学习过程; $(\cdot)^j$ 表示向量 (\cdot) 的第 j 个分量):

STEP1. 以 $\mathbf{x}(0) \approx \mathbf{x}_d(0)$, 开始进行第 1 趟正向学习, 选取正向学习系数矩阵, 使满足

$$|\mathbf{x}^j(0) - \mathbf{x}_d^j(0)| > |\mathbf{x}_F^{j(1)}(T) - \mathbf{x}_d^j(T)|, \quad j = 1, 2, \dots, n. \quad (11)$$

STEP2. 以 $\mathbf{x}_F^{(1)}(T)$ 为初值, 开始进行第 1 趟反向学习, 选择反向学习系数矩阵, 使满足

$$|\mathbf{x}_F^{j(1)}(T) - \mathbf{x}_d^j(T)| > |\mathbf{x}_B^{j(1)}(0) - \mathbf{x}_d^j(0)|, \quad j = 1, 2, \dots, n. \quad (12)$$

STEP3. 置 $k = 2$.

STEP4. 以 $\mathbf{x}_B^{k-1}(0)$ 为初值进行第 k 趟正向学习, 得到 $\mathbf{x}_F^{(k)}(T)$.

STEP5. 以 $\mathbf{x}_F^{(k)}(T)$ 为初值进行第 k 趟反向学习, 得到 $\mathbf{x}_B^{(k)}(0)$.

STEP6. 令 $k = k + 1$, 转 STEP4, 直至正向学习过程达到精度要求.

3、仿真结果

考虑下述问题

$$\left. \begin{aligned} \dot{x}(t) &= -5[1 + x(t - \tau)]^{x(t-\tau)} \tanh(u(t)), \\ y(t) &= x(t), \end{aligned} \right\} \quad (13)$$

$$y_d(t) = \begin{cases} t, & t \in [0, 0.2], \\ -t + 0.4, & t \in (0.2, 0.4], \\ 0, & t \in (0.4, 0.6], \\ t - 0.6, & t \in (0.6, 0.8], \\ -t + 1, & t \in (0.8, 1.0]. \end{cases} \quad (14)$$

取允许误差 0.005, $e_b \triangleq \max_{t \in [0, T]} |y_d(t) - y(t)|$, $\text{RMS}(e)$ 表示误差的均方根值。学习系数取

$N = 2$; $p_1 = 0.98$, $p_2 = 0.02$; $q_1 = 0.005$, $q_2 = 0.001$; $r_1 = -0.145$, $r_2 = -0.005$; $s_1 = -0.8$, $s_2 = -0.1$ 。

有延时算法的收敛比较见表 1, 当输出加入扰动 $d(t)$ 时, 其恢复过程如图 1 所示; 无延时初态有偏差的反复学习过程参见图 2。

表 1 不同延时收敛比较

ILC 次数	$\tau = 0$		$\tau = 0.6$		$\tau = 1.25$	
	$\text{RMS}(e)$	e_b	$\text{RMS}(e)$	e_b	$\text{RMS}(e)$	e_b
1	.0116	.0196	.0118	.0199	.0120	.0196
2	.0021	.0048	.0022	.0049	.0023	.0049

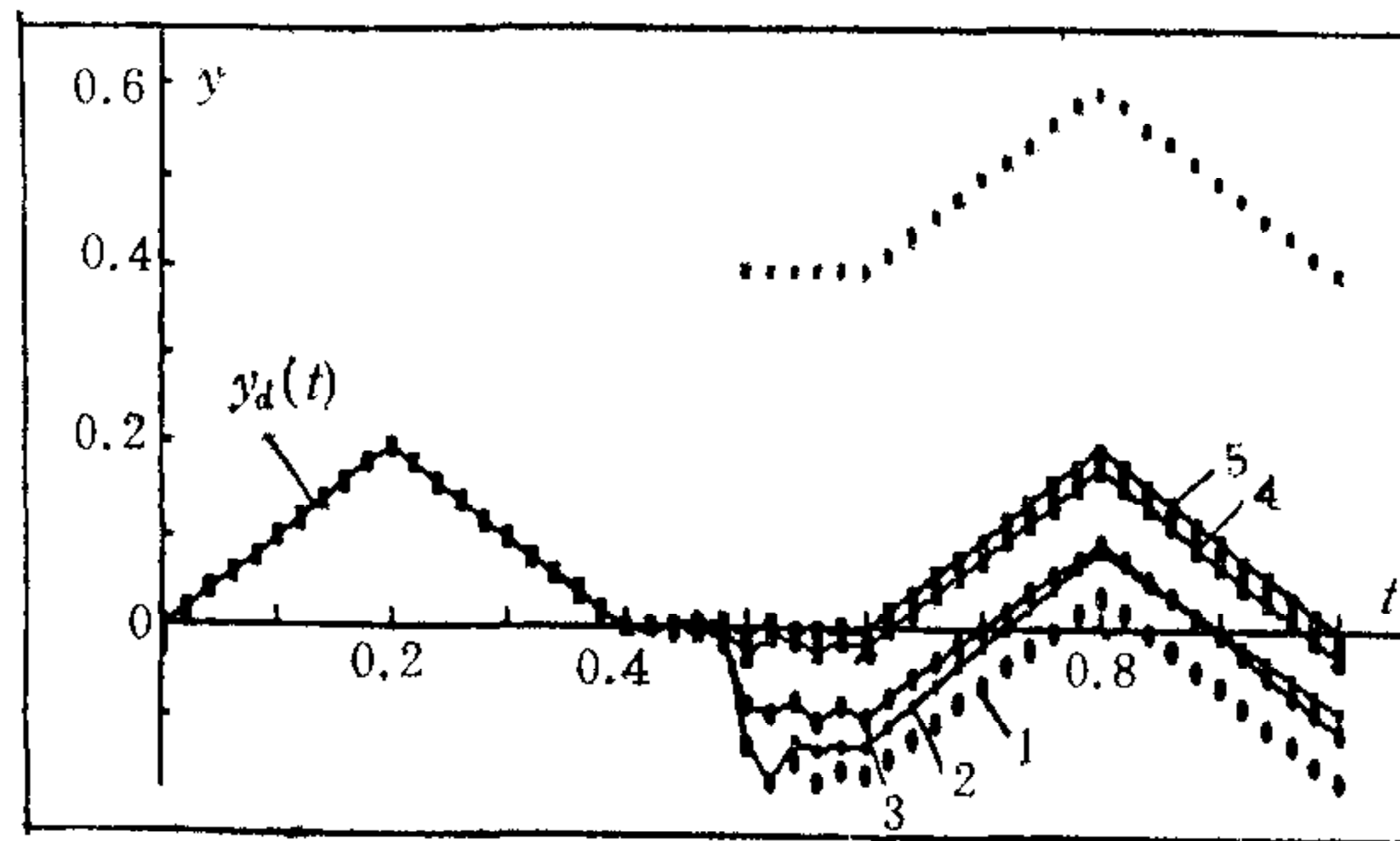


图 1 无延时输出扰动恢复过程

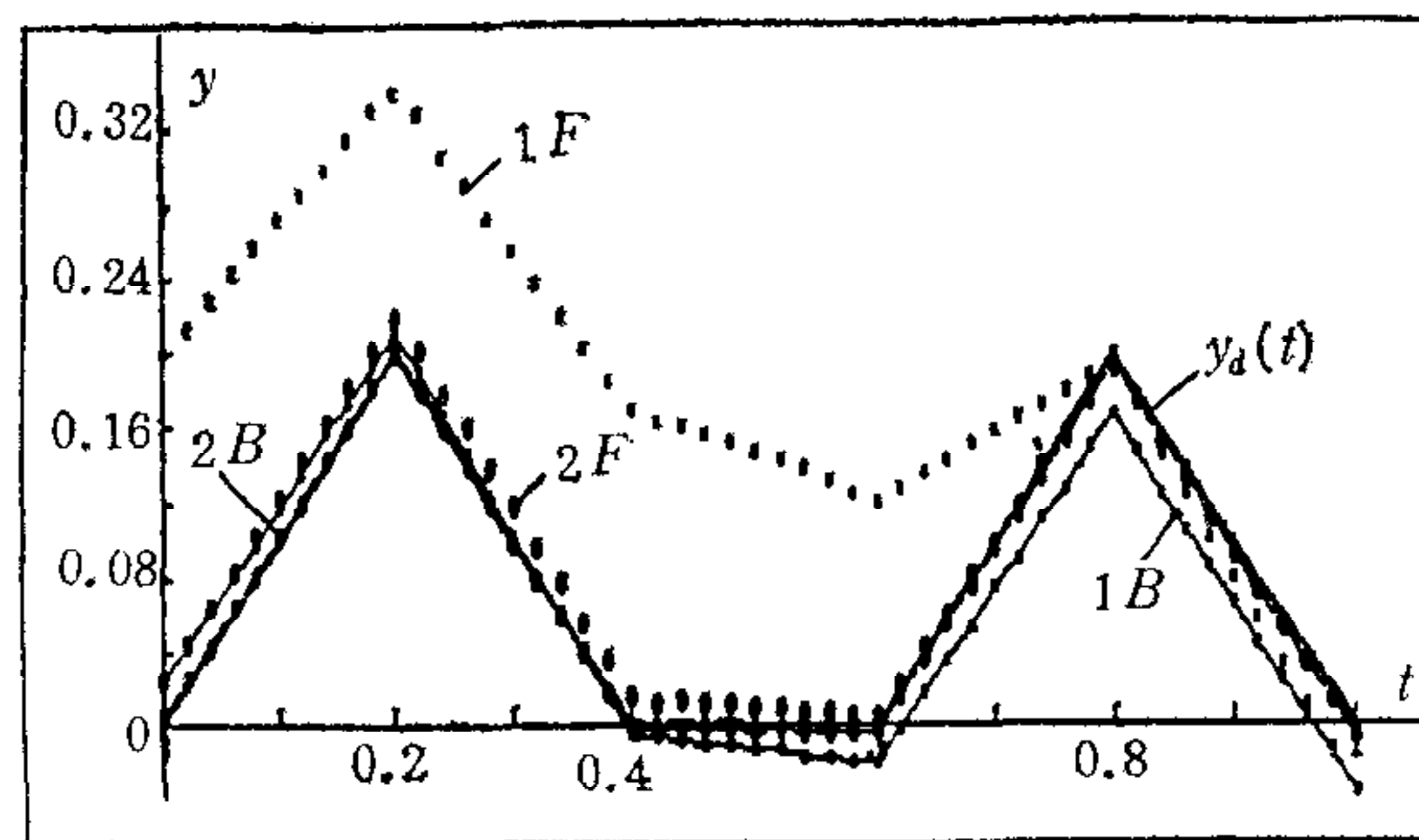


图 2 无延时反复学习过程

从仿真结果看, 高阶 PID 型算法由于多利用了 D, I 信息, 收敛更快。对扰动和时滞有较强的抑制能力。反复迭代学习方案对克服初态偏差是有效的。

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ROBUST HIGHER ORDER ITERATIVE LEARNING CONTROL ALGORITHM FOR TRACKING CONTROL OF DELAYED REPE- ATED SYSTEMS

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ABSTRACT

A robust higher order iterative learning control algorithm is proposed to perform the output tracking control of delayed repetitive systems. The proof of the convergence for this kind of iterative learning control algorithm is given in more general case. When there exists bias in initial states, a repetitive learning scheme is proposed to make the algorithm more robust with respect to this bias. Compound numerical simulation results indicate the effectiveness and usefulness of the algorithm.

Key words: Iterative algorithm; tracking control; delayed systems.