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Hybrid Direct Model Reference Adaptive Control[1)]

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Abstract Direct model reference adaptive control (MRAC) with hybrid adaptive law is studied in this paper. For the hybrid MRAC scheme, we prove rigorously that all signals in the closed-loop system are bounded, meanwhile the tracking error satisfies $e_1 \in S(\mu^2(\Delta^2+\Delta_\infty^2)+d_0^2+1/a_0^2$. Compared with control schemes in [1], the hybrid MRAC has the following advantages: (1) the smaller computational effort during implementation, and (2) the better robustness properties in the presence of measurement noise.

Key words Hybrid model reference adaptive control, direct, normalization

1 Introduction

Many good results on robust model reference adaptive control (RMRAC) have been obtained[1]. In [1], RMRAC schemes for continuous systems are classified as direct and indirect according to different choices of parameters. The parameter estimate $\theta(t)$ obtained by the two approaches has the same character: $\theta(t)$ is continuous with time t, i.e., there is a new estimate at each t.

Direct model reference adaptive control with hybrid adaptive law is considered in this paper. The idea of the control scheme is to update the parameter estimate at some instances of time. Let $t_k = kT_s$, where $T_s = t_{k+1} - t_k (k=0,1,2,\cdots)$ is the sampling period; the parameter estimate on the unknown parameter θ^* (parameter of the controller) is to generate only at discrete instances of time $t=0, T_s, 2T_S, \cdots$. Compared with the control schemes in [1], the hybrid MRAC has the following advantages: 1) the smaller computational effort during implementation, and 2) the better robustness properties in the presence of measurement noise. For the control scheme combining the continuous system with discrete adaptive laws, since the parameter estimate $\theta(t)$ is not differential, some key techniques used to analyze stability of adaptive systems in [1] (such as the swapping lemma) can not be used. Meanwhile some properties of parameter estimation may be changed accordingly, hence, how to analyze stability of this kind of adaptive systems theoretically constitutes the main work in this paper.

The paper is organized as follows. The first section is introduction. The design scheme of hybrid MRAC is given in section 2. In section 3, we prove rigorously that all signals in the closed-loop system are bounded, meanwhile the tracking error satisfies $e_1 \in S(\mu^2(\Delta^2+\Delta_\infty^2)+d_0^2+1/a_0{}^2$.

2 Design of hybrid direct MRAC

2.1 Problem statement and assumptions

Let us consider the following plant:

$$y_p = G_p(s)(1+\mu\Delta_m(s))(u_p+d_u) \tag{1}$$

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where $G_p(s)=\bar{Z}_p(s)/R_p(s)=k_pZ_p(s)/R_p(s)$, $R_p(s)=s^n+a_{n-1}s^{n-1}+\cdots+a_0$, $\bar{Z}_p(s)=b_ms^m+b_{m-1}s^{m-1}+\cdots+b_0$, $b_m=k_p$, $G_p(s)$ is the transfer function of the modeled part of the plant, $\bar{Z}_p(s)=k_pZ_p(s)$, $\Delta_m(s)$ is unmodeling dynamics, and d_u is a bounded disturbance, i. e., there exists a constant $d_0>0$ such that $|d_u|\leqslant d_0$. a_i, $b_j(i=0,1,\cdots,n-1,j=0,1,\cdots,m)$ are unknown parameters, and $\mu>0$ is finite.

The control objective is to choose u_p such that all signals in the closed-loop system are bounded, and the output approaches to the output y_m of the reference model as close as possible

$$y_m = W_m(s)r = k_m\frac{Z_m(s)}{R_m(s)}r \tag{2}$$

where r is a bounded reference signal.

For the above plant and reference model, we still adopt the assumptions for the ideal case (i. e., $\Delta_m(s)=0$, $d_u=0$):

Plant assumptions:

P_1: $Z_p(s)$ is monic Hurwitz polynomial.

P_2: The relative degree $n^*=n-m\geqslant 1$ is known.

P_3: The sign of k_p is known.

Reference model assumptions:

M_1: $Z_m(s)$ and $R_m(s)$ are monic Hurwitz polynomials of degree q_m, $p_m(p_m\leqslant n)$, respectively, and $W_m(s)$ is analytic in Re $[s]\geqslant-\delta_0/2$ for some known $\delta_0>0$.

M_2: The relative degree is n^*.

For unmodeled dynamics $\Delta_m(s)$, we need the following assumptions:

S_1: For the above δ_0, $\Delta_m(s)$ is analytic in Re $[s]\geqslant-\delta_0/2$.

S_2: $W_m(s)\Delta_m(s)$ is strictly proper.

2.2 Parametric model

We develop an appropriate parametric model according to the desired controller parameter θ^*, then choose an adaptive law for unknown parameter θ^*. (1) can be expressed as

$$R_p(s)y_p = k_pZ_p(s)u_p + \eta_0 \tag{3}$$

where $\eta_0=k_pZ_p(s)(\mu\Delta_m(s)(u_p+d_u)+d_u)$. Adding and subtracting $k_pZ_p\theta^{*T}\omega$ for (3), we obtain

$$\begin{aligned} R_p(s)y_p &= k_pZ_p(s)(u_p-\theta^{*T}\omega)+k_pZ_p(s)\theta^{*T}\omega+\eta_0 = \\ &k_pZ_p(s)(u_p-\theta^{*T}\omega)+k_pZ_p(s)(\theta_1^{*T}\omega_1+\theta_2^{*T}\omega_2+\theta_3^*y_p+c_0^*r)+\eta_0 \end{aligned} \tag{4}$$

where $\theta^*=(\theta_1^{*T},\theta_2^{*T},\theta_3^*,c_0^*)^T$ is parameter of the controller, $\omega=(\omega_1^T,\omega_2^T,y_p,r)^T$, $\omega_1=\frac{a(s)}{\Lambda(s)}u_p$, $\omega_2=\frac{a(s)}{\Lambda(s)}y_p$, $a(s)=(s^{n-2},s^{n-3},\cdots,s,1)^T$, $\Lambda(s)$ is known Hurwitz polynomial of degree $n-1$, and $\frac{1}{\Lambda(s)}$ is analytic in Re$[s]\geqslant-\delta_0/2$. From the matching equations (please see [1] eqs. (6.3.12)~(6.3.13))

$$(\Lambda(s)-\theta_1^{*T}a(s))R_p(s)-k_p(\theta_2^{*T}a(s)+\theta_3^*\Lambda(s))Z_p(s) = Z_p(s)\Lambda_0(s)R_m(s) \tag{5}$$

$$c_0^* = \frac{k_m}{k_p} \tag{6}$$

where $\Lambda(s)=\Lambda_0(s)Z_m(s)$. From (3)~(6) it is easy to see that

$$y_p-y_m = \frac{1}{c_0^*}W_m(s)(u_p-\theta^{*T}\omega)+\frac{(\Lambda(s)-\theta_1^{*T}a(s))}{\Lambda(s)c_0^*}W_m(s)\eta_1 \tag{7}$$

where $\eta_1=\mu\Delta_m(s)(u_p+d_u)+d_u$. Since θ^* is known, the certainty equivalence controller is chosen as

$$u_p = \theta^T\omega \tag{8}$$

where θ is the estimate of θ^*. Noticing (2), as the expression of ω and θ^* are constant, we can rewrite (7) as the following parametric model

$$z = \theta^{*T}\varphi_p - \eta \tag{9}$$

where

$$z = W_m(s)u_p, \varphi_p = (W_m(s)\omega_1^T, W_m(s)\omega_2^T, W_m(s)y_p, y_p)^T, \eta = \frac{(\Lambda(s) - \theta_1^{*T}a(s))}{\Lambda(s)}W_m(s)\eta_1 \tag{10}$$

Remark 1. The detailed deduction procedure of the equations can be found in chapters 6 and 9 in [1].

2.3 Hybrid adaptive law

Since θ^* is unknown, we now give an algorithm to estimate θ^*. From the plant assumption P_3, the sign of c_0^* is known. Without loss of generality, we assume that there exists a constant $\underline{c}>0$ such that $c_0^* > \underline{c}$. Set $t_k = kT_s$, where $T_s = t_{k+1} - t_k (k=0,1,2,\cdots)$ is the sampling period, and denote $\theta_k = \theta(kT_s), k=0,1,2,\cdots$. Then the adaptive law on θ^* is chosen as

$$\theta_{k+1} = \theta_{k+1}^p + \Delta_{k+1} \tag{11}$$

$$\theta_{k+1}^p = \theta_k + \Gamma\int_{t_k}^{t_{k+1}} (\epsilon(\tau)\varphi_p(\tau) - \delta\theta_k)\mathrm{d}\tau, \quad \Delta_{k+1} = \begin{cases} 0 & c_{0(k+1)}^p \geqslant \underline{c} \\ \frac{\tau_1}{\tau_2}(\underline{c} - c_{0(k+1)}^p) & \text{otherwise} \end{cases} \tag{12}$$

$$\epsilon(t) = \frac{z(t) - \theta_k^T\varphi_p(t)}{m^2(t)}, \quad m^2(t) = 1 + \|u_{pt}\|_{2\delta_0}^2 + \|y_{pt}\|_{2\delta_0}^2 + y_p^2(t) \tag{13}$$

where $\forall t \in [t_k, t_{k+1})$, $\|x_t\|_{2\delta} = (\int_0^t e^{-\delta(t-\tau)}|x(\tau)|^2\mathrm{d}\tau)^{1/2}$, τ_1 is the last column of Γ, τ_2 is the last element of τ_1. The estimation algorithm (11)~(13) has the following properties.

Lemma 1. Let m, σ_0, T_s, and Γ be chosen so that:

1) $\frac{\eta}{m} \in L_\infty$, $\frac{\varphi_p^T\varphi_p}{m^2} < c_1$

2) $2c_1T_s\lambda_m < 1$, $2\sigma_0T_s\lambda_m < 1$

Then

i) $\epsilon, \epsilon m, \theta_k$, $\Delta\theta_k \in L_\infty$, $c_{0(k+1)} \geqslant \underline{c}$

ii) $\epsilon, \epsilon m \in X\left(\frac{\eta^2}{m^2}\right)$, $\Delta\theta_k \in D\left(\frac{\eta^2}{m^2}\right)$

where

$\lambda_m = \lambda_{\max}(\Gamma)$, $\Delta\theta_k = \theta_{k+1} - \theta_k$, $X(u) = \left\{x, u \mid \int_t^{t+T}|x(\tau)^2|\mathrm{d}\tau \leqslant c\int_t^{t+T}|u(\tau)|\mathrm{d}\tau + c,\ \forall t, T>0\right\}$, $D(v) = \left\{\{x_k\} \in R^n \,\middle|\, \sum_{k=k_0}^{k_0+N}|x(k)|^2 \leqslant c\int_{t_{k_0}}^{t_{k_0}+NT_s}|v(\tau)|\mathrm{d}\tau + c,\ \forall k_0,\ N \in N^+\right\}$, c is a finite constant, $\bar{\eta} = \sup\left|\frac{\eta}{m}\right|$.

Proof. The projection algorithm is used in this algorithm. Since the projection algorithm can only make the derivative of Lyapunov function more negative, we can prove Lemma 1 using the method of Theorem 8.5.9 in [1]. □

Remark 2. The reason of using the projection algorithm is that $c_{0(k+1)} \geqslant \underline{c}$ can be satisfied, the result can avoid the problem of zero divider, therefore (24) can be obtained.

3 Main results

We give the main results in this paper.

Theorem 1. Assume that the plant (1), the reference model (2) and unmodeled dynamics $\Delta_m(s)$ satisfy assumptions $P_1 - P_3$, $M_1 - M_2$, and $S_1 - S_2$, respectively. The hybrid model reference adaptive controller consists of (8), (10)~(13). If the following conditions are satisfied:

$$2c_1 T_s \lambda_m < 1, \quad 2\sigma_0 T_s \lambda_m < 1,$$

then there exists $\mu^* > 0$ such that $\forall \mu \in [0, \mu^*)$, and all signals in the closed-loop system are bounded and the tracking error $e_1 = y_p - y_m$ satisfies

$$e_1 \in S(\mu^2(\Delta^2 - \Delta_\infty^2) + d_0^2 + 1/a_0^2),$$

where c_1 is a constant, $\Delta = \left\| \frac{\Lambda(s) - \theta_1^{*T} a(s)}{\Lambda(s)} W(s) \Delta_m(s) \right\|_{2\delta_0}$, $\Delta_\infty = \| W_m(s) \Delta_m(s) \|_{\infty\delta}$, $\delta \in (0, \delta_0]$, $S(\lambda) = \{x : [0, \infty) \mapsto R^n \mid \int_t^{t+T} |x(\tau)|^2 d\tau \leqslant c\lambda T + c, \forall t, T \geqslant 0, c, \lambda$ are constants$\}$.

For simplicity, in the procedure of the proof, sometimes the polynomial operator $X(s)$ is denoted as X, $x(t)$ is denoted as x. Before giving the proof of Theorem 1, we need a lemma.

Lemma 2. For the HMRAC scheme, for any $\delta \in (0, \delta_0]$, define $m_f^2(t) = 1 + \|u_{pt}\|_{2\delta}^2 + \|y_{pt}\|_{2\delta}^2 + y_p^2(t)$, then

i) $\frac{|\omega_i|}{m_f}$, $\frac{\|\omega\|_{2\delta}}{m_f}$, $i = 1, 2$ and $\frac{n_s}{m_f} \in L_\infty$,

ii) If $\theta \in L_\infty$, then $\frac{u_p}{m_f}$, $\frac{y_p}{m_f}$, $\frac{\omega}{m_f}$, $\frac{W(s)\omega}{m_f}$, $\frac{\|u_p\|}{m_f}$, $\frac{\|\dot{y}_p\|}{m_f} \in L_\infty$,

iii) If θ, $\dot{r} \in L_\infty$, then $\frac{\|\dot{\omega}\|}{m_f} \in L_\infty$,

iv) $\frac{m}{m_f}$, $\frac{\varphi_p}{m_f}$, $\frac{\|\varphi_p\|}{m_f}$, $\frac{W(s)\varphi_p}{m_f} \in L_\infty$,

v) For $\delta = \delta_0$, the above conclusions are still true,

where $W(s)$ is any proper transfer function that is analytic in $\mathrm{Re}[s] \geqslant -\delta_0/2$. $\|x\|$ denotes the norm of $\|x_t\|_{2\delta}$, m^2 is defined by (13), $n_s^2 = \|u_{pt}\|_{2\delta_0}^2 + \|y_{pt}\|_{2\delta_0}^2$, and φ_p is defined by (10).

Proof. Similar to the proof of Lemma 9.8.1 in [1].

We now give the proof of Theorem 1. Without special explanation, $\|x_t\|_{2\delta}$ is denoted as $\|x\|$. In addition, in the procedure of enlarging inequality, the coefficient is often expressed by c.

The proof of theorem.

Step1. Express the plant input and output according to the parameter error term $\tilde{\theta}^T \omega$. From (7) and (8) and since c_0^* is a constant, the closed-loop system is obtained as

$$y_p = W_m(s)\left(r + \frac{\tilde{\theta}^T \omega}{c_0^*}\right) + \eta_y \tag{14}$$

By (1), (5), (6) and (14) we get

$$u_p = G_p^{-1}(s) W_m(s)\left(r + \frac{\tilde{\theta}^T \omega}{c_0^*}\right) + \eta_u \tag{15}$$

where $c_0^* = \frac{k_m}{k_p}$, $\eta_u = \frac{\theta_2^{*T} a(s) + \theta_3^* \Lambda(s)}{c_0^* \Lambda(s)} W_m(s) \eta_1$, $\eta_y = \frac{\Lambda(s) - \theta_1^{*T} a(s)}{c_0^* \Lambda(s)} W_m(s) \eta_1$. From assumption P_1, we know that there exists a constant $\delta > 0$ ($\delta \leqslant \delta_0$) such that $G_p^{-1}(s)$ is analytic in $\mathrm{Re}[s] \geqslant -\delta/2$. Define

$$m_f^2(t) = 1 + \|u_{pt}\|^2 + \|y_{pt}\|^2 + y_p^2(t) \tag{16}$$

Since $G_p^{-1}(s)$ is analytic in $\mathrm{Re}[s] \geqslant -\delta/2$, $\eta_1 = \mu\Delta_m(s)(u_p + d_u) + d_u$, (14)~(16), assumptions M_1, M_2, S_1, S_2 and $\Lambda(s)$ is Hurwitz polynomial, and $1/\Lambda(s)$ is analytic in $\mathrm{Re}[s] \geqslant -\delta_0/2$, using Lemma 2 and Lemma 3.3.2 in [1], we obtain

$$m_f^2 \leqslant c + c\|\tilde{\theta}^T \omega\|^2 + c\mu^2 \Delta_\infty^2 m_f^2 + c d_0^2 \tag{17}$$

Step2. Consider the properties of parameter estimation, and determine the upper bound of $\|\tilde{\theta}^T \omega\|$. From (10), $\eta_1 = \mu\Delta_m(s)(u_p + d_u) + d_u$, assumptions M_1, S_1, S_2, and Lemma 2 and Lemma 3.3.2 in [1], we have

$$|\eta| \leqslant \mu \left\| \frac{\Lambda(s) - \theta_1^{*T} a(s)}{\Lambda(s)} W_m(s) \Delta_m(s) \right\|_{2\delta_0} \|u_p\|_{2\delta_0} + cd_0 \leqslant c\mu\Delta m + cd_0 \tag{18}$$

where m is defined by (13). Noticing $m > 1$, μ is finite, and $\Delta < \infty$, we have $\frac{\eta}{m} \in L_\infty$. From iv) and v) of Lemma 2, $\frac{\varphi_p}{m} \in L_\infty$. Therefore from the assumptions of Theorem 1 and Lemma 1, it is easy to see that the algorithm (11~13) has the properties i)~ii) of Lemma 1.

For $\forall t \in [t_k, t_{k+1})$, constitute function

$$\bar{\theta}(t) = \theta_k + \frac{\theta_{k+1} - \theta_k}{T_s}(t - t_k) \tag{19}$$

It is obvious that for $\forall t \in [t_k, t_{k+1})$ $\bar{\theta}(t)$ has the properties:

$$1)\ \bar{\theta}(t) \text{ is continuous; } 2)\ |\bar{\theta}(t) - \theta_k| \leqslant |\theta_{k+1} - \theta_k|\ ;\ 3)\ \dot{\bar{\theta}}(t) = \frac{\theta_{k+1} - \theta_k}{T_s} \tag{20}$$

For $\forall t \geqslant 0$, there exists positive integer k' such that $t \in [t'_k, t'_{k+1})$. From the conclusion 2) of (20) and $\theta_k = \theta(t_k)$, we know

$$|\bar{\theta}(t) - \theta(t)| = |\bar{\theta}(t) - \theta_{k'}| \leqslant |\theta_{k'+1} - \theta_{k'}| = |\Delta\theta_{k'}|,$$

$$|\bar{\theta}(t)| \leqslant |\bar{\theta}(t) - \theta_{k'}| + |\theta_{k'}| \leqslant |\Delta\theta_{k'}| + |\theta_{k'}|,$$

$\Delta\theta_{k'} = \theta_{k'+1} - \theta_{k'}$. By conclusion i) of Lemma 1, we know that for $\forall t \geqslant 0$

$$\bar{\theta}(t),\ \bar{\theta}(t) - \theta(t) \in L_\infty \tag{21}$$

For $\forall t, T \geqslant 0$, there exist positive integers k' and n such that $k'T_s \leqslant t < (k'+1)T_s$, $(k'+n-1)T_s < t + T \leqslant (k'+n)T_s$. Thus by $\frac{\eta}{m} \in L_\infty$. The conclusion 2) of (20), and ii) of Lemma 1, we have

$$\int_t^{t+T} |\bar{\theta}(\tau) - \theta(\tau)|^2 \mathrm{d}\tau \leqslant \sum_{i=0}^{n-1} \int_{(k'+i)T_s}^{(k'+i+1)T_s} |\bar{\theta}(\tau) - \theta(\tau)|^2 \mathrm{d}\tau \leqslant$$
$$\sum_{i=0}^{n-1} T_s |\Delta\theta_{k'+i}|^2 \leqslant cT_s \sum_{i=0}^{n-1} \int_{k'T_s}^{(k'+n)T_s} \frac{\eta^2(\tau)}{m^2(\tau)} \mathrm{d}\tau + c \leqslant$$
$$cT_s \int_t^{t+T} \frac{\eta^2(\tau)}{m^2(\tau)} \mathrm{d}\tau + cT_s^2,$$

where c is some constant. By the same way, it follows from 3) of (20) and ii) of Lemma 1 that

$$\int_t^{t+T} |\dot{\bar{\theta}}(\tau)^2| \mathrm{d}\tau \leqslant c \int_t^{t+T} \frac{\eta^2(\tau)}{m^2(\tau)} \mathrm{d}\tau + c,$$

where c is some constant. Hence from the definition of $X(\cdot)$, we know that for $\forall t \geqslant 0$

$$\bar{\theta}(t) - \theta(t), \dot{\bar{\theta}}(t) \in X\left(\frac{\eta^2}{m^2}\right) \tag{22}$$

Define $\tilde{\bar{\theta}}(t) = \bar{\theta}(t) - \theta^*$, $\bar{\theta}(t) = (\bar{\theta}_1^{\mathrm{T}}(t), \bar{\theta}_2^{\mathrm{T}}(t), \bar{\theta}_3(t), \bar{c}_0(t))^{\mathrm{T}}$. Since $\bar{\theta}(t)$ is differentiable, using swapping lemma A1 [1,P774], we get

$$W_m(s)(\tilde{\bar{\theta}}^{\mathrm{T}}\omega) = \tilde{\bar{\theta}}^{\mathrm{T}}(W_m(s)\omega) + W_c(s)((W_b(s)\omega^{\mathrm{T}})\dot{\tilde{\bar{\theta}}}) \tag{23}$$

where $W_c(s)$ and $W_b(s)$ are strictly proper and have the same poles as $W_m(s)$. Define $\tilde{\theta}(t) = \theta(t) - \theta^*$, obviously, $\theta(t) = \theta_k$, $\forall t \in [t_k, t_{k+1})$. By (12),(14), (23), and the definition of ω, and since c_0^* is constant, we have

$$-\varepsilon m^2 - \eta = \tilde{\theta}^{\mathrm{T}}\varphi_p = (\tilde{\theta} - \tilde{\bar{\theta}})^{\mathrm{T}}\varphi_p + \tilde{\bar{\theta}}^{\mathrm{T}}\varphi_p =$$
$$(\tilde{\theta} - \tilde{\bar{\theta}})^{\mathrm{T}}\varphi_p + \frac{\bar{c}_0}{c_0^*}W_m(s)(\tilde{\bar{\theta}}^{\mathrm{T}}\omega) - W_c(s)((W_b(s)\omega^{\mathrm{T}})\dot{\tilde{\bar{\theta}}}) + \frac{\tilde{\bar{c}}_0}{c_0^*}W_m(s)((\tilde{\theta} - \tilde{\bar{\theta}})^{\mathrm{T}}\omega) + \tilde{\bar{c}}_0\eta_y$$

for any $t > 0$, where $\tilde{\bar{\theta}}_0 = (\tilde{\bar{\theta}}_1^{\mathrm{T}}, \tilde{\bar{\theta}}_2^{\mathrm{T}}, \tilde{\bar{\theta}}_3)^{\mathrm{T}}$, $\varphi_0 = (W_m(s)\omega_1^{\mathrm{T}}, W_m(s)\omega_2^{\mathrm{T}}, W_m(s)y_p)^{\mathrm{T}}$. From Remark 2, we know $c_0(t) \geqslant \underline{c}$, $\forall t \geqslant 0$. Hence from (10) and the above equality

$$W_m(s)(\bar{\tilde{\theta}}^{\mathrm{T}}\omega)=\frac{c_0^*}{\bar{c}_0}\Big[-\varepsilon m^2-(\tilde{\theta}-\bar{\tilde{\theta}})^{\mathrm{T}}\varphi_p+W_c(s)((W_b(s)\omega^{\mathrm{T}})\dot{\bar{\tilde{\theta}}})-\frac{\bar{\tilde{c}}_0}{c_0^*}W_m(s)((\tilde{\theta}-\bar{\tilde{\theta}})^{\mathrm{T}}\omega)-\bar{c}_0\eta_y\Big] \tag{24}$$

From (24) and swapping lemma A2 [1,P775], it follows that

$$\tilde{\theta}^{\mathrm{T}}\omega=\bar{\theta}^{\mathrm{T}}\omega+(\tilde{\theta}-\bar{\tilde{\theta}})^{\mathrm{T}}\omega=F_1(s,a_0)(\dot{\bar{\tilde{\theta}}}^{\mathrm{T}}\omega+\bar{\tilde{\theta}}^{\mathrm{T}}\dot{\omega})+F(s,a_0)W_m^{-1}(s)\cdot \frac{c_0^*}{\bar{c}_0}\Big[-\varepsilon m^2-(\tilde{\theta}-\bar{\tilde{\theta}})^{\mathrm{T}}\varphi_p+W_c(s)((W_b(s)\omega^{\mathrm{T}})\dot{\bar{\tilde{\theta}}})-\frac{\bar{\tilde{c}}_0}{c_0^*}W_m(s)((\tilde{\theta}-\bar{\tilde{\theta}})^{\mathrm{T}}\omega)-\bar{c}_0\eta_y\Big]+(\tilde{\theta}-\bar{\tilde{\theta}})^{\mathrm{T}}\omega \tag{25}$$

where $F(s,a_0)=\frac{a_0^{n^*}}{(s+a_0)^{n^*}}$, $F_1(s,a_0)=\frac{1-F(s,a_0)}{s}$, and $\forall a_0>\delta_0$. By swapping lemma A2 [1,P775], $\|F_1(s,a_0)\|_{\infty\delta}\leqslant\frac{c}{a_0}$, where c is a constant independent of a_0. Since $a_0>\delta_0$, there exists a constant c independent of a_0 such that

$$\|F_1(s,a_0)W_m^{-1}(s)\|_{\infty\delta}\leqslant ca_0^{n^*} \tag{26}$$

By (21),(22),(25),(26), Lemma 2 and Lemma 3. 3. 2 in [1], we get

$$\|\tilde{\theta}^{\mathrm{T}}\omega\|\leqslant\frac{c}{a_0}(\|\dot{\bar{\tilde{\theta}}}m_f\|+m_f)+ca_0^{n^*}(\|\varepsilon mm_f\|+\|(\bar{\tilde{\theta}}-\tilde{\theta})m_f\|+\|\dot{\bar{\tilde{\theta}}}m_f\|+\mu\Delta_\infty m_f+d_0) \tag{27}$$

Step3. Use the B-G lemma to establish boundedness of m_f. Substituting (27) into (17), we have

$$m_f^2\leqslant c+\frac{c}{a_0^2}(\|\dot{\bar{\tilde{\theta}}}m_f\|^2+m_f^2)+ca_0^{2n^*}(\|\varepsilon mm_f\|^2+\|(\bar{\tilde{\theta}}-\tilde{\theta})m_f\|^2+\|\dot{\bar{\tilde{\theta}}}m_f\|^2+\mu^2\Delta_\infty^2m_f^2+d_0^2)+c\mu^2\Delta_\infty^2m_f^2+cd_0^2\leqslant c+c\Big(\frac{1}{a_0^2}+a_0^{2n^*}\mu^2\Delta_\infty^2\Big)m_f^2+c\|gm_f\|^2 \tag{28}$$

where the constant c of the second term in the last inequality is independent of a_0, and

$$g^2=\frac{1}{a_0^2}|\dot{\bar{\tilde{\theta}}}|^2+a_0^{2n^*}(|\varepsilon m|^2+|\dot{\bar{\tilde{\theta}}}|^2+|\bar{\tilde{\theta}}-\tilde{\theta}|^2) \tag{29}$$

Note in (28), that the coefficient c of the second term in the last inequality is independent of a_0, and choose a_0 appropriately large such that $\frac{c}{a_0^2}<1$. Then there exists $\mu_1^*>0$, for $\mu\in[0,\mu_1^*)$,

$$m_f^2\leqslant c+c\|gm_f\|^2 \tag{30}$$

By (10), assumptions S_1, S_2 and Lemma 3. 3. 2 in [1] we have

$$|\eta|\leqslant\left\|\frac{(\Lambda(s)-\theta_1^{*T}a(s))}{\Lambda(s)c_0^*}\mu W_m(s)\Delta_m(s)\right\|_{2\delta_0}\|u_p\|_{2\delta_0}+cd_0\leqslant\mu\Delta m+cd_0 \tag{31}$$

where $\Delta=\left\|\frac{(\Lambda(s)-\theta_1^{*T}a(s))}{\Lambda(s)c_0^*}\mu W_m(s)\Delta_m(s)\right\|_{2\delta_0}$. According to (22),(29),(31),and Lemma 1,

$$\int_t^{t+T}g^2(\tau)\mathrm{d}\tau\leqslant c\Big(\frac{1}{a_0^2}+a_0^{2n^*}\Big)\int_t^{t+T}\Big(\mu^2\Delta^2+\frac{d_0^2}{m^2(\tau)}\Big)\mathrm{d}\tau+c \tag{32}$$

where c is independent of a_0. Similar to the proof of P755 in [1], we have $\frac{cd_0^2}{m^2(\tau)}\leqslant\frac{\delta}{2}$. From (30),(32), B-G lemma, there exists $\mu_2^*=\sqrt{\frac{a_0^2\delta}{2c(1+a_0^{2(n^*+1)})\Delta^2}}$ such that for $\mu\in[0,\mu_2^*)$,

$$m_f^2\leqslant ce^{-\delta t}e^{c\int_0^t g^2(\tau)\mathrm{d}\tau}+c\delta\int_0^t e^{-\delta(t-s)}e^{c\int_s^t g^2(\tau)\mathrm{d}\tau}\mathrm{d}s\leqslant$$

$$ce^{\frac{-\delta t}{2}+c\left(\frac{1}{a_0^2}+a_0^{2n^*}\right)\mu^2\Delta^2 t}+c\delta\int_0^t e^{\frac{-\delta(t-s)}{2}+c\left(\frac{1}{a_0^2}+a_0^{2n^*}\right)\mu^2\Delta^2(t-s)}\leqslant c.$$

Choosing $\mu^*=\min(\mu_1^*,\mu_2^*)$, for $\mu\in[0,\mu^*)$, we conclude with $m_f^2<\infty$.

Step4. Consider the tracking error. From (27) and $m_f<\infty$, it follows that

$$\|\tilde{\theta}^{\mathrm{T}}\omega\|^2\leqslant c\left(\frac{1}{a_0^2}+a_0^{2n^*}\mu^2\Delta_\infty^2\right)m_f^2+c\|gm_f\|^2+cd_0^2\leqslant$$
$$c\left(\frac{1}{a_0^2}+a_0^{2n^*}\mu^2\Delta_\infty^2\right)+c\|g\|^2+cd_0^2 \tag{33}$$

where g is defined by (29). By (7), (14), (32), (33) and $m_f<\infty$, it is easy to establish $e_1\in S(\mu^2(\Delta^2+\Delta_\infty^2)+d_0^2+1/a_0^2)$. □

4 Conclusion

Direct model reference adaptive control with hybrid adaptive law is studied in this paper. Compared with control schemes in [1], the scheme has the following advantages: (1) the smaller computational effort; (2) the better robustness properties. The key point studied in this paper is how to design direct model reference adaptive controller with hybrid adaptive law, and rigorous proof from theory is given.

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混合的直接型模型参考自适应控制

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摘　要　本文研究了具有混合自适应律的直接型模型参考自适应控制. 对这种混合的 MRAC 方案，我们严格地证明了闭环系统的所有信号都有界，同时得到了跟踪误差满足 $e_1\in S(\mu^2(\Delta^2+\Delta_\infty^2)+d_0^2+1/a_0^2)$. 同文献[1]的控制方案相比，这种具有混合自适应律的直接型模型参考自适应控制具有如下优点：1) 实现过程中计算量大大减小；2) 具有更好的鲁棒性.

关键词　混合的模型参考自适应控制，直接型，规范化

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